

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)



Estimate Reliability Function of Inverse Lindley for Strength – Stress Models

Ali N.Hssein*

Basrah Univirsity , Basrah, Iraq.Email:ali.hussien@uobasrah.edu.iq

ARTICLE INFO

Article history:

Received: 10 /12/2020

Revised form: 20 /12/2020

Accepted : 27 /12/2020

Available online: 05 /01/2021

Keywords:

Lindley distribution, Inverse Lindley distribution, Cascade, Stress -Strength Reliability, Maximum Likelihood Methods, Bayes method.

ABSTRACT

Cascade models are considered as a special case of standby redundancy of stress strength modes. Inverse Lindley distribution belong to an exponential family distribution and it can be written as invers of Lindley distribution

This study concerns some Inverse Lindley Distribution's properties. cascade models with stress and strength variables that follow inverse Lindley distribution has been simulated. Reliability function of inverse Lindsley distribution for stress strength cascade models was estimated using both Likelihood and Bayes method. Compresence between these two methods has been made

MSC. 41A25; 41A35; 41A36...

DOI : <https://doi.org/10.29304/jqcm.2020.12.4.725>

1. Introduction

Stress-Strength model plays important role in reliability theory and its applications. These models describe the relation between two independent random variables (x and y) (3). The variable (x) represents strength whereas (y) represents stress. Strength is defined as the ability of a component or a system to achieve what is required when it is exposed to external load. While, stress is defined as the amount of load that leads to system failure. Stress may be temperature, pressure or load ...etc.

The reliability of Stress-strength model is defined as probability that the strength of a system or a component is greater than the pressure of stress and it is defined as (2)

$$R = p(y > x)$$

*Corresponding author:Ali N. Hussein

Email addresses:ali.hussien@uobasrah.edu.iq

Communicated by : Alaa Hussein Hamadi

Cascade models were early used and studied by S.N.N. Pandit and G.L. Srivastav (10). This use increased with the development of technology to include scientific, engineering and artificial fields. Reliability theory plays the main role in evaluating the work of the different components in the systems.

This study concentrates on cascades having inverse Lindley distribution for random variable of strength, whereas the random variable of stress follows inverse Lindley distribution with different parameters

2-Objective

The aim is studying cascade models for inverse Lindley models as well as comparing two methods of statistical estimation. Maximum likelihood estimating method and Bayes method.

3-Stress-Strength Model

The standby model is a procedure for increasing the reliability of the system or component by replacing the failure component by another one. It is not necessary that the new component is exposed to the same stress as the failure component which has been exchanged.

A cascade (10;11) model is a spatial type of redundancy standby of stress -strength model. Cascade redundancy system is also known as Hierarchical standby redundancy in which only one component works while the others in a standby situation

There are several kinds of standby systems such as the standby system with one active component while (n-1) components are in standby. Normally this system is known as (cascade (1+1)). The reliability for this system is under the joint effect of both stress and strength. Another type is a multi - standby system, in this system each component is regarded as a single system. The total reliability of the system is found by using probability. The cascade system in this case keeps more than one component in standby situation

4-Reliability of Stress-Strength

The reliability of stress-strength model expresses relation between two random variables. The first represents the strength and the other represents the stress. The failure of any component in a stress-strength system happens when the stress is greater than strength. Another component is activated to be in standby situation. The stress must be reset again after the failure of any component,

Cascade system for stress-strength model works when the probability of strength is more than that of stress. The exerted stress on an active component is equal to constant value (k) multiplied by the stress of the failure component. (k) is known as the attenuation factor.

Assuming $(x_1; x_2; \dots x_n)$ are random variables that represent strength for (n) of components. Similarly, $(y_1; y_2; \dots y_n)$ represent stress variables respectively. After any failure the stress is rested by factor (k) such as

$$y_2 = ky_1; y_3 = ky_2; \dots y_n = ky_{n-1}$$

$$y_i = k^{i-1}y_1$$

Strength is also rested after any failure by factor (m) as

$$x_2 = mx_1; x_3 = mx_2; \dots x_n = mx_{n-1}$$

$$x_i = m^{i-1}x_1$$

The system with n-components keeps working even though (n-1) components fail. And it fails after the failure of the nth component.

The marginal reliability of the first component (R1) assuming (x_1) for strength and (y_1) for stress is given by

$$R_1 = \text{pr}(x_1 > y_1) = \int_0^\infty \int_y^\infty f(x)g(y)dx dy \quad (1)$$

Whereas, the reliability of the second component (R2) assuming $(x_1; x_2)$ representing strength of the first and second components respectively and $(y_1; y_2)$ representing stress of the first and second components respectively will be

$$R_2 = \text{pr}(x_1 < y_1; x_2 \geq y_2)$$

$$R_2 = \text{pr}(x_1 < y_1; mx_1 \geq ky_1)$$

$$R_2 = \int_0^\infty \text{pr}(x_1 < y_1) \text{pr}(mx_1 \geq ky_1) g(y_1) dy_1 \quad (2)$$

In the same way, if $(x_1; x_2; \dots; x_n)$ are random independent variables representing strength for n components and $(y_1; y_2; \dots; y_n)$ are random independent variables representing stress for n components respectively, the reliability of the nth component is

$$R_n = \text{pr}(x_1 < y_1; x_2 < y_2; \dots; x_n \geq y_n)$$

$$R_n = P[x_1 < y_1, mx_1 \leq ky_1, \dots, m^{n-2}x_1 \leq k^{n-2}y_1, k^{n-1}x_1 > k^{n-1}y_1]$$

$$R_n = \int_0^\infty \text{pr}(x_1 < y_1) \dots \text{pr}(m^{n-1}x_n \geq k^{n-1}y_n) g(y_1) dy \quad (3)$$

The reliability of n cascade system is

$$R(n) = R_1 + R_2 + \dots + R_n \quad (4)$$

5-The Inverse Lindley Distribution

Lindley (1; 8) distribution considered as one of the continuous distributions suggested by (D.V. Lindley) while studying building of Bayesian theory. This distribution is obtained by maxing both exponential distribution with shape parameter (α) and gamma distribution with shape parameter equal (2); scale parameter (α) and mixture parameter $(\frac{\alpha}{\alpha+1})$. The probability density, cumulative and hazard functions for Lindley distribution are (1;2;8)

$$f(x; \alpha) = \frac{\alpha^2}{(1+\alpha)} \left(\frac{1+x}{x^3} \right) e^{-\frac{\alpha}{x}} \quad (5)$$

$$F(x; \alpha) = \left(1 + \frac{\alpha}{1+\alpha} \frac{1}{x} \right) e^{-\frac{\alpha}{x}} \quad (6)$$

The probability models of Lindley distribution in many applications are better than exponential distribution models (7). Lindley distribution has been widely used for studying survival functions, cancer data, stress-strength models and load-sharing system models. Whereas the use of Linley distribution is limited on applications with Monotonic increasing hazard function curve (7). That wise Vikas K.S suggested(c7) inverse Linley distribution which can be used with applications having non monotone hazard function. The random variable $(y = 1/x)$ is said to be following the inverse Lindley distribution if the random variable (x) is following Lindley distribution. The probability distribution function of inverse Lindsey is

$$f(x; \alpha) = \frac{\alpha^2}{1+\alpha} \left(\frac{1+x}{x^3} \right) e^{-\frac{\alpha}{x}} \quad (7)$$

Where $(x; \alpha > 0)$; The distribution and hazard functions are given by

$$F(x; \alpha) = \left(1 + \frac{1}{x+\alpha x} \right) e^{-\frac{\alpha}{x}} \quad (8)$$

$$h(x) = \frac{\alpha^2(1+x)}{x^2 \left(\alpha + x(1+\alpha) \left(e^{-\frac{\alpha}{x}-1} \right) \right)} \quad (9)$$

6-The Reliability of cascade system based on Inverse Lindley distribution

This study will be concentrated on reliability of a cascade system composed of one component. In this case the reliability of stress-strength system is equal to $\text{pr}(x > y)$ assuming independency of the two random variables (x and y). In this case the reliability will take the following form

$$\begin{aligned} R_1 &= \text{p}(x > y) = \int_y^\infty \int_0^\infty f(x / \theta_1) g(y / \theta_2) dy dx \\ R_1 &= \int_0^\infty \left(1 - \int_0^y f(x / \theta_1) \right) g(y / \theta_2) dx dy \\ R_1 &= \int_0^\infty (g(y / \theta_2) dy - F(y / \theta_1) g(y / \theta_2) dy) \\ R_1 &= 1 - \int_0^\infty F(y / \theta_1) g(y / \theta_2) dy \end{aligned} \quad (10)$$

The cascade stress-strength system for inverse Lindley distribution can be obtained by substituting distribution function and PDF function in equation (10) to get

$$R_1 = 1 - \int_0^\infty \left(1 + \frac{\alpha}{y(1+\alpha)} \right) e^{-\alpha/y} \frac{\beta^2}{1+\beta} \left(\frac{1+y}{y^3} \right) e^{-\beta/y} dy$$

Implementing some algebraic operations, we get

$$R_1 = 1 - \left(\frac{\beta^2(1+\alpha+\beta)}{(1+\beta)(\alpha+\beta)^2} + \frac{\beta^2\alpha(2+\alpha+\beta)}{(\alpha+\beta)^3(1+\alpha)(1+\beta)} \right) \quad (11)$$

7-Estimation of Reliability for Stress -Strength Model for inverse Lindley

Two methods will be used to obtain an estimation for reliability function of inverse Lindley distribution function for stress-strength system. The first method is the Maximum Likelihood Estimate which is regarded as the most important method of classical estimation. The other method is the Bayes Method.

7-1-The Maximum Likelihood Method

The joint probability density function of strength (x) and stress (y) for inverse Lindley distribution is

$$f(x, y, \alpha, \beta) = \frac{\alpha^2\beta^2}{(1+\alpha)(1+\beta)} \left(\frac{1+x}{x^3} \right) \left(\frac{1+y}{y^3} \right) e^{-\frac{\alpha}{x}-\frac{\beta}{y}} \quad (12)$$

The likelihood function of the joint probability function for inverse Lindley distribution for strength - stress system is

$$L = \frac{\alpha^{2n}\beta^{2m}}{(1+\alpha)^n(1+\beta)^m} \prod_{i=1}^n \left(\frac{1+x_i}{x_i^3} \right) \prod_{j=1}^m \left(\frac{1+y_j}{y_j^3} \right) e^{-\sum_{i=1}^n \frac{\alpha}{x_i} - \sum_{j=1}^m \frac{\beta}{y_j}} \quad (13)$$

The log of likelihood function of equation (13) is

$$\mathcal{L} = 2n \ln(\alpha) + 2m \ln(\beta) - n \ln(1 + \alpha) - m \ln(1 + \beta) + \sum_{i=1}^n \ln\left(\frac{1+x_i}{x_i^3}\right) + \sum_{j=1}^m \ln\left(\frac{1+y_j}{y_j^3}\right) - \alpha \sum_{i=1}^n \frac{1}{x_i} - \beta \sum_{j=1}^m \frac{1}{y_j} \quad (14)$$

To estimate (α) and (β) ; The Maximum likelihood function (14) is derived with respect to parameter as follows

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \frac{2n}{\alpha} - \frac{n}{1 + \alpha} - \sum_{i=1}^n \frac{1}{x_i} \quad (15)$$

$$\frac{\partial \mathcal{L}}{\partial \beta} = \frac{2m}{\beta} - \frac{m}{1 + \beta} - \sum_{j=1}^m \frac{1}{y_j} \quad (16)$$

By equalizing equation (15) to zero; and implementing some algebraic operations we get

$$\hat{\alpha} = \left(\sum_{i=1}^n \frac{1}{x_i}\right)^{-1} \left(n - \sum_{i=1}^n \frac{1}{x_i}\right) \left(n^2 + 6n \sum_{i=1}^n \frac{1}{x_i} + \left(\sum_{i=1}^n \frac{1}{x_i}\right)^2\right)^{1/2} \quad (17)$$

Similarly; Equalization of the equation (16) to zero; and repeated the algebra operation on equation (16) get

$$\hat{\beta} = \left(\sum_{j=1}^m \frac{1}{y_j}\right)^{-1} \left(m - \sum_{j=1}^m \frac{1}{y_j}\right) \left(m^2 + 6m \sum_{j=1}^m \frac{1}{y_j} + \left(\sum_{j=1}^m \frac{1}{y_j}\right)^2\right)^{1/2} \quad (18)$$

7-2-Bayes Method

Bayes method of estimation is based on the principle that the parameters of distribution are random variables following probability distribution.

The most important step in Bayes method is obtaining prior function. Two cases are possible. If information about the parameters is available, the prior function is obtained from the information. But in case of non-information or if the information is rare, non-information prior function is used.

7-2-1-Non-information Bayes method

To estimate the strength- stress reliability when both random variables strength and stress are following the inverse Lindley distribution with parameters $(\alpha; \beta)$ respectively, the non-information Bayes estimation is used. To obtained prior function, the Sinha (5) and keta method will be followed. They suggested Jeffery formal which is independent on Fisher information as follows

$$\pi(\theta) \propto \frac{1}{\theta^c} \quad c > 0; 0 < \theta < \infty$$

$$\pi(\theta) = \frac{k}{\theta^c} \quad c > 0; 0 < \theta < \infty$$

Where k is Proportionality constant.

The prior marginal function for stress parameter is

$$\pi_1(\alpha) = \frac{k}{\alpha^c} \quad c > 0; 0 < \alpha < \infty$$

Also, the prior marginal function for strength is given by

$$\pi_2(\beta) = \frac{k}{\beta^c} \quad c > 0; 0 < \beta < \infty$$

As the two random variables of strength – stress are independent, then the joint prior function will be as

$$\pi(\alpha; \beta) = \left(\frac{1}{\alpha^c \beta^c}\right) \quad (19)$$

The joint Posterior function of $(\alpha; \beta)$ is obtained based on Likelihood function (14) and prior function (19). The joint posterior is given as

$$p(\alpha; \beta/x; y) \propto \left(\frac{1}{\alpha^c \beta^c}\right) \left(\frac{\alpha^{2n} \beta^{2m}}{(1 + \alpha)^n (1 + \beta)^m} \prod_{i=1}^n \left(\frac{1 + x_i}{x_i^3}\right) \prod_{j=1}^m \left(\frac{1 + y}{y^3}\right) e^{-\sum_{i=1}^n \frac{\alpha}{1+x} - \sum_{j=1}^m \frac{\beta}{1+y}}\right) \quad (20)$$

Squared error loss function is assumed to be as follows

$$\mathcal{L}(\hat{\theta} - \theta) \propto (\hat{\theta} - \theta)^2 \quad (21)$$

The posterior expectation of square error loss function is

$$E[\mathcal{L}(\hat{\theta} - \theta)] \propto \int_0^\infty \int_0^\infty \mathcal{L}(\hat{\theta} - \theta) p(\alpha; \beta/x; y) d\alpha d\beta \quad (22)$$

$$E[\mathcal{L}(\hat{\theta} - \theta)/x; y] = \frac{\int_0^\infty \int_0^\infty \mathcal{L}(\hat{\theta} - \theta) p(\alpha; \beta/x; y) d\alpha d\beta}{\int_0^\infty \int_0^\infty p(\alpha; \beta/x; y) d\alpha d\beta} \quad (23)$$

It can be observed that equation (23) cannot be solved by analytical methods. To solve this equation, Lindley approximation will be used.

7-3-Lindley Approximation:

The posterior expectation function equation (23) will be reformulated as the form of ratio of integrals according to Lindley approximation. as

$$E[\mathcal{L}(\hat{\theta} - \theta)/x; y] = \frac{\int w(\theta)L(\theta) d\theta}{\int v(\theta)L(\theta) d\theta} \quad (24)$$

Where $L(\theta)$ represents log of Likelihood function; $w(\theta)$ and $v(\theta)$ are functions of (θ) ; $u(\theta)$ is loss function

$$w(\theta) = u(\theta)v(\theta)$$

For large samples, Lindley suggests approximation for equation (24) as below

$$\hat{\theta} = u(\hat{\theta}) + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m (u_{ij} + 2u_i \rho_j) \hat{\sigma}_{ij} + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \sum_{k=1}^m \sum_{l=1}^m L_{ijkl} u_i \sigma_{ij} \sigma_{kl} \quad (24)$$

When $(m = 2)$, equation (24) simplified as follows

$$\hat{\theta} = u(\hat{\theta}) + \frac{1}{2} (u_{11} \sigma_{11} + u_{22} \sigma_{22}) + u_1 \rho_1 \hat{\sigma}_{11} + u_2 \rho_2 \hat{\sigma}_{22} + \frac{1}{2} (L_{30} u_1 \sigma_{11}^2 + L_{03} u_2 \sigma_{22}^2) \quad (25)$$

When

$$\begin{aligned}
 u(\hat{\alpha}) &= \alpha; \quad u_1 = \frac{\partial u}{\partial \alpha} = 1; \quad u_{11} = \frac{\partial^2 u}{\partial \alpha^2} = 0 \\
 L_{20} &= \frac{\partial^2 L}{\partial \alpha^2} = \frac{n}{(1+\alpha)^2} - \frac{2n}{\alpha^2}; \quad L_{30} = \frac{\partial^3 L}{\partial \alpha^3} = \frac{4n}{\alpha^3} - \frac{2n}{(1+\alpha)^3} \\
 u(\hat{\beta}) &= \beta; \quad u_2 = \frac{\partial u}{\partial \beta} = 1; \quad u_{22} = \frac{\partial^2 u}{\partial \beta^2} = 0 \\
 \sigma_{11} &= (-L_{20})^{-1} = \frac{\alpha^2(1+\alpha)^2}{2n\alpha + 4\alpha^2 + \alpha^3} \\
 L_{20} &= \frac{\partial^2 L}{\partial \beta^2} = \frac{n}{(1+\beta)^2} - \frac{2n}{\beta^2}; \quad L_{03} = \frac{\partial^3 L}{\partial \beta^3} = \frac{4m}{\beta^3} - \frac{2m}{(1+\beta)^3} \\
 \sigma_{22} &= (-L_{03})^{-1} = \frac{\beta^2(1+\beta)^2}{2m\beta + 4\beta^2 + \beta^3} \\
 \rho(\alpha; \beta) &= \ln\left(\frac{1}{\alpha\beta}\right) \\
 \rho_1 &= \frac{\partial \rho}{\partial \alpha} = -\frac{1}{\alpha}; \quad \rho_2 = \frac{\partial \rho}{\partial \beta} = -\frac{1}{\beta}
 \end{aligned}$$

$$\hat{\alpha} = \hat{\alpha}_{ML} - \frac{c\alpha(1+\alpha)^2}{2n\alpha + 4\alpha^2 + \alpha^3} + \frac{1}{2} \left(\left(\frac{4n}{\alpha^3} - \frac{2n}{(1+\alpha)^3} \right) \left(\frac{\alpha^2(1+\alpha)^2}{2n\alpha + 4\alpha^2 + \alpha^3} \right) \right) \quad (25)$$

$$\hat{\beta} = \hat{\alpha}_{ML} - \frac{c\beta(1+\beta)^2}{2m\beta + 4\beta^2 + \beta^3} + \frac{1}{2} \left(\left(\frac{4m}{\beta^3} - \frac{2m}{(1+\beta)^3} \right) \left(\frac{\beta^2(1+\beta)^2}{2m\beta + 4\beta^2 + \beta^3} \right) \right) \quad (26)$$

8- Simulation

Simulation model is used to generate a duplicate copy of the system under study. There is more than a method for simulation. Monte Carlo method is the most common one for this reason this study will be based on this method; we have taken samples sizes which are tabled as in table below:

Table: (1) samples sizes for experimental simulation

	Small samples		Big samples		
n	10	35	50	75	100

While the values for strength and stress parameters are as shown in table

Table (2): values of parameters for experimental simulation

α	0.5	1	2	2
β	0.2	0.5	1	2

Also assumed that attenuation factors for strength and stress equal one. Simulation was executed by MATLAB programming. It was repeated 1000 times. Mean square error (MSE) have been used for compering results of simulation

$$\text{MSE}(\hat{R}) = \frac{1}{L} \sum_{i=1}^L (\hat{R}_i - R)^2 \quad (27)$$

Where L is a value of replications (L=1000); \hat{R}_i is Estimation a Reliability;

R is a Reliability.

9-Results:

Monte Carlo simulation was used to generate samples of different sizes for stress and strength random variables as mentioned in table (1) and default parameters values for studying random behavior of stress-strength systems with variables following inverse distribution as in table (2). Methods of estimation which has been used in this study were compared with each other. Results of simulation were tabled in tables (3) ;(4) ;(5) and (6). The means of estimated values by both maximum likelihood and Bayes methods of reliability were calculated for all default models and for all sizes of samples in addition to MSE for MLM and Byes methods.

9-conclusion:

Simulation results show convergence between MLM and BM for most simulation experiments as well as decrement in MSE values as increasing sizes of samples. For small samples sizes , simulation shows better results for Bayes method, while for big samples sizes MLM was the best.

Table (3): Mean values of reliability and MSE for MLM and Bayes methods with parameters ($\alpha = 0.50; \beta = 0.20$)

n	R MLE	R Bayes	MSE MLE	MSE Bayes	Best
10	0.44761	0.44803	0.05919	0.05719	Bayes
25	0.41902	0.41932	0.04583	0.04582	Bayes
50	0.41578	0.41588	0.0443	0.04432	MLE
75	0.41295	0.41297	0.04311	0.04312	MLE
100	0.41154	0.41155	0.04253	0.04253	MLE

Table (4): Mean values of reliability and MSE for MLM and Bayes methods with parameters ($\alpha = 1.0; \beta = 0.50$)

n	R MLE	R Bayes	MSE MLE	MSE Bayes	Best
10	0.463415	0.433623	0.199848	0.194643	Bayes
25	0.439208	0.431017	0.199680	0.196835	Bayes
50	0.436570	0.430790	0.191959	0.197032	MLE
75	0.434180	0.430527	0.194041	0.197263	MLE
100	0.433357	0.430416	0.194761	0.197360	MLE

Table (5): Mean values of reliability and MSE for MLM and Bayes methods with parameters ($\alpha = 2.0; \beta = 1$)

n	R MLE	R Bayes	MSE MLE	MSE Bayes	Best
10	0.405117	0.381148	0.197212	0.112258	Bayes
25	0.384049	0.377698	0.120266	0.114640	Bayes
50	0.381639	0.377224	0.111850	0.114807	MLE
75	0.379722	0.376733	0.113125	0.115137	MLE
100	0.378717	0.376535	0.113611	0.115228	MLE

Table (6): Mean values of reliability and MSE for MLM and Bayes methods with parameters ($\alpha = 2.0; \beta = 2.0$)

n	R MLE	R Bayes	MSE MLE	MSE Bayes	Best
10	0.321763	0.305741	0.042067	0.037801	Bayes
25	0.305156	0.300899	0.039999	0.039647	Bayes
50	0.303149	0.300162	0.038761	0.039938	MLE
75	0.301736	0.299732	0.039314	0.040109	MLE
100	0.301077	0.299442	0.039573	0.040224	MLE

References

- 1- Al-Mutairi, D.K, Ghitany, M.E. & Kundu, D. (2013) "Inferences on the stress–strength reliability from Lindley distributions" *Comm Stat Theory Methods*, Vol. 42, NO.8, pp1443–1463.
- 2- Ashour, S. K., Eltehiwy, M. A. (2015) "Exponentiated power Lindley distribution. *Journal of advanced research*" Vol.6, No.6, pp 895-905.
- 3- Devi, M.T, Maheswari, T.U. & Swathi, N. (2016) "Cascade System Reliability with Stress and Strength Follow Lindley Distribution" *International Journal of Engineering Research & Technology (IJERT)*; Vol. 5; No. 9, pp 708-714.
- 4- Ghitany, M.E. Atieh, B. & Nadarajah, S. (2008) "Lindley distribution and its application" *Mathematics and Computers in Simulation*, Vol. 78, pp493–506.
- 5- Guure, C.B. Ibrahim, N.A., Adam, M.B. Bosomprah, S., Mohammed, A. (2014) "Bayesian parameter and reliability estimate of Weibull failure time distribution" *Bull. Malays. Math. Sci. Soc. (2)* 37(3), 611–632 (2014).
- 6- Kanaparhi, R. Palakurthi, J. Narayana, L. & Sanka ,G.D. (2019)" Cascade and System Reliability for the New Rayleigh-Pareto Distribution" *International Journal of Science and Research (IJSR)*; Vol.8; Issue;2;pp 1864-1867.

- 7- Sharma, V.K. Singh, S.K. Singh, U. & Agiwal, V. (2014) "The inverse Lindley distribution: a stress-strength reliability model with application to head and neck cancer data" *Journal Of Industrial and Production Engineering*; Vol.32; No.3; pp.162-173.
- 8- Sharma, V.K., Singh, S.K., Singh, U. (2017) "Classical and Bayesian methods of estimation for power Lindley distribution with application to waiting time data" *Commun. Stat. Appl. Methods*, vol. 24, No. 3, pp 193–209.
- 9- Sundar, T.S (2012) "Case Study of Cascade Reliability with Weibull Distribution" *International Journal of Engineering and Innovative Technology*, Vol. 1, No. 6, 2012, pp. 103-110.
- 10-Uma Maheswari, T.S. & Swathi, N. (2013)" "Cascade Reliability for Generalized Exponential Distribution" *International Journal of Computational Engineering Research (ijceronline.com)* Vol. 3; No. 1; pp. 132-136.
- 11-Uma Maheswari, T.S. & Swathi, N. (2013) "Cascade Reliability of Stress-Strength system when Strength follows mixed Exponential distribution" *IOSR Journal of Mathematics (IOSR-JM)*; Vol.4; No.5; pp 27-31