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On Mixed Fuzzy Topological Ring

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ABSTRACT

The theory of fuzzy topological ring has wide scope of applicability than order topological ring theory. The reason is fuzzy can provide better result. Therefore, fuzzy topological ring has been found in Robotics, computer, artificial intelligent, etc. In this paper, we induce mixed fuzzy topological ring space and fuzzy neighborhoods system of mixed fuzzy topological ring space. Also, we study fuzzy continuity of mixed fuzzy topological ring.

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1 . Introduction

In 1965 [9], Zadeh L. A. gave the definition of fuzziness. After three years C. Chang [2] gave the notion of fuzzy topology. In 1990[1], Ahsanullah and Ganguli, depended on the convergent in fuzzy topological space in the sense of Lowen[7, 8] to introduce the concept of fuzzy nbhd rings which gives the necessary and sufficient condition for a prefilter basis to be fuzzy nbhd prefilter of 0 in fuzzy topological ring. Also they are study the notions of right and left bounded fuzzy set and precompact fuzzy set fuzzy nbhd rings.

In 2009, Deb Ray, A. and Chettri, P [3] introduced fuzzy topology on a ring. Also in [4] they introduced fuzzy continuous function and studied left fuzzy topological ring

Analogy to bi- fuzzy topological space we define bi- fuzzy topological ring. We apply the results on fuzzy nbhd systems in a fuzzy topological ring developed so far, to construct a mixed fuzzy topological ring. Also we study the relationship between fuzzy continuities of fuzzy homo. with respect to different fuzzy topologies

For rich the paper, some basic concept of fuzzy set , fuzzy topology and fuzzy topological ring are given below. The symbol I will denote to the closed interval [0,1]. Let R be a non-empty set:

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Definition 1.1[9]

A fuzzy set in R is a map $\partial: R \to I$ and, that is, belonging to I^R(the set of all fuzzy set of R) . Let $E \in I^R$, for every $r \in R$, we expressed by $E(r)$ of the degree of membership of r in R . If $E(r)$ be an element of $\{0, 1\}$, then E is said a crisp set.

Definition 1.2[2]

A class $\mu \in I^R$ of fuzzy set is called a fuzzy topology for R if the following are satisfied

1) $\emptyset, R \in \mu$
2) $\forall E , H \in \mu$ *implies* $E \wedge H \in \mu$
3) $\forall (E_j)_{j\in J} \in \mu$ *implies* $\vee_{j\in J} E_j \in \mu$

(R, μ) is called fuzzy topological space. if $A \in \mu$ Then A is fuzzy open and A^c (complement of A)is a fuzzy closed set.

Definition 1.3 [1, 3 and 4]

A pair (R , μ) , where R a ring and μ a fuzzy topology on R, is called fuzzy topological ring if the following functions are fuzzy continuous:

1) $R \times R \to R , (r , k) \to r + k.$
2) $R \to R , \ r \to -r$
3) $R \times R \to R , (r , k) \to r. k$

Definition 1.4 [4]

A family B of fuzzy nbhds of r_α, for $0 < \alpha \leq 1$, is called a fund. system of fuzzy nbhds of r_α iff for any fuzzy nbhd V of r_α, there is $U \in B$ such that $r_\alpha \leq U \leq V$

Definition 1.5[4]

Let R be a ring and μ a FZT on R. Let U and V are fuzzy sets in R. We define $U + V$, $-V$ and $U. V$ as follows

$(U + V)(k) = \sup_{k=k_1+k_2} min\{U(k_1), V(k_2)\}$
$-V(k) = V(-k)$
$(U. V)(k) = \sup_{k=k_1+k_2} min\{U(k_1), V(k_2)\}$

Theorem 1.6[4]

If R is a fuzzy topological ring then there is a fundamental system of fuzzy nbhds B of 0 $(0 < \alpha \leq 1)$, such that the conditions:
(i) $\forall U \in B$, then $-U \in B$
(ii) $\forall U \in B$, then U is symmetric
(iii) $\forall U, V \in B$, then $U \wedge V \in B$
(iv) $\forall U \in B$, there is $V \in B$ such that $V + V \leq U$
(v) $\forall U \in B$, there is $V \in B$ such that $V. V \leq U$
(vi) $\forall r \in R, \forall U \in B$, there is $V \in B$ such that a $r. V \leq U$ and $V. r \leq U$.

Definition 1.7[7]

(R, μ) is fully stratified fuzzy topology on R if the fuzzy topology μ on R contain all constant fuzzy set

Theorem 1.8 [5]

Let (R, μ) and (R, ρ) be two fuzzy topological spaces and let $\mu(\rho) = \{E \in I^R: \exists\, U \in \rho\ s.t\ cl_\mu(U) \leq E\}$. Then $\mu(\rho)$ is a mixed fuzzy topology on R

2. bi-fuzzy topological rings and mixed fuzzy topological rings

We study mixed fuzzy topological ring and fuzzy nbhds of mixed fuzzy topological ring

Definition 2.1{5}

Let R be any ring equipped with two fuzzy topological ring space μ and ρ. Then the triplet (R, μ, ρ) is defined as a bi- fuzzy topological ring space.

Example 2.2

Let R be any ring with the indiscrete fuzzy topology I and the discrete fuzzy topology D. Then, (R, I, D) is a bi- fuzzy topological ring

Theorem 2.3

Let (R, μ) and (R, ρ) be two fuzzy topological rings such that $\rho \leq \mu$. Let β_1, β_2 be a fundamental system of fuzzy nbhds of the identity element $0 \in R$ in the fuzzy topological spaces μ, ρ respectively. Then $\beta_1(\beta_2) = \{U \in I^R : \exists V \in \rho\ s.t\ cl_\mu(V) \leq U\}$ is a fundamental system of fuzzy nbhds of 0

Proof

We want to obtain that the conditions of Theorem 1.6 are satisfied by the collection $\mu(\rho)$.

(***i***) For each $\alpha \in I\backslash\{0\}$, the constant fuzzy set K_α and $1 - K_\alpha$ are belonging to both μ and ρ, therefore $K_\alpha = cl_\mu(K_\alpha)$ for each $\alpha \in I\backslash\{0\}$, we have, $K_\alpha \in \beta_1(\beta_2)$

(***ii***) Let $U \in \beta_1(\beta_2)$, then there exists $V \in \beta_2$ s.t $cl_\mu(V) \leq U$, by condition 2 theorem 1.6

$$-V \in \beta_2$$

Now

$cl_\mu(-V) = -cl_\mu(V) \leq -U$. Also $-U(0) = U(-0) = U(0) > 0$

Thus

$$-U \in \beta_1(\beta_2),$$

(***iii***) Let $U \in \beta_1(\beta_2)$, by above condition $-U \in \beta_2$. Let $U_1 = U \wedge (-U)$, Now

$$U_1(0) = \min\{U(0), (-U)(0)\} = \min\{U(0), U(-0)\}$$
$$= U(0) > 0$$
$$U_1(r) = \min\{U(r), (-U)(r)\} = \min\{U(r), U(-r)\}$$
$$= U(r)\text{ . Thus } U \text{ is symmetric}$$

(***iv***) Let $U_1, U_2 \in \beta_1(\beta_2)$ then there exists $V_1, V_2 \in \beta_2$ s.t $cl_\mu(V_1) \leq U_1$ and $cl_\mu(V_2) \leq U_2$. Since $V_1, V_2 \in \beta_2$ then by condition 4 theorem1.6, we have $V_1 \wedge V_2 \in \beta_2$.

Now

$$cl_\mu(V_1 \wedge V_2) \leq cl_\mu(V_1) \wedge cl_\mu(V_2) \leq U_1 \wedge U_2$$

Thus,

$$U_1 \wedge U_2 \in \beta_1(\beta_2).\text{ Also } (U_1 \wedge U_2)(0) = \min\{U_1(0), U_2(0)\} > 0$$

(***v***)

Let $U \in \beta_1(\beta_2)$. Then then there exists $V \in \beta_2$. By condition 5 theorem1.6, $V + V \in \beta_2$. Now,

$$cl_\mu(V) + cl_\mu(V) \subseteq U + U$$

Also $(U + U)(0) = supmin\{U(0), U(0)\} = U(0) > 0$

Thus, $U + U \in \beta_1(\beta_2)$

(***vi***)

By the same way of (v) we can prove that $U.U \in \beta_1(\beta_2)$

(***vii***)

Let $U \in \beta_1(\beta_2)$. To show that there exists $V \in \beta_2$ s.t $cl_\mu(V) \subseteq U$, implies $(V.r) \in \beta_2$. Now, $cl_\mu(V.r) = cl_\mu(V).r \leq U.r$,

Also

$$(U.r)(0) = U(0) > 0$$

Thus, $U.r \in \beta_1(\beta_2)$

Theorem 2.4

Let $V_\mu(V_\rho) = \{U \in I^R : \exists V \in \rho \; s.t \; cl_\mu(V) \leq U\}$ is a fundamental system of fuzzy nbhds of 0 satisfied the conditions of theorem 2.3, then there exist a unique fuzzy topology denoted by $\mu(\rho)$ such that $(R, \mu(\rho))$ is fuzzy topological ring.

Proof

(i)

By condition 1 of theorem 2.3 we have 0,1 and K_α are belonging to $\mu(\rho)$

(ii)

Let $U_1, U_2 \in \mu(\rho)$ with $U_1 > 0$ and $U_2 > 0$ then there is

$$V_1, V_2 \in \rho, \text{with } V_1 > 0 \text{ and } V_2 > 0 \text{ s.t } V_1 \leq U_1, V_2 \leq U_2$$

Since $V_1, V_2 \in \rho$ implies $V_1 \wedge V_2 \in \rho$

Also

$(V_1 \wedge V_2)(0) = \min\{V_1(0), V_2(0)\} > 0$

Now

$$cl_\mu(V_1 \wedge V_2) \leq cl_\mu(V_1) \wedge cl_\mu(V_2) \leq U_1 \wedge U_2$$

Thus $U_1 \wedge U_2 \in \mu(\rho)$

(iii)

Let J be an indexed set and for each $j \in J$ s.t $U_j \in \mu(\rho)$ and

$U_j(0) > 0$.

Then there is $V_j \in \rho$ with $V_j(0) > 0$ s.t $cl_\mu(V_j) \leq U_j$.

Also $cl_\mu(V_j)(0) \geq V_j(0) > 0$

Now $cl_\mu(V_j) \leq U_j < \vee\{U_j\}$

Thus $\vee U_j \in \mu(\rho)$

(iv)

We claim $g: (R, \mu(\rho)) \times (R, \mu(\rho)) \to (R, \mu(\rho))$, $g(r, k) = r + k$ is fuzzy cont.

Let U be a fuzzy nbhd of $r + k$ then there is $V \in \rho$ with $V(r + k) > 0$ s.t $cl_\mu(V) \leq U$.

$$g^{-1}(cl_\mu(V)(r, k) = cl_\mu(V)\big(g(r, k)\big) = cl_\mu(V)(r + k) > 0$$

Since (R, ρ) is fuzzy topological ring implies there is $V_1, V_2 \in \rho$ with $V_1(r) > 0$ and $V_2(k) > 0$ s.t $V_1 + V_2 \leq V$. And $cl_\mu(V_1 + V_2) \leq cl_\mu(V)$

$$g(\big(cl_\mu(V_1) \times cl_\mu(V_2)\big) = cl_\mu(V_1) + cl_\mu(V_2)$$

$$\leq cl_\mu(V_1 + V_2)$$

$$\leq cl_{\mu}(V) \leq U$$

Thus $g(r,k) = r + k$ is fuzzy continuous

(v) We claim$g: \big(R, \mu(\rho)\big) \times \big(R, \mu(\rho)\big) \to \big(R, \mu(\rho)\big),\ g(r,k) = r.k$ is fuzzy cont.

Let U be a fuzzy nbhd of $r.k$ then there is $V \in \rho$ with $V(r.k) > 0$ s.t $cl_{\mu}(V) \leq U$.

$$g^{-1}\big(cl_{\mu}(V)(r,k) = cl_{\mu}(V)\big(g(r,k)\big) = cl_{\mu}(V)(r.k) > 0$$

Since (R,ρ) is fuzzy topological ring implies there is $V_1, V_2 \in \rho$ with $V_1(r) > 0$ and $V_2(k) > 0$ s.t $V_1.V_2 \leq V$. And $cl_{\mu}(\,V_1.V_2) \leq cl_{\mu}(V)$

$$g(\big(cl_{\mu}(V_1) \times cl_{\mu}(V_2)\big) = cl_{\mu}(V_1).cl_{\mu}(V_2)$$

$$\leq cl_{\mu}(\,V_1.V_2)$$

$$\leq cl_{\mu}(V) \leq U$$

Thus $g(r,k) = r.k$ is fuzzy continuous.

(vi) We claim $\big(R, \mu(\rho)\big) \to \big(R, \mu(\rho)\big), g(r) = -r$ is fuzzy cont.

Let U be a fuzzy nbhd of $-r$ then there is $V \in \rho$ with $-V(r) > 0$ s.t $-cl_{\mu}(V) \leq U$. By condition 3 theorem 2.3 we have $cl_{\mu}(V) = -cl_{\mu}(V)$

$$g\left(cl_{\mu}(V)\right) = -cl_{\mu}(V) = cl_{\mu}(V) < U$$

Thus $g(r) = -r$ is fuzzy continuous.

Definition 2.5

Let (R, μ, ρ) be any bi- fuzzy topological ring. The fuzzy topology $\boldsymbol{\mu(\rho)}$determined on R by the collection $\{U \in I^R : \exists V \in \mu\ s.t\ cl_{\mu}(V) \leq U\}$ of all fuzzy open nbhds of 0 such that $(R, \boldsymbol{\mu(\rho)})$ is a fuzzy topological ring, is defined as a mixed fuzzy topology ring

Example 2.6

In the bi- fuzzy topological ring (R, μ, ρ)**,** let us put $\mu = I$, the indiscrete fuzzy topology on R and $\rho = D$, the discrete fuzzy topology on R, then in both cases, we have $\boldsymbol{\mu(\rho)} = \boldsymbol{\mu}$.

Example 2.7

In the bi- fuzzy topological ring (R, μ, ρ), let us put $\mu = D$, the discrete fuzzy topology and $\rho = I$, the indiscrete fuzzy topology, then in both cases, we have $\mu(\rho) = \rho$

3. Fuzzy continuity

This section deals with some results of fuzzy continuity of mixed fuzzy topological spaces.

Theorem 3.1

L et (R, μ, ρ) be any bi- fuzzy topological ring. I f $\mu < \rho$, then

$\mu < \ \mu(\rho) < \rho$

Proof

Let us consider the identity map

$$i : (R, \rho) \rightarrow (G, \mu(\rho)).$$

For $cl_{\mu}(E) \in N_{\mu(\rho)}$,

$$i^{-1}\left(cl_{\mu}(E)\right) = cl_{\mu}(E) \supseteq E \in \rho$$

So, i is fuzzy continuous and consequently,

$$\rho > \mu(\rho)$$

For the other part, let $N_{\mu} = \{U\}$ be a fuzzy fundamental system of μ-fuzzy closed fuzzy nbhds of 0 in (R, μ). Since $\mu < \rho$, for each $U \in N_{\mu}$, there is a $V \in N_{\rho}$such that

$$V \subseteq U$$

Therefore,

$$cl_{\mu}(V) \subseteq cl_{\rho}(U)$$

Thus, for each $U \in N_{\mu}$ there exists $Cl_{\mu}(V) \in N_{\mu(\rho)})$ such that

$$Cl_{\mu}(V) \subseteq U$$

This implies that

$$\rho < \mu(\rho)$$

Combining (1) and (2), the result follows.

Theorem 3.2

Let $f : R_1 \rightarrow R_2$ be a fuzzy homo. from a bi- fuzzy topological ring (R_1, μ_1, μ_2) to another bi- fuzzy topological ring(R_2, ρ_1, ρ_2). Then

(1) if $f: \mu_1 \rightarrow \rho_1$ and $f: \mu_2 \rightarrow \rho_2$ are fuzzy cont., it is also $f: \mu_1(\mu_2) \rightarrow \rho_1(\rho_2)$ fuzzy continuous.

(2) if $f: \mu_1(\mu_2) \rightarrow \rho_2$ is fuzzy cont., it is also $f: \mu_2 \rightarrow \rho_2$ fuzzy cont.

(3) if $f: \mu_1(\mu_2) \rightarrow \rho_1$ is fuzzy cont., it is also $f: \mu_2 \rightarrow \rho_1$ fuzzy cont.

Proof

(1) Let $V \in \rho_1(\rho_2)$ with $V(f(0_1)) = V(0_2) > 0$ where 0_1 and 0_2 are identity elements of $R_1$and R_2 respectively. Then for some $V_1 \in \rho_2$ s.t

$$cl_{\rho_1}(V_1) \subseteq V \text{ and } V_1(0_2) = V(0_2)$$

Therefore, $f^{-1}(V_1) \in \mu_2$ and $f^{-1}(V_1)(0_1) > 0$

Since $f: \mu_1 \rightarrow \rho_1$ is fuzzy cont. then $f^{-1}(cl_{\rho_1}(V_1))$ is μ_1-fuzzy closed set and

$$cl_{\mu_1}(f^{-1}(V_1)) \subseteq f^{-1}(cl_{\rho_1}(V_1)) \subseteq f^{-1}(V)$$

Hence $f^{-1}(V) \in \mu_1(\mu_2)$

Thus $f: \mu_1(\mu_2) \rightarrow \rho_1(\rho_2)$ is fuzzy cont.

Proofs of (2) and (3) are clearly.

Theorem 3.3

If $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ fuzzy continuous homomorphism, from a bi- fuzzy topological ring (R_1, μ_1, μ_2) to another bi- fuzzy topological ring (R_2, ρ_1, ρ_2) with $\mu_1 < \mu_2$ and $\rho_1 < \rho_2$, then

(1) $f: \mu_2 \to \rho_1$ is fuzzy cont.

(2) $f: \mu_2 \to \rho_1(\rho_2)$ is fuzzy cont.

Proof

(1) Let $V \in \rho_1$ with $V(0_2) > 0$, By Theorem 3.1,we have

$\mu_1 < \mu_1(\mu_2) < \mu_2$ and $\rho_1 < \rho_1(\rho_2) < \rho_2$

Therefore, $V \in \rho_1(\rho_2)$. Since $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ is fuzzy cont., then $f^{-1}(V) \in \mu_1(\mu_2)$ and so $f^{-1}(V) \in \mu_2$.

Also

$f^{-1}(V)(0_1) = V\big(f(0_1)\big) = V(0_2) > 0.$

Thus, $f: \mu_2 \to \rho_1$ is fuzzy cont.

(2) Let $V \in \rho_1(\rho_2)$ with $V(0_2) > o.$

Since $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ is fuzzy cont., then $f^{-1}(V) \in \mu_1(\mu_2)$ and so $f^{-1}(V) \in \mu_2$.

Also

$f^{-1}(V)(0_1) = V\big(f(0_1)\big) = V(0_2) > 0.$

Thus, $f: \mu_2 \to \rho_1(\rho_2)$ is fuzzy cont.

Theorem 3.4

Let f be any fuzzy homo. from a bi- fuzzy topological ring (R_1, μ_1, μ_2) to another bi- fuzzy topological ring(R_2, ρ_1, ρ_2) with $\mu_1 < \mu_2$. Then

(1) if $f: \mu_1 \to \rho_2$ fuzzy cont., then $f: \mu_1(\mu_2) \to \rho_2$and $f: \mu_2 \to \rho_2$are fuzzy cont.

(2) if $f: \mu_1 \to \rho_1$ fuzzy cont., then $f: \mu_1(\mu_2) \to \rho_1$ fuzzy cont..

Proof

Clearly

Proposition 3.5

Let f be a fuzzy homo from a bi- fuzzy topological ring (R_1, μ_1, μ_2) to another bi- fuzzy topological ring (R_2, ρ_1, ρ_2)). If $f: \mu_1 \to \rho_1$is fuzzy closed set and $f: \mu_2 \to \rho_2$ is fuzzy open set then, $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ is fuzzy open set.

Proof

Let $U \in \mu_1(\mu_2)$ with $U(0_1) > 0$. Then there is exists $V \in \mu_2$ with $V(0_1) > 0$, such that $cl_{\mu_1}(V) \subseteq U$. Therefore $f(V) \subseteq f(cl_{\mu_1}(V)) \subseteq f(U)$ Since $f: \mu_1 \to \rho_1$ is fuzzy closed set, $cl_{\rho_1}(f(V)) \subseteq f(cl_{\mu_1}(V))$.

Making use of the facts that $f: \mu_2 \to \rho_2$ is fuzzy open and V is μ_2-fuzzy open, we have $Q = f(V)$ is ρ_2–fuzzy open set. Thus,

$$cl_{\mu_1}(Q) = cl_{\mu_1}(f(V)) \subseteq f(cl_{\mu_1}(V))$$

i.e

$$cl_{\mu_1}(V) = cl_{\mu_1}(f(V)) \subseteq f(cl_{\mu_1}(V)) \subseteq f(U)$$

$f(U) \in \rho_1(\rho_2)$

Also $f(U)(0_2) = U(0_1) > 0$

Thus $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ is fuzzy open set.

Theorem 3.6

Let f be a fuzzy homo. from a bi- fuzzy topological ring (R_1, μ_1, μ_2) to another bi- fuzzy topological ring (R_2, ρ_1, ρ_2). I f $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ is fuzzy closed and $\mu_1 < \mu_2$, $\rho_1 < \rho_2$ then $f: \mu_1 \to \rho_2$ is fuzzy closed.

Proof

Let U be any μ_1-fuzzy closed set with $U(0_1) > 0$. In view of Theorem 3.1 U is also $\mu_1(\mu_2)$-fuzzy closed and hence $f(U)$ is $\rho_1(\rho_2)$-fuzzy closed and hence ρ_2-fuzzy closed.

Theorem 3.7

Let f be a fuzzy homo. from a bi- fuzzy topological ring (R_1, μ_1, μ_2) to another bi- fuzzy topological ring(R_2, ρ_1, ρ_2). Then

(1) $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ is fuzzy open then $f: \mu_1(\mu_2) \to \rho_2$ is fuzzy open.

(2) $f: \mu_2 \to \rho_1(\rho_2)$ is fuzzy open then $f: \mu_1(\mu_2) \to \rho_2$ is fuzzy open.

(3) $f: \mu_1 \to \rho_1$ is fuzzy open and $\rho_1 < \rho_2$ then $f: \mu_1 \to \rho_2$ is fuzzy open.

(4) $f: \mu_1(\mu_2) \to \rho_1(\rho_2)$ is fuzzy open and $\mu_1 < \mu_2$ then $f: \mu_1 \to \rho_1(\rho_2)$ is fuzzy open.

Proof: By the same way of previous theorems.

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