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Investment of Trigonometric B-Spline to Solve I. V. Ps.

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ABSTRACT

This paper is devoted for finding approximate solution of the second order differential equation with initial conditions by using quadratic trigonometric basic spline function. It turns out through the examples analyzed (numerical results) in this field that the results are very close to the real values, given that the absolute error is very small, thus the trigonometric B-spline function investment for solving I. V. Ps is acceptable.

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1-Introduction

Likewise, B-Spline, a trigonometric B-Spline has minimal support with respect to given degree, smoothness, and domain partition. A trigonometric B-Spline as B-spline defined as base spline interpolation [Vijay-2015]. [T. lych-1979] studied a stable recurrence relation for trigonometric B-Spline, and [Per Erik-1992] introduced the normalized trigonometric B-Spline associated with knot point sequence by recursion. Quasi interpolants based on trigonometric B-Spline [Tom lych-1992]. Approximate solution for Boundary Value Problems by Trigonometric Spline Interpolation has given by [Najim-2016]. [M. Abas-2014] discussed Numerical method using cubic trigonometric B-Spline to find approximate solution for non-classical diffusion problems. Also the numerical method employed

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the cubic trigonometric B-Spline are set up to solve a class of linear two points singular boundary value problem in the cubic B-Spline algorithm for Burgers' equation by [Dag, O and Ö-2014].

In the present paper, we employ a quadratic trigonometric B-Spline function in order to solve the second order ordinary differential equation with respect to initial value problems, and we obtained the maximum error bound is very small and so the proximate error is acceptable.

Definition [4]:

The iterating formula $T_i^k(x) = \frac{\sin(\frac{x-x_i}{2})}{\sin(\frac{x_{i+k-1}-x_i}{2})} T_i^{k-1}(x) + \frac{\sin(\frac{x_{i+k}-x}{2})}{\sin(\frac{x_{i+k}-x_{i+1}}{2})} T_{i+1}^{k-1}(x), k=2, 3, \ldots$

gives several degree of trigonometric B-spline function starting with first order normalized of trigonometric B-spline function

$$T_i^2(x) = \begin{cases} 1 & for\ x_i \le x \le x_{i+1} \\ 0 & other\ wise \end{cases}$$

The trigonometric B-spline function share many properties of B-spline polynomial, such as support T_i^k is $[x_i, x_{i+1})$, $T_i^k>0$ for $x \in (x_i, x_{i+1})$, $T_1^k = 0$ for $x \notin (x_i, x_{i+1})$, and $\sum_{k=-\infty}^{\infty} T_i^k=1$ $\forall x \in R$ moreover, the spline function $s(x)$ with respect to the given trigonometric B-spline has a unique representation of the form

$$s(x)= \sum_{i=1}^{n} c_i T_i^k(x), \quad c_i \in R, \ i=1, 2, \ldots, n.$$

Table 1 The value of $TB_{i,2}$, $TB'_{i,2}$, and $TB''_{i,2}$

X	X_{i-1}	X_i	X_{i+1}	X_{i+2}
$B_{i,2}$	0	$sin(\frac{h}{2})csc(h)$	$sin(\frac{h}{2})csc(h)$	0
$B'_{i,2}$	0	$\frac{1}{2}csc(\frac{h}{2})$	$-\frac{1}{2}csc(\frac{h}{2})$	0
$B''_{i,2}$	0	$\frac{1}{2}\cot(\frac{h}{2})csc(\frac{h}{2})$	$-\frac{1}{2}\cot(\frac{h}{2})csc(\frac{h}{2})$	0

Table 1 gives coefficient of quadratic trigonometric B-Spline and its first and second derivatives.

2-Description of the method:

Consider the self-ad joint second order singularly perturbed problems of the form

$$Lu(x)= -\in Lu''(x)+ a(x)u(x)= f(x) \qquad 1$$

With initial value problem $u(0)=\alpha$, $u'(0)=\lambda$.

Such that α, and λ are real constants and $(0<\in\le 1)$, $\in$ is small positive parameter; a(x), and f(x) are sufficiently smooth functions. Let a(x)=a constant, and

$u(x)=s(x)=\sum_{j=-1}^{n} c_j TB_{j,2}(x)$, be approximate solution of equation (1). Then let $x_0, x_1, \ldots,$

x_n be n+1 grid points in the interval [0, 1]. Thus we have $x_1=x_0+ih$ where $h=\frac{b-a}{n}$ or $h=x_{i+1}-x_i=\frac{1}{n}$, and $x_0=0$, $x_n=1$, i=0, 1, …, n-1, we get:

$$s(x_i)=\sum_{j=-1}^{n} c_j\, TB_{j,2}\ (x_i) \tag{2}$$

$$s'(x_i)=\sum_{j=-1}^{n} c_j\, TB'_{j,2}\ (x_i) \tag{3}$$

$$s''(x_i)=\sum_{j=-1}^{n} c_j\, TB''_{j,2}\ (x_i) \tag{4}$$

substituting the value of equations (2) and (4) in equation (1) we get:

$$-\epsilon \sum_{j=-1}^{n} c_j\, TB''_{j,2}\ (x_i)+ a(x_i) \sum_{j=-1}^{n} c_j\, TB_{j,2}\ (x_i)= f(x),\ i=0, 1, \dots, n \tag{5}$$

While the initial condition becomes

$$\sum_{j=-1}^{n} c_j\, TB_{j,2}\ (x_0)= \alpha, \sum_{j=-1}^{n} c_j\, TB_{j,2}\ (x_0)= \lambda \tag{6}$$

The value of spline functions at the knots are determined by use of table (1), and substituting equations (5,6) we get a system of (n+1) equations with (n+1) unknown.

Now the above system of equation will be written in the form $SX_n = I_n$ where

$X_n =(c_{-2}, c_{-1}, \dots ,c_{n-1})^T$ are unknowns

$I_n = (\alpha, \lambda, f(x_0), \dots, f(x_n),)^T$,

Now since

$$TB_j^2(x) = \frac{1}{\gamma}\begin{cases} sin^2\left(\frac{x - x_{i-1}}{2}\right) & [x_{i-1}, x_i] \\ sin\left(\frac{x - x_{i-1}}{2}\right) sin\left(\frac{x_{i+1} - x}{2}\right) + sin\left(\frac{x_{i+2} - x}{2}\right) sin\left(\frac{x - x_i}{2}\right) & [x_i, x_{i+1}] \\ sin^2\left(\frac{x_{i+2} - x}{2}\right) & [x_{i+1}, x_{i+2}] \\ 0 & otherwise \end{cases}$$

Where $\gamma = sin(h)\, sin(\frac{h}{2})$.

The other trigonometric B-splines $TB_{j,2}$ are generated from the above trigonometric B-spline by translating of the intervals, so that

$$TB_{-1}^2(x) = \frac{1}{\gamma}\begin{cases} sin^2\left(\frac{x_1 - x}{2}\right) & [x_1, x_0] \\ 0 & otherwise \end{cases}$$

$$TB_0^2(x) = \frac{1}{\gamma}\begin{cases} sin\left(\frac{x-x_{i+h}}{2}\right) sin\left(\frac{x_1-x}{2}\right) + sin\left(\frac{x_2-x}{2}\right) sin\left(\frac{x-x_0}{2}\right) & [x_0, x_1] \\ sin^2\left(\frac{x_2-x}{2}\right) & [x_1, x_2] \\ 0 & otherwise \end{cases}$$

$$TB_1^2(x) = \frac{1}{\gamma}\begin{cases} sin^2\left(\frac{x-x_0}{2}\right) & [x_0, x_1] \\ sin\left(\frac{x-x_0}{2}\right) sin\left(\frac{x_2-x}{2}\right) + sin\left(\frac{x_3-x}{2}\right) sin\left(\frac{x-x_1}{2}\right) & [x_1, x_2] \\ sin^2\left(\frac{x_3-x}{2}\right) & [x_2, x_3] \\ 0 & otherwise \end{cases}$$

$$TB_{n-1}^2(x) = \frac{1}{\gamma}\begin{cases} sin^2\left(\frac{x-x_{n-2}}{2}\right) & [x_{n-1}, x_n] \\ sin\left(\frac{x-x_{n-2}}{2}\right) sin\left(\frac{x_n-x}{2}\right) + sin\left(\frac{x_{n+1}-x}{2}\right) sin\left(\frac{x-x_{n-1}}{2}\right) & [x_n, x_{n+1}] \\ 0 & otherwise \end{cases}$$

$$TB_n^2(x) = \frac{1}{\gamma}\begin{cases} sin^2\left(\frac{x-x_{n-1}}{2}\right) & [x_n, x_{n+1}] \\ 0 & otherwise \end{cases}$$

Now from equation (6)

$-\in \sum_{j=-1}^{n} c_j\, TB''_{j,2}\ (x_i)+ a(x_i) \sum_{j=-1}^{n} c_j\, TB_{j,2}\ (x_i)= f(x)$, i=0, 1, …, n

We get:

For i=0

$-\in\left(\frac{1}{2} c_{-1} \cot(h) \csc\left(\frac{h}{2}\right) - \frac{1}{2} c_0 \cot\left(\frac{h}{2}\right) \csc\left(\frac{h}{2}\right)\right) + a(x_0)\left(c_{-1} \sin\left(\frac{h}{2}\right) \csc(h) + c_0 \sin\left(\frac{h}{2}\right) \csc(h)\right.$
$= f(x_0)$ 7

i=1

$-\in\left(\frac{1}{2} c_0 \cot(h) \csc\left(\frac{h}{2}\right) - \frac{1}{2} c_1 \cot\left(\frac{h}{2}\right) \csc\left(\frac{h}{2}\right)\right) + a(x_1)\left(c_0 \sin\left(\frac{h}{2}\right) \csc(h) + c_1 \sin\left(\frac{h}{2}\right) \csc(h)\right.$
$= f(x_1)$ 8

i=2

$-\frac{1}{2} \in c_1 \cot(h) \csc\left(\frac{h}{2}\right) - \frac{1}{2} c_1 a(x_2) \csc\left(\frac{h}{2}\right) = f(x_2)$ 9

i=n-1

$-\in\left(\frac{1}{2} c_{n-1} \cot(h) \csc\left(\frac{h}{2}\right)\right) + a(x_{n-1})\left(c_{n-1} \sin\left(\frac{h}{2}\right) \csc(h) = f(x_{n-1})\right.$ 10

i=n

$-\in\left(\frac{-1}{2} c_{n-1} \cot\left(\frac{h}{2}\right) \csc\left(\frac{h}{2}\right) + \frac{1}{2} c_n \cot(h) \csc\left(\frac{h}{2}\right)\right) + a(x_n)\left(c_{n-1} \sin\left(\frac{h}{2}\right) \csc(h)\right.$
$+ c_n \sin\left(\frac{h}{2}\right) \csc(h) = f(x_n)$ 11

And from initial conditions:

$$(c_{-1}+c_0)\sin\left(\frac{h}{2}\right)\csc(h)=\alpha; \qquad \frac{-1}{2}(c_{-1}-c_0)\csc\left(\frac{h}{2}\right)=\lambda \tag{12}$$

From the equations (9- 14) consequently the coefficients matrix is given by

$$\begin{bmatrix} U & U & 0 & 0 & 0 & 0 \\ -V & V & 0 & \vdots & \vdots & \vdots \\ S_0 & T_0 & 0 & \vdots & \vdots & \vdots \\ 0 & S_1 & T_1 & 0 & S_{n-1} & 0 \\ 0 & \cdots & \cdots & 0 & T_n & S_n \\ 0 & \cdots & \cdots & 0 & U & U \end{bmatrix} + \begin{bmatrix} C_{-1} \\ C_0 \\ \vdots \\ \vdots \\ \vdots \\ C_{n-1} \\ C_n \end{bmatrix} = \begin{bmatrix} \alpha \\ \lambda \\ f(x_0) \\ f(x_1) \\ \vdots \\ \vdots \\ f(x_n) \end{bmatrix};$$

Where

U=$\sin\left(\frac{h}{2}\right)\csc(h)$

V=$\frac{1}{2}\csc\left(\frac{h}{2}\right)$

S_i=$\frac{-1}{2}\in\cot(h)\csc\left(\frac{h}{2}\right)$)+ $a(x_i)$($\sin\left(\frac{h}{2}\right)\csc(h)$

T_i= $\frac{1}{2}\in\cot\left(\frac{h}{2}\right)\csc\left(\frac{h}{2}\right)$)+ $a(x_i)$($\sin\left(\frac{h}{2}\right)\csc(h)$

3-Numerical Results:

To find maximum absolute error at the knot points by using quadratic trigonometric B-spline to solving initial value problems of ordinary differential equation. Take the following examples.

Example 1:

Consider the following second order differential equation subject to initial value conditions.

-∈y”+6y=$\frac{(0.05x)^{10}}{832}$x, where y(0)=0, y'(0)=0, for x∈[0, 1].

N \ ∈	10^{-1}	10^{-2}	10^{-4}
10	$8.35530474*10^{-18}$	$1.982482938*10^{-14}$	$1.955920911*10^{-17}$
20	$8.053038727*10^{-18}$	$1.932692708*10^{-14}$	$1.934568318*10^{-17}$
40	$1.146460340*10^{-17}$	$1.979332940*10^{-14}$	$1.795344161*10^{-17}$

Example 2:

$-\in y''-8y=\frac{(0.02)^4}{12}x^3$, for initial conditions $y(0)=0, y'(0)=0, for\ x \in [0,1]$.

N \ ∈	10^{-1}	10^{-2}	10^{-4}
10	$3.26719582 * 10^{-10}$	$1.917270161 * 10^{-9}$	$4.159803 * 10^{-12}$
20	$1.042154853 * 10^{-9}$	$1.905988790 * 10^{-9}$	$2.5591131 * 10^{-11}$
40	$1.397219475 * 10^{-9}$	$1.075079715 * 10^{-9}$	$1.07401584 * 10^{-10}$

4-Conclusion

In this paper we used $TB_{j,2}$ interpolation to solve differential equation with I. V. Ps. It seems that the absolute errors are small enough so is acceptable and gives more accurate results.

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