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Multi Discrete Laguerre Wavelets Transforms with The Mathematical aspects

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ABSTRACT

The Linear approximation, means the order of smoothness of the void function. As for non-linear approximation, when compared to linear approximation, a significant improvement in the arrangement of approximation can be observed.
In this paper the theorems of linear approximation and non-linear approximation are demonstrated, Moreover, some hypotheses were orthogonal theorem is proven
many important theorems will be proven and proven in formation Multi Discrete Laguerre Wavelets Transform (MDLWT) so that they take their role in signal or image processing, this makes them possess properties like those they have those used for the same purpose and with high efficiency.
In second part in this paper, the new wavelet MDLWT was used after extracting the filter high pass filter and low pass filter by extracting the decision trees for the transactions where the new wavelet was used in image processing and mean square error (MSE), Peak signal-to-noise ratio (PSNR), Bits per pixel ratio (BPP) and Compression ratio (CR) were calculated, and the applications were done on 4 samples which good results have been found to prove the efficiency of the new wavelet

MSC :

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1 . Introduction

It is worth mentioning that the analysis of data from the signals or images is done by wavelets in the decisions as know that wavelet algorithms that deal with signals are working in the same way as the human eye or camera and capture signals cell phone and even digital color images used in medicine are all processed By waveforms as approximation of data and intermittent signals or images with too many edges, [1], [2].

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The basis for wavelet transformation is the requirements for analyzing multiple solutions.

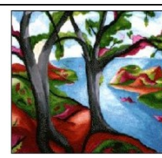
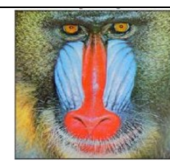
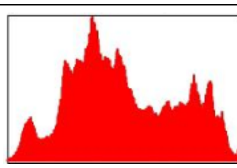
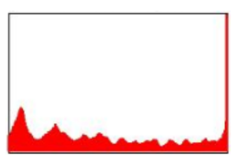
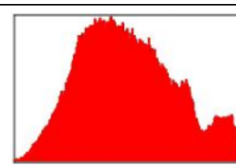
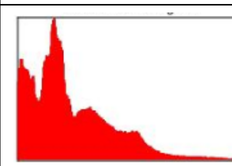
This is why found a large group of wavelet families ready to do various applications, including image compression, which depends on the image content.

The characteristic of a discrete wavelet transform has the efficiency of image coding due to its ability to activate data wavelet analysis is a powerful mathematical tool [3],[4], so it has been widely used in image digital processing, [5] .

The approximation by wavelet was used to solve many problems in the fields of mathematics, physics, and engineering, [6-10]Many scholars have an interest in this topic through Debnath ,Meyer ,Morlet, [11]

In this work constructed the new wavelet depended of the mother wavelet with the tow scalars c, d are parameters for translating and dilating in section 3 transform is constructed Multi Discrete Laguerre Wavelets Transform (MDLWT)**,** with proved the orthogonal of the MDLWT in section 4 finally in section 5 proved the approximation Multi Discrete Laguerre Wavelets Transform (MDLWT)**,** in linear and non linear approximation the theorems illustrated The Linear approximation, means the order of smoothness of the void function. As for non-linear approximation, when compared to linear approximation, a significant improvement in the arrangement of approximation can be observed. Image processing using MDLWT were done on 4 samples are displayed in table1 used high pass filter and low pass filter for analyses image and processing it during calculate important criteria that indicate the efficiency of the proposed wavelets.

Table1: The original samples with its normalized histogram

	Image 1	Image 2	Image 3	Image 4
Original image				
Original image normalized histogram				

2. Wavelets Transformation

The wavelets are created from expansion and contraction through the two parameters a and b which are represented by the parent function from the continuous wavelets.

$$\chi_{c,d}(t) = |c|^{-\frac{1}{2}} \chi\left(\frac{x-d}{c}\right) \qquad c,d \in R \qquad c \neq 0 \tag{1}$$

where $\chi(t) = [\chi_0(t), \chi_1(t), \ldots, \chi_{M-1}]^T$

The basis for the above function consists of the elements $\chi_0(t), \chi_1(t), \ldots, \chi_{M-1}(t)$ are orthogonal on the [0,1].

3. Multi Discrete Laguerre Wavelets Transform (MDLWT)

By transfer the parameters c, d to specific values, the wavelets will turn into discrete wavelets transform as, Laguerre wavelet $L_{u,v}(t) = L_{t,u,v,k}$ have four arguments; $k = 1,2,\ldots,$ $u = 1,2,\ldots,2^{k-1}$, v is order for Laguerre polynomials and t is normalized time.

The dilation by parameter $c = 2^{-(k+1)}$ and translation by parameter $d = 2^{-(k+1)}(2u-1)$ and use transform x , $x = 2^{-(k+1)}(2^k t)$, substituted these transfers in (1) with replace $\chi_{c,d}$ by $L_{u,v}$ will be get

$$L_{u,v}(t)=2^{-(k+1)}\widetilde{L}_{u,v}\left(\frac{2^{-(k+1)}\left(2^{k}t\right)-2^{-(k+1)}(2u-1)}{2^{-(k+1)}}\right) \quad (2)$$

$$L_{u,v}(t)=\begin{cases}2^{k+1/2}\widetilde{L}_{v}\left(2^{k}t-2u+1\right) & \frac{u-1}{2^{k-1}}\le t\le\frac{u}{2^{k-1}}\\ 0 & otherwise\end{cases} \quad (3)$$

where $\widetilde{L}_v=\frac{1}{v!}L_v$ v is the order of Laguerre polynomials

In used k=2 gotten from (3) many functions but if used k=3 will be get multi numbers functions from (3), therefore, the wavelet in this work was called the multi wavelet

$$L_{u,v}(t)=\begin{cases}\frac{4}{v!}L_{v}\left(8t-2u+1\right) & \frac{u-1}{2^{k-1}}\le t\le\frac{u}{2^{k-1}}\\ 0 & otherwise\end{cases} \quad (4)$$

The atoms obtained in equation (4) from Laguerre polynomials with weight function e^{-x^2} on the interval $[0,\infty]$ are as follows Initial values

$$L_0(x)=1$$
$$L_1(x)=1-x$$
$$L_{v+1}(x)=(2v+1-x)L_v(x)-v^2L_{v-1}(x) \quad (5)$$

$v=0,1,2,\ldots.$

The function approximate with dilation and translation the weight function $e^{-(8t-2u+1)^2}$ By expansion of the period $[0,1)$ it is obtained $f(t)$,

$$f(t)=\sum_{u=1}^{\infty}\sum_{v=0}^{\infty}C_{u,v}L_{u,u}(t) \quad (6)$$

where

$$C_{u,v}=\left\langle f(t),L_{u,v}(t)\right\rangle e^{-(8t-2u+1)^2}=\int_0^1 e^{-(8t-2u+1)^2}L_{u,v}(t)f(t)dt \quad (7)$$

$\langle .,.\rangle$ denoted the inner product with weight function $w_u(8t-2u-1)$ on the Hilbert Space $[1,0)$, if finite equation(6) obtained

$$f(t)=\sum_{u=1}^{2^k-1}\sum_{v=0}^{M-1}C_{u,v}L_{u,u}(t) \quad (8)$$

From the above equations will be get two vectors $A_{u,v}$, $L_{u,v}$ $k=3$ with dimension $2^{k-1}M\times 1$

$$C=\left[C_{1,0},C_{1,1},\ldots C_{1,M-1},C_{2,0},\ldots,C_{2,(M-1)},C_{3,0},C_{3,1},\ldots,C_{3,M-1},\ldots.C_{2^{k-1},0},\ldots,C_{2^{k-1},M-1}\right]^T$$
$$L=\left[L_{1,0},L_{1,1},\ldots L_{1,M-1},L_{2,0},\ldots,L_{2,(M-1)},L_{3,0},L_{3,1},\ldots,L_{3,M-1},\ldots.L_{2^{k-1},0},\ldots,L_{2^{k-1},M-1}\right]^T \quad (9)$$

4. Orthogonality for MDLWT

In this section the orthogonality of the wavelet constructed in section 2 will be demonstrated.

$L_u(x)$ has orthogonality with $w(t)=e^{-t}$ is weight function on $[0,\infty)$, the series of laguerre wavelets are the orthogonal with respect weight function $w_u(t)=e^{\left[-\left(2^k t-2u+1\right)\right]}$ the functions taking values wavelets belong to [0,1), then $L_{u,v}(t)$ define an orthogonal basis in $L^2(R)$.

Theorem 4.1

Assume f(t) in $L^2[0,1]$ the orthogonal of MDLWT

$$\int_0^\infty L_{u,v}(t)L_{u',v'}(t)dt=\begin{cases}0 & if \quad v\neq v' \\ 2 & if \quad v=v'\end{cases} \tag{10}$$

Proof

The orthogonal of Laguerre polynomials with weight function e^{-x}

$$\int_0^\infty e^{-x}L_m(x)L_n(x)dx=\begin{cases}0 & if \quad n\neq m \\ (n!)^2 & if \quad n=m\end{cases} \tag{11}$$

The MDLWT

$$L_{u,v}(t)=\begin{cases}2^{k+1/2}\,\widetilde{L}_v\left(2^k t-2u+1\right) & \frac{u-1}{2^{k-1}}\leq t\leq\frac{u}{2^{k-1}} \\ 0 & otherwise\end{cases} \tag{12}$$

where $\widetilde{L}_v=\frac{1}{v!}L_v$ $\quad v$ is the order of Laguerre polynomials v is the order of Laguerre polynomials with weight function $\exp\left(-\left[2^k t-2u+1\right]\right)$, function approximate in [0,1) obtained f(t)

$$f(t)=\sum_{u=1}^{\infty}\sum_{v=0}^{\infty}C_{u,v}L_{u,v}(t) \tag{13}$$

$$C_{u,v}=\int_0^\infty f(t)L_{u,v}(t)dt \tag{14}$$

Finite equation(3) and (4)

$$C_{u,v}=\int_{\frac{u-1}{2^{k-1}}}^{\frac{u}{2^{k-1}}} f(t)L_{u,v}(t)dt$$

Let

$$f(t)=L_{u',v'}(t)$$

$$C_{u,v}=\left(\frac{1}{v!}\right)^2\int_0^1\left(2^{\frac{k+1}{2}}\right)^2 L_{u,v}\left(2^k t-2u+1\right)L_{u',v'}\left(2^k t-2u+1\right)e^{-\left(2^k t-2u+1\right)}dt$$

Supposed

$$N=\frac{1}{(v!)^2}2^{k+1},\quad x=2^k t-2u+1,\quad t=\frac{x+2u-1}{2^k},\quad dt=\frac{1}{2^k}dx$$

$$C_{uv}=\frac{N}{2^k}\int_0^1 e^{-\frac{(x+2u-1)}{2^k}}L_{u,v}\left(\frac{(x+2u-1)}{2^k}\right)L_{u',v'}\left(\frac{(x+2u-1)}{2^k}\right)dx$$

From equation(1) if $v=u,\ u'=v'$ then

$$C_{u,v}=\frac{N}{2^k}(v!)^2 \ \ if\ \ u=u'\ C_{u,v}=0\ \ if\ \ u\neq u'$$

Then $C_{u,v}=\frac{1}{2^k}\frac{1}{(v!)^2}2^{k+1}(v!)^2=2$

Theorem is hold

5. Multi Discrete Laguerre Wavelets Transform (MDLWT) with Mathematical aspects

Definition The interval $I_{u,v}\ \forall u,v\in Z$ defined by $I_{u,v}=\left[\frac{v}{2^u},\frac{v+1}{2^u}\right]$ is dyadic interval all interval in group is called dyadic sub interval

5.1 Scaling function in Multi Discrete Laguerre Wavelets Transform (MDLWT)

$$\{\omega_{u,v}(t)\}_{u,v\in Z}=2^{-\frac{u}{2}}\omega\left(2^{-\frac{u}{2}}t-v\right)\forall u,v\in Z$$

$$\omega(t)=\begin{cases}1\ if\ \ t\in[0,1)\\ 0\ otherwise\end{cases}$$

The wavelet function in MDLWT $\{L_{u,v}(t)\}_{u,v\in Z}=2^{-\frac{u}{2}}L\left(2^{-\frac{u}{2}}t-v\right)\forall u,v\in Z$

From above equation it is the MDLWT system if $f(t)\subseteq L^2[0,1)$

Then

$$f(t)=\sum_{v=0}^{2^u-1}\langle f(t),\omega_{u,v}(t)\rangle+\sum_{u=U}^{\infty}\sum_{v=0}^{2^u-1}\langle f(t),L_{u,v}(t)\rangle L_{u,v}(t)$$

The coefficients of wavelets are approximate coefficients denoted by $a_{u,v}$ and the details coefficients denoted by $d_{u,v}$ then

$$f(t)=\sum_{v=0}^{2^u-1}a_{u,v}\omega_{u,v}(t)+\sum_{u=U}^{\infty}\sum_{v=0}^{2^u-1}d_{u,v}L$$

5.2 Multi Resolution Analysis (MRA) of Multi Discrete Laguerre Wavelets Transform (MDLWT)

The analysis of DMLWT is a system for determent of basis coefficients in $L^2(R)$

And the MRA in wavelet space

$$f\in V_u=\left\{f(t)\middle|f(t)=\frac{1}{2^{u/2}}h(2^{-u}t),\ h(t)\in V_0\right\}$$

Where $f(t)=\sum_{u\in z}\langle f,\omega(\bullet-v)\rangle\omega(t-v)$

Then a multi resolution analysis of MDLWT on R is a sequence of subspaces $\{V_u\}$, $u\in Z$ of functions L^2 on R, First and foremost, It has the following important characteristics to clarify MRA that it has MDLWT to be ready to work in important areas of image processing such as image analysis

$\forall u,v\in Z,V_u\subseteq V_{u+1}\ \ f(t)\in R$, then $f(t)\in span\{V_u\}\,u\in Z$, with $\epsilon>0$, there is an $u\in Z$

and a function such that $\|f - g\|_2 < \epsilon$. $\bigcap_{u\in z} V_u = \phi$. $f(t) \in V_0 \leftrightarrow 2^{-u/2} f(2^{-u}t) \in V_u$.
The scalar function $\omega(t) \subseteq L^2$ on R→ $\omega(t-u)$, $V_0 = span\{\omega(t-u)\}$.

6. Approximation by MDLWT in different Spaces

In this section it is established that the waveforms belong to the approximation area, which qualifies the smoothness of the wave in many uses in the field of image processing by finding approximate and details coefficients, which leads to extracting a suitable filter for use in analyzing the image to approximate and details coefficients then compressing the image and removing noise from it

6.1. Approximation in $L^2(R)$ Space

In this section some theorems proved MDLWT in $L^2(R)$ is a square integrable functions over R belong to different approximate spaces, $L_P(R)$, $Lip_M(\delta, P)$, and Besov Space $B_r^{\delta}(L_P(R))F^s(R)$

Theorem (6.1)

$f \in L_P(R)$ the partial sum of MDLWT of f is

$$Q = \sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f, L_{u,v}\rangle L_{u,v}(t) \ \ u \in U \to E_{app} = \eta\left(2^{\frac{-u}{2}}\right)$$

Proof:

E_{app} - Which determine the approximate error in $L_P(R)$

$$E_{aPP} = \|f - Q\|_{L_P} = \left\| f - \sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f, L_{u,v}\rangle L_{u,v}(t)\right\|_{L_P}$$

$$= \left\|\sum_{u=I}^{\infty}\sum_{v=0}^{U-1}\langle f, L_{u,v}\rangle L_{u,v}(t)\right\|_{L_P} = \left(\sum_{u=I}^{\infty}\sum_{v=0}^{U-1}\left|\langle f, L_{u,v}\rangle\right|^P\right)^{\frac{1}{P}}$$

$$= \left(\sum_{u=I}^{\infty} 2^{\frac{-IP}{2}}\right)^{\frac{1}{P}} = 2^{\frac{-I}{2}} = \eta 2^{\frac{-I}{2}} \ \bullet$$

Theorem (6.2)

If $f \in LipU(\sigma, P)$, $\sigma \in (0,1]$, $P \in (1,\infty]$, $U \geq 0$, $Q(t) = \sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f, L_{u,v}\rangle L_{u,v}(t)$ is the MDLWT of f for some $I \in U$, also $E_{aPP} = \eta\left(2^{-I\sigma}\right)$

Proof:

See [11],

Theorem (6.3)

Let $f \in F^s$ and $Q(t) = \sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f, L_{u,v}\rangle L_{u,v}(t)$ is the finite MDLWT of f then $\exists : I \in U$ then $E_{aPP} = \eta\left(2^{\frac{-srU}{2}}\right)$

where $U = 2^I$

Proof:

The approximate error of in $F^s L_2(R)$ will be E_{app}

$$E_{aPP}=\|f-Q\|_{L_2}=\left\|f-\sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f,L_{u,v}\rangle L_{u,v}(t)\right\|_{L_P}\leq=\left(\sum_{u=I}^{\infty}\sum_{v=0}^{U-1}\left|\langle f,L_{u,v}\rangle\right|^2\right)^{\frac{1}{2}}\leq\left(\sum_{u=I}^{\infty}\sum_{v=0}^{U-1}\frac{2^{rv}}{2^{rU}}\left|\langle f,L_{u,v}\rangle\right|^2\right)^{\frac{1}{2}}$$

$$\left(2^{-srU}\sum_{u=I}^{\infty}\sum_{v=0}^{U-1}2^{sm}\left|\langle f,L_{u,v}\rangle\right|^2\right)^{\frac{1}{2}}\Rightarrow\|f\|_{P^r}(L_2(R))\cong\left(\left(U\sum_{u=I}^{\infty}\sum_{v=0}^{N-1}2^{rv}\left|\langle f,L_{u,v}\rangle\right|^2\right)^{\frac{1}{2}}\right)$$

Therefore $E_{app}\leq 2^{\frac{-rU}{2}}\|f\|_{P^s}(L_2(R))=\eta 2^{\frac{-rU}{2}}$.

The following theorem with respect of Approximation in Besov Space $B_r^\sigma(L_P(R))$

Theorem (6.4)

If $f\in B_s^\sigma(L_P(R))$ $\sigma>0, 0<s\leq\infty$ and $Q(t)=\sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f,L_{u,v}\rangle L_{u,v}(t)$

MDLWT limited of f $\exists\, I\in U$, $\Rightarrow$ $E_{aPP}=\eta\left(2^{\frac{-\sigma U}{2}}\right)$ where $U=2^I$.

Proof:

same above theorems, can be used properties, the Besov Space its properties

$$\|f\|_{B_s^\sigma(L_P(R))}\cong\left(\left(\sum_{u=I}^{\infty}\sum_{v=0}^{U-1}|<f,L_{u,v}>|^s\right)^{\frac{1}{s}}\right)$$

Therefore

$$E_{app}\leq 2^{\frac{-\sigma U}{s}}\|f\|_{B_s^\sigma(L_P(R))}=\eta\left(2^{\frac{-\sigma U}{2}}\right)$$

Where

$$\frac{1}{s}=\frac{\sigma}{2}+\frac{1}{2}\;\blacksquare$$

Theorem (6.5): Non linear approximation in $L_P(R)$

$f\in L_P(R)$ then $Q(t)=\sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f,L_{u,v}\rangle L_{u,v}(t)\forall u,v\in Z\Rightarrow E_{app}=\eta\left(2^{\frac{-u}{2}}\right)$

Proof:

$$E_{aPp} = \|f - Q\|_{L_2} = \left\| f - \sum_{u=0}^{I-1}\sum_{v=0}^{U-1}\langle f, L_{u,v}\rangle L_{u,v}(t) \right\|_{L_P} =$$

$$\left\| \sum_{u=I+1}^{\infty}\sum_{v=0}^{U-1}\langle f, L_{u,v}\rangle L_{u,v}(t) \right\|_{L_P} \left(\sum_{u=I+1}^{\infty}\sum_{v=0}^{U-1}\left|\langle f, L_{u,v}\rangle\right|^{P} \right)^{\frac{1}{P}} = \left(\sum_{u=I+1}^{\infty} 2^{\frac{-IP}{2}} \right)^{\frac{1}{P}} = 2^{\frac{-I}{2}} = \eta 2^{\frac{-I}{2}} \bullet$$

7. Tree decision Multi Discrete Laguerre Wavelets Transform (MDLWT)

In this section used filters MDLWT in Image Processing after find it by decision Tree of Signal from MDLWT, the signal S is found its root started analysis signal from the approximation $A_0 = S$ in level n=0 second step will get the detail in level 1

$$D_1 = A_0 - A_1 = s - A_1$$
$$S = D_1 + A_1$$

And in level 2

$$D_2 = A_1 - A_2$$
$$= S - D_1 - A_2$$
$$S = D_2 + D_1 + A_2$$

$S = P_{u,v}$ and $\eta_{u,v} = A_{u,v}$ from section three used equations (6-9) $S = \sum_{u=1}^{2^{k-1}}\sum_{v=0}^{M-1}\eta_{u,v}L_{u,v}(t)$

u=0,1,2,3,… v=1,2,….,2^u-1

Table.1:Shows the MDLWT u=0,1,2 $t \in [0,1)$

$$\omega_{u,v}(t) = \begin{cases} \frac{1}{4} & 0 \le t < 1 \\ 0 & otherwise \end{cases}$$

	u=0	u=1	u=2
$L_{0,0}$	$\frac{1}{4}$	0	.
$L_{1,0}$	0	$\frac{1}{4}$	.
$L_{1,1}$	0	$\frac{1}{4}(1-(8t-1))$	0
$L_{2,0}$	0	0	$\frac{1}{4}$
$L_{2,1}$	0	0	$\frac{1}{4}(1-(8t-3))$
$L_{2,2}$	0	0	$\frac{1}{4}((8t-3)^2-4(8t-3)+2)$
$L_{2,3}$	0	0	$\frac{1}{4}(-(8t-3)^3+9(4t-3)^2-18(8t-3)+6)$

7. Two-Dimensional Multi Discrete Laguerre Wavelet Transform Analysis

In this section, the filters of Multi Discrete Laguerre Wavelet Transform MDLWT

high pass filter and low pass filter are obtained from Approximation coefficients and Details coefficients figure1 illustrated the decomposition operation

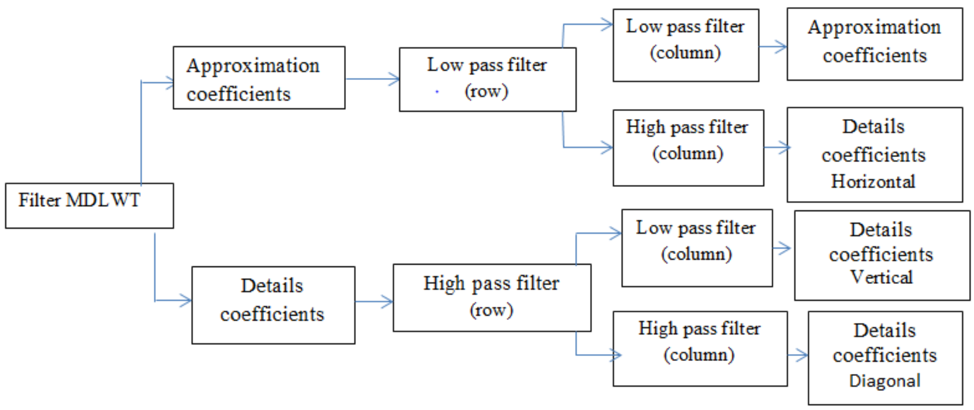
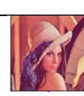
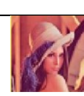
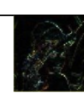
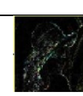
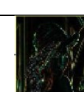
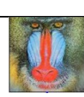
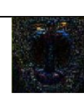
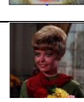
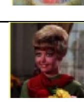
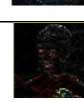
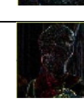


Figure1: The decomposition of MDLWT filters

The decomposition of MDLWT filters are applied of 4 samples table2 shows the effected MDLWT on analyses image

Table2: the effected MDLWT on analyses image

Image	original	Approximation	Horizontal Details	Diagonal Details	Vertical Details
Image1					
Image2					
Image3					
Image4					

The filters of MDLWT after add in MATLAB program used it in analyses image (decomposition) in compressed image with got the results of PSNR depended of MSE and PBB of the CR in level 8 table2 illustrated the effected of MDLWT in image processing in table3 shows the results of compression images in size 256×256 with pixels in x, y dimension

$$MSE = \frac{1}{MN}\sum_{x=0}^{M=1}\sum_{y=0}^{N-1}\left(I_{x,y} - O_{x,y}\right)^2$$

$$\text{PSNR}=10*\log_{10}\left(\frac{255^2}{MSE}\right)$$

$$CR = \frac{compression\ image}{uncompression\ image}$$

$$B.P.P = \frac{number\ of\ each\ bit}{CR}$$

Table3: the results MDLWT on analyses image

	MSE	PSNR	CR	PBB
Image1	0.106	57.88	77.79%	18.668000
Image2	0.03276	62.28	112.11%	26.9061
Image3	0.03723	62.42	111.16%	26.6781
Image4	0.1094	57.74	70.11%	16.8271

8. Conclusion

In this work, the orthogonality theory is given to give the function or wavelet constructed from Laguerre polynomials and obtained the new wavelet Multi Discrete Laguerre Wavelets Transform (MDLWT)

The Linear approximation, means the order of smoothness of the void function. As for non-linear approximation, when compared to linear approximation, a significant improvement in the arrangement of approximation can be observed.

The second part of the work has been demonstrated by a number of theorems in linear and non-linear approximation in order to prove that the new wave is smooth in its use in many applications such as solutions to many numerical problems such as Variational problems, integro differential and integral equations.

Moreover to this it enhances the possibility and smoothness of its use in many areas of image processing such as pressure and de noise from the image to the end using a process

new wavelet MDLWT was used after extracting the filter high pass filter and low pass filter by extracting the decision trees for the transactions where the new wavelet was used in image processing and mean square error (MSE), Peak signal-to-noise ratio (PSNR), Bits per pixel ratio (BPP) and Compression ratio (CR) were calculated, and the applications were done on 4 samples which good results have been found to prove the efficiency of the new wavelet

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