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On The Decomposition Matrix Of Spin Characters Of S_{19} Modulo p=5

Marwa Mohammed Jawad Al-musawi[a] and Mohanad Abdulkareem Hasan Hasab [b]

[a] *University Basrah, College of Science, Department of Mathematics, Basrah-Iraq, Email : marwamohammadjawad@gmail.com*
[b] *University Basrah, College of Education for Human Sciences, Department of Psychological Counseling, Basrah-Iraq, Email: mohanadhasab@yahoo.com*

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ABSTRACT

In this paper, we calculate the decomposition matrix of spin characters for the symmetric group S_n when n=19 and p=5 by fond the decomposition matrices for all blocks of the symmetric group S_n since the decomposition matrix of S_n is the direct sum of all blocks

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1 . Introduction

In symmetric group , the covering group has characters [1] called spin characters and the relation between ordinary and modular characters is matrix called decomposition matrix [2],[3],[4] . when we calculate the decomposition matrix for the symmetric group S_{19} we will depend on the decomposition matrix for S_{18} which is calculated by Lukas Alexander Maas [5]. From of the researchers worked on the decomposition matrix are :

1. N. S. Abdullah found the decomposition matrix of S_{16} for p=5 and S_{19} for p=7 [6].
2. S. A. Taban found the decomposition matrix of S_{14} for p=5 , S_{15} for p=5 and S_n where n=16,17,18 for p=7 [7]

Corresponding author *: Marwa Mohammed , Mohanad Abdulkareem*

Email address: *marwamohammadjawad@gmail.com*

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2. Notation:-

$D^{(i)}_{(19,5)}$	The decomposition matrix of the block B_i
$d_i \downarrow S_{n-1}$	The restriction of d_i in S_n to S_{n-1}
$D_j \uparrow S_n$	The inducing of D_j in S_{n-1} to S_n
‹ α ›*	The double spin characters
‹ α ›, ‹ α ›'	The associate spin characters

3.The Spin Blocks of S_{19}:

In the symmetric group S_{19} there are 81 irreducible spin characters and the covering group has 45 of 5-regular classes that is mean the decomposition matrix of S_{19} has 81 rows and 45 columns.

The symmetric group S_{19} has four blocks (B_1 , B_2 of defect three)and (B_3 , B_4 of defect two) such that :

B_1={‹19›*, ‹15,4›, ‹15,4›', ‹14,5›, ‹14,5›', ‹14,4,1›*, ‹14,3,2›*, ‹13,4,2›*, ‹12,4,3›*, ‹10,9›, ‹10,9›', ‹10,5,4›*, ‹10,4,3,2›, ‹10,4,3,2›', ‹9,8,2›*, ‹9,7,3›*, ‹9,6,4›*, ‹9,5,4,1›, ‹9,5,4,1›', ‹9,5,3,2›, ‹9,5,3,2›', ‹9,4,3,2,1›*, ‹8,7,4›*, ‹8,5,4,2›, ‹8,5,4,2›', ‹7,5,4,3›, ‹7,5,4,3›'}

B_2 ={ ‹18,1›, ‹18,1›', ‹16,3›, ‹16,3›', ‹15,3,1›*, ‹13,6›, ‹13,6›', ‹13,5,1›*, ‹13,3,2,1›, ‹13,3,2,1›', ‹11,8›, ‹11,8›', ‹11,5,3›*, ‹11,4,3,1›, ‹11,4,3,1›', ‹10,8,1›*, ‹10,6,3›*, ‹10,5,3,1›, ‹10,5,3,1›', ‹9,6,3,1›, ‹9,6,3,1›', ‹8,7,3,1›, ‹8,7,3,1›', ‹8,6,5›*, ‹8,6,4,1›, ‹8,6,4,1›', ‹8,6,3,2›, ‹8,6,3,2›', ‹8,5,3,2,1›*, ‹6,5,4,3,1›*}

B_3 ={‹17,2›, ‹17,2›', ‹12,7›, ‹12,7›', ‹12,5,2›*, ‹12,4,2,1›, ‹12,4,2,1›', ‹10,7,2›*, ‹10,7,2›*, ‹9,7,2,1›, ‹9,7,2,1›', ‹7,6,4,2›, ‹7,6,4,2›', ‹7,5,4,2,1›*}

B_4 ={‹16,2,1›*, ‹12,6,1›*, ‹11,7,1›*, ‹11,6,2›*, ‹11,5,2,1›,‹11,5,2,1›', ‹10,6,2,1›, ‹10,6,2,1›', ‹7,6,5,1›, ‹7,6,5,1›', ‹7,6,3,2,1›*}

4. The block B_1

In this block all the irreducible spin characters of the decomposition matrix are double and ‹$^{\beta}$›=‹$^{\beta}$›' on 5-regular classes.

lemma (4.1):$D^{(1)}_{(19,5)}$is the decomposition matrix of the block B_1 .

proof: by the inducing of D_1 , D_{31} , D_7 , D_5 , D_{33} , D_{11} , D_{13} , D_{32} , D_{35} and D_{19} in S_{18} to S_{19} we have the columns a_1 , d_2 , d_3 , d_4 , d_5 , d_6 , a_2 , d_8 , d_9 and d_{10} respectively then we get the following approximation matrix :

Spin characters	The approximation matrix for B_1								
‹19›*	1								
‹15,4›	1	1							
‹15,4›'	1	1							
‹14,5›	1	1	1						
‹14,5›'	1	1	1						
‹14,4,1›*	2	1	1	1					
‹14,3,2›*				1					
‹13,4,2›*	2		1	1	1				
‹12,4,3›*	2		1		1	1			
‹10,9›	1		1				1		
‹10,9›'	1		1				1		
‹10,5,4›*	4	2	2			2	2	2	
‹10,4,3,2›						1		1	
‹10,4,3,2›'						1		1	
‹9,8,2›*	2		1		1		1		1	
‹9,7,3›*	6		1	1	1	1	2		1	1
‹9,6,4›*	6	2	1	1		1	4	2		1
‹9,5,4,1›	4					1	2	1		1
‹9,5,4,1›'	4	1				1	2	1		1
‹9,5,3,2›	4	1				1	3	1		1
‹9,5,3,2›'	4					1	3	1		1
‹9,4,3,2,1›*							2	1		
‹8,7,4›*	4			1			2			1
‹8,5,4,2›	4						3		1	1
‹8,5,4,2›'	4						3		1	1

‹7,5,4,3›									1	
‹7,5,4,3›'									1	
	a_1	d_2	d_3	d_4	d_5	d_6	a_2	d_8	d_9	d_{10}

Now , we will discuss the cases of the subtraction of columns :

Case (1) : d_{10} is subtracted from a_2 such that $a_2 - d_{10} = d_7$

Case (2) : d_{10} is subtracted from a_1 such that $a_1 - d_{10} = d_1$

Case (3) : $(d_3 - d_5)\downarrow S_{18} = D_5 + D_6 + D_7 + D_8 - D_9 - D_{10} + 3D_{31}$

That is mean d_5 is not subtracted from d_3 .

Case (4) : $(d_1 - d_3)\downarrow S_{18} = = D_1 + D_2 - D_7 - D_8 + D_5 + D_6 + 4D_{19} + 4D_{20} + 3D_{31} + 6D_{32} + 2D_{33} + 2D_{34} + 3D_{35} + D_{36}$

That is mean d_3 is not subtracted from d_1 .

Case (5) : $(d_1 - d_5)\downarrow S_{18} = D_1 + D_2 + 2D_5 + 2D_6 - D_9 - D_{10} + 4D_{19} + 4D_{20} + 2D_{33} + 2D_{34} + 3D_{35} + 3D_{36} + 6D_{31} + 6D_{32}$

That is mean d_5 is not subtracted from d_1 .

Then the decomposition matrix is $D^{(1)}_{(19,5)}$:

Spin characters	The approximation matrix for B_1									
‹19›*	1									
‹15,4›	1	1								
‹15,4›'	1	1								
‹14,5›	1	1	1							
‹14,5›'	1	1	1							
‹14,4,1›*	2	1	1	1						
‹14,3,2›*				1						
‹13,4,2›*	2		1	1	1					
‹12,4,3›*	2		1		1	1				
‹10,9›	1		1				1			
‹10,9›'	1		1				1			

‹10,5,4›*	4	2	2			2	2	2		
‹10,4,3,2›						1		1		
‹10,4,3,2›'						1		1		
‹9,8,2›*	2		1		1		1		1	
‹9,7,3›*	5		1	1	1	1	1		1	1
‹9,6,4›*	5	2	1	1		1	3	2		1
‹9,5,4,1›	3					1	1	1		1
‹9,5,4,1›'	3	1				1	1	1		1
‹9,5,3,2›	3	1				1	2	1		1
‹9,5,3,2›'	3					1	2	1		1
‹9,4,3,2,1›*							2	1		
‹8,7,4›*	3			1			1			1
‹8,5,4,2›	3						2		1	1
‹8,5,4,2›'	3						2		1	1
‹7,5,4,3›									1	
‹7,5,4,3›'									1	
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}

5. The block B_2

In this block all the irreducible spin characters of the decomposition matrix are associate and $\langle\beta\rangle \neq \langle\beta\rangle'$ on 5-regular classes.

lemma (5.1):$D^{(2)}_{(19,5)}$ is the decomposition matrix of the block B_2 .

proof: by the inducing of D_{21} , D_3 , D_5 , D_6 , D_7 , D_8 , D_9 , D_{10} , D_{13} , D_{11} , D_{12} , D_{17} , D_{18} , D_{19} , D_{20} and D_{30} in S_{18} to S_{19} we have the columns ($K_1 = d_{11}+d_{12}$), K_2 , d_{15} , d_{16} , d_{17} , d_{18} , d_{19} , d_{20} , K_3 , d_{23} , d_{24} , d_{25} , d_{26} , d_{27} , d_{28} and K_4 respectively then we get the following approximation matrix :

Spin characters	The approximation matrix for B_2																	
‹18,1›	1																	
‹18,1›'		1																
‹16,3›	1		1														a	
‹16,3›'		1	1															a
‹15,3,1›*			2	1	1												b	b
‹13,6›			1			1	1										c	
‹13,6›'			1			1	1											c
‹13,5,1›*	1	1	2	1	1	2	2	1	1								d	d
‹13,3,2,1›				1				1									e	
‹13,3,2,1›'					1				1									e
‹11,8›						1	1			1							f	
‹11,8›'						1	1			1								f
‹11,5,3›*	1	1	2			3	3	1	1	2	3	3					g	g
‹11,4,3,1›							1		1		2	1					h	
‹11,4,3,1›'						1		1			1	2						h
‹10,8,1›*	1	1				2	2	1	1	2			1	1			i	i
‹10,6,3›*	1	1	4	1	1	3	3	1	1	4	3	3	1	1	1	1		j	j
‹10,5,3,1›			2			1	1			2	3	3			1	1		k	
‹10,5,3,1›'			2			1	1			2	3	3			1	1			k
‹9,6,3,1›	1	1	2	1		1		1		2	1	2		1	2	1	1	l	
‹9,6,3,1›'	1	1	2		1		1		1	2	2	1	1		1	2	1		l
‹8,7,3,1›	1	1			1				1				1			1	1	m	
‹8,7,3,1›'	1	1		1				1						1	1		1		m
‹8,6,5›*			2	1	1					2					1	1		n	n
‹8,6,4,1›	1	1	2		1					2			1	1	1	2	1	o	

‹8,6,4,1›'	1	1	2	1						2			1	1	2	1	1		o
‹8,6,3,2›	1	1								2			1	1	1	1	1	p	
‹8,6,3,2›'	1	1								2			1	1	1	1	1		p
‹8,5,3,2,1›*										4			1	1	1	1		q	q
‹6,5,4,3,1›*													1	1				r	r
	d_{11}	d_{12}	K_2	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	K_3	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	K_4	Y_1	Y_2

Since ‹16,3› ≠ ‹16,3›'on (5,α) – regular classes then either K_2 is split or there are other two columns ; if we suppose there are two columns such that Y_1 and Y_2 .
By using the (r,$\bar{r}$)- restriction of ‹16,3›, ‹15,3,1›*, ‹13,6›, ‹13,5,1›*, ‹13,3,2,1›, ‹11,8› , ‹11,5,3›* , ‹11,4,3,1›, ‹10,8,1›*, ‹10,6,3›*, ‹10,5,3,1› , ‹9,6,3,1› , ‹8,7,3,1› , ‹8,6,5›* , ‹8,6,4,1› , ‹8,6,3,2› , ‹8,5,3,2,1›* and ‹6,5,4,3,1›*we have a , b , c , f ϵ {0,1,2,3} , d , m ϵ {0,1,…,6} , e ϵ {0,1,2} , g ϵ {0,1,…,11} , h ϵ {0,1,…,4} , i ϵ {0,1,…,7} , j ϵ {0,1,…,19} , k ,o ϵ {0,1,…,20} , l ϵ {0,1,…,21} , n , q ϵ {0,1,…,8} , p ϵ {0,1,…,12} and r ϵ {0,1}

If a ϵ {1,2,3} we have:

Y_1= a ‹16,3› + b ‹15,3,1›* + c ‹13,6› + d ‹13,5,1›* + g ‹11,5,3›* + j ‹10,6,3›* + k ‹10,5,3,1› + l ‹9,6,3,1› + n ‹8,6,5›* + o ‹8,6,4,1›
Y_1= a ‹16,3›' + b ‹15,3,1›* + c ‹13,6›' + d ‹13,5,1›* + g ‹11,5,3›* + j ‹10,6,3›* + k ‹10,5,3,1›' + l ‹9,6,3,1›' + n ‹8,6,5›* + o ‹8,6,4,1›'

deg $Y_1 \equiv 0 \bmod 5^3$ and deg $Y_2 \equiv 0 \bmod 5^3$ only when a=b= c=d= g=j=k=l=n=o= 1 and e=f=h=i=m=p=q=r=0 .

so k_2 split to d_{13} and d_{14} .

we will discuss the subtract of columns :

$(d_{11} - K_4)\downarrow S_{18} = D_1 - D_{19} - D_{20}$

That is mean K_4 is not subtracted from d_{11} .

Since d_{11} , d_{12} are associate , that is mean K_4 is not subtracted from d_{12} .

Finally , by the same way , the other columns K_3 and K_4 are split to (d_{21} , d_{22}) and (d_{29} , d_{30})

Then the decomposition matrix is $D^{(2)}_{(19,5)}$:

Spin characters	The decomposition matrix for B_2																			
‹18,1›	1																			
‹18,1›'		1																		
‹16,3›	1		1																	
‹16,3›'		1		1																
‹15,3,1›*			1	1	1	1														
‹13,6›			1				1	1												
‹13,6›'				1			1	1												
‹13,5,1›*	1	1	1	1	1	1	2	2	1	1										
‹13,3,2,1›					1				1											
‹13,3,2,1›'						1				1										
‹11,8›							1	1			1									
‹11,8›'							1	1				1								
‹11,5,3›*	1	1	1	1			3	3	1	1	1	1	3	3						
‹11,4,3,1›								1		1			2	1						
‹11,4,3,1›'							1		1				1	2						
‹10,8,1›*	1	1					2	2	1	1	1	1			1	1				
‹10,6,3›*	1	1	2	2	1	1	3	3	1	1	2	2	3	3	1	1	1	1		
‹10,5,3,1›			1	1			1	1			1	1	3	3			1	1		
‹10,5,3,1›'			1	1			1	1			1	1	3	3			1	1		
‹9,6,3,1›	1	1	1	1	1		1		1		1	1	1	2		1	2	1	1	
‹9,6,3,1›'	1	1	1	1		1		1		1	1	1	2	1	1		1	2		1
‹8,7,3,1›	1	1				1				1					1			1	1	
‹8,7,3,1›'	1	1			1				1							1	1			1
‹8,6,5›*			1	1	1	1					1	1					1	1		
‹8,6,4,1›	1	1	1	1		1					1	1			1	1	1	2	1	

‹8,6,4,1›'	1	1	1	1	1						1	1			1	1	2	1		1
‹8,6,3,2›	1	1									1	1			1	1	1	1	1	
‹8,6,3,2›'	1	1									1	1			1	1	1	1		1
‹8,5,3,2,1›*											2	2			1	1	1	1		
‹6,5,4,3,1›*															1	1				
	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	d_{26}	d_{27}	d_{28}	d_{29}	d_{30}

6. The block B_3 :

In this block all the irreducible spin characters of the decomposition matrix are associate and $\langle\beta\rangle \neq \langle\beta\rangle'$ on 5-regular classes.

lemma (6.1): $D^{(3)}_{(19,5)}$ is the decomposition matrix of the block B_3 .

proof: by the inducing of D_{21} , D_{24} , D_{33} , D_{34} , D_{35} , D_{36} and D_{30} in S_{18} to S_{19} we have the columns K_1 , K_2 , d_{35} , d_{36} , d_{37} , d_{38} and K_3 respectively then we get the following approximation matrix :

Spin characters	The approximation matrix for B_3								
‹17,2›	1							a	
‹17,2›'	1								a
‹12,7›		1						b	
‹12,7›'		1							b
‹12,5,2›*	2	2	1	1				c	c
‹12,4,2,1›			1					d	
‹12,4,2,1›'				1					d
‹10,7,2›*	2	2	1	1	1	1		e	e
‹9,7,2,1›	2		1		1		1	f	
‹9,7,2,1›'	2			1		1	1		f
‹7,6,4,2›	2				1	1	1	g	
‹7,6,4,2›'	2				1	1	1		g
‹7,5,4,2,1›*					1	1		h	h
	K_1	K_2	d_{35}	d_{36}	d_{37}	d_{38}	K_3	Y_1	Y_2

Since ‹17,2›≠‹17,2›'on (5,α) – regular classes then either K_1 is split or there are other two columns ; if we suppose there are two columns such that Y_1 and Y_2 .
By using the $(r,\bar{r})$- restriction of ‹17,2›, ‹12,7›, ‹12,5,2›*, ‹12,4,2,1›, ‹10,7,2›*, ‹9,7,2,1› , ‹7,6,4,2› and ‹7,5,4,2,1›*we have a ϵ {0,1,2} , b ϵ {0,1,2,3} , c ϵ {0,1,…,9} , d ϵ {0,1,…,4} , e ϵ {0,1,…,14} , f ϵ {0,1,…,16} , g ϵ {0,1,…,7} and h ϵ {0,1,…,5} .
If a ϵ {1,2} then the other determined as :
(1)‹17,2› $\downarrow S_{18}$ ∩ ‹12,7› $\downarrow S_{18}$ = 1 of i.m.s. and b ∈ {0,1,2,3}.
(2)‹17,2› $\downarrow S_{18}$ ∩ ‹12,5,2›* $\downarrow S_{18}$ = 12 of i.m.s. and c ϵ {0,1,…,9} .
(3)‹17,2› $\downarrow S_{18}$ ∩ ‹12,4,2,1› $\downarrow S_{18}$ = 0 of i.m.s. and d=0.
(4)‹17,2› $\downarrow S_{18}$ ∩ ‹10,7,2›* $\downarrow S_{18}$ = 10 of i.m.s. and e ϵ {0,1,…,14} .
(5)‹17,2› $\downarrow S_{18}$ ∩ ‹9,7,2,1› $\downarrow S_{18}$ = 5 of i.m.s. and f ϵ {0,1,…,16}.
(6)‹17,2› $\downarrow S_{18}$ ∩ ‹7,6,4,2› $\downarrow S_{18}$ = 5 of i.m.s. andg ϵ {0,1,…,7} .
(7)‹17,2› $\downarrow S_{18}$ ∩ ‹7,5,4,2,1›* $\downarrow S_{18}$ = 0 of i.m.s. andh ϵ {0,1,…,5}.
Now we have :
Y_1= a‹17,2› + b‹12,7› + c‹12,5,2›* +d‹12,4,2,1› + e‹9,7,2,1› +f‹9,7,2,1›+ g‹7,6,4,2› +h‹7,5,4,2,1›*
Y_2= a‹17,2›' + b‹12,7›' + c‹12,5,2›* +d‹12,4,2,1›' +e‹9,7,2,1›' +f‹9,7,2,1›' +g‹7,6,4,2›' +h‹7,5,4,2,1›*
deg $Y_1 \equiv 0 \bmod 5^3$ and deg $Y_2 \equiv 0 \bmod 5^3$ only when a= c=e= f= g= 1 and b=d=h=0 .
so k_1 split to d_{31} and d_{32}

by the same way , the other columns K_2 and K_3 are split to (d_{33} , d_{34}) and (d_{39} , d_{40})

Finally , we will discuss the subtract of columns :

case (1) :

$(d_{31} - d_{39})\downarrow S_{18} = D_{21} - D_{30} + D_{33} + D_{34} + D_{36}$

That is mean d_{39} is not subtracted from d_{31} .

Since d_{31} , d_{32} are associate and d_{39} , d_{40} also associate that is mean d_{40} is not subtracted from d_{32}.

case (2) :

$(d_{37} - d_{39})\downarrow S_{18} = D_{28} - D_{30} + D_{35} + D_{36}$

That is mean d_{39} is not subtracted from d_{37} .

Since d_{37} , d_{328} are associate and d_{39} , d_{40} also associate that is mean d_{40} is not subtracted from d_{32} .

Then the decomposition matrix $D^{(i)}_{(19,5)}$is :

Spin characters	The decomposition matrix for B_3									
‹17,2›	1									
‹17,2›'		1								
‹12,7›			1							

‹12,7›'				1						
‹12,5,2›*	1	1	1	1	1	1				
‹12,4,2,1›					1					
‹12,4,2,1›'						1				
‹10,7,2›*	1	1	1	1	1	1	1	1		
‹9,7,2,1›	1	1			1		1		1	
‹9,7,2,1›'	1	1				1		1		1
‹7,6,4,2›	1	1					1	1	1	
‹7,6,4,2›'	1	1					1	1		1
‹7,5,4,2,1›*							1	1		
	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	d_{36}	d_{37}	d_{38}	d_{39}	d_{40}

7. The block B_4:

In this block all the irreducible spin characters of the decomposition matrix are double and $\langle^{\beta}\rangle = \langle^{\beta}\rangle'$ on 5-regular classes.

lemma (7.1):$D_{19,5}^{(4)}$ is the decomposition matrix of the block B_4 .

proof: by the inducing of D_{23} , D_{38} , D_{28} , D_{27} and D_{30} in S_{18} to S_{19} we have the columns d_{41} , d_{42} , d_{43} , d_{44} and d_{45} respectively and there are one case of subtraction such that :

$(d_{43} - d_{45})\downarrow S_{18} = D_{26} - D_{29} + 2\langle 11,6,1\rangle + 2\langle 11,6,1\rangle'$

Then d_{45} is not subtracted from d_{43} .

So, the decomposition matrix for B_4 is $D_{19,5}^{(4)}$:

Spin characters	The decomposition matrix for B_4				
‹16,2,1›*	1				
‹12,6,1›*	1	1			
‹11,7,1›*		1	1		
‹11,6,2›*	1	1	1	1	
‹11,5,2,1›				1	

‹11,5,2,1›'				1	
‹10,6,2,1›	1		1	1	1
‹10,6,2,1›'	1		1	1	1
‹7,6,5,1›	1		1		1
‹7,6,5,1›'	1		1		1
‹7,6,3,2,1›*			2		1
	d_{41}	d_{42}	d_{43}	d_{44}	d_{45}

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