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Methods For Estimating $R_{(S,K)}$ Based On Rayleigh-Pareto Distribution

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ABSTRACT

This paper considers with the reliability of a multicomponent system of k components $R_{(S,K)}$ estimation problem of a stress-strength model. $R_{(S,K)}$ is obtained when the strength and stress variables have the two-parameters Rayleigh-Pareto distribution $RP(\sigma,\rho)$. (σ) is the known scale parameter and (ρ) is an unknown shape parameter for stress - strength distribution of Rayleigh-Pareto. The system contains (K) components with its strength $(Y_1, Y_2, \ldots\ldots, Y_K)$, which represent random variables distributed independently and symmetrically, and each component suffers from random stress is (X). The system regards as active system only if at least strength components exceed the stress. Parameter estimation using Least Squares (LS) , Relative Least Squares RLS , Wight Least Squares (WLS) and Ridge Regression Method (RRM) have discussed. The estimating of reliability parameters obtained from all the approaches above are compared with the Mean Square Error (MSE) and Mean Absolute Percentage Error (MAPE) criteria based on Monte-Carlo simulation experiment. Significantly, WLS and LS estimators have shown better performance compared with other methods.

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1- Introduction*:*

The Stress – Strength system is one of the most important models used frequently in data analysis in different areas such as industrial engineering, military applications, health and applied Sciences[10]. The reliability of the stress- strength system is the evaluation of the reliability of a component in terms of the random variable (X) that represents the stress the component is exposed to, (Y) represents the strength of the component available to overcome the potential stress. The system fails when the stress surpasses the strength. The impression of the reliability of the stress resistance $R = P(Y > X)$ was presented in Birnbaum [1] and settled in Birnbaum and McCarty [2]. Bhattacharyya and Johnson was the first researchers interested in studying and deriving the reliability of the stress strength model due to its practical applications

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[3]. The reliability of the multicomponent stress- strength (S-out-of-K) indicated by $R_{(S,K)}$ was widely recognized by many researchers, as the system operates at a minimum (1≤ S ≤ K) of components, and it works better when (K) components resist common stress. Noted that, when S = 1 and S = K is respectively referring to parallel and series systems. When strength and stress variables for parallel and series systems follow exponential distribution, Pandit and Kantu [7], discussed the estimation of multicomponent stress-strength reliability When K components being independently and identically generalized Pareto distributed random variables, Pandit and Joshi, they arrangements with the estimation of multicomponent system reliability, Abdulateef and Salman, they estimated the reliability of the multicomponent system in stress–strength model for Exponentiated Pareto distribution, using; Maximum Likelihood and Shrinkage methods. Recently, Hassan and Basheikh.[6] They considered estimation of the multicomponent stress – strength system reliability underline Weibull distribution based on upper record values data. In a multi-component stress and strength system estimating the survival probability does not receive much attention from specialists, especially when tracking stress variables - the strength of the Rayleigh-Pareto distribution. Therefore, in this research we estimate $R_{(S,K)}$ based on a two-parameter Rayleigh-Pareto RP(σ, ρ) distribution whereas σ is scale known parameter and ρ is an unknown shape parameter using regression estimation methods such as, LS , RLS , WLS and RRM. The simulation study by Monte Carlo method is conducted to compare the reliability estimates via mean squared error (MSE) and Mean Absolute Percentage Error (MAPE) criteria. The probability density (p. d.f) and cumulative distribution (c. d .f) functions based on Rayleigh-Pareto distribution are known as:

$$d(x;\sigma,\rho) = \frac{\rho}{\sigma^{\rho}} x^{\rho-1} e^{-(\frac{x}{\sigma})^{\rho}} \qquad \text{for } 0 < x < \infty , \rho > 0 , \sigma > 0 \tag{1}$$

and

$$D(x;\, \sigma,\rho) = 1 - e^{-(\frac{x}{\sigma})^{\rho}} \tag{2}$$

where $0 < x < \infty , \rho > 0 , \sigma > 0$, σ is a scale parameter and ρ is a shape parameter.

2- Experimental Aspect of Reliability in multicomponent stress-strength

A multicomponent system of k component having Y strengths such as that $Y_1, Y_2, \ldots\ldots Y_K$ are identity independent Rayleigh-Pareto random variables with parameters (ρ_1,σ) and subject to common stress random variable X which is distributed as Rayleigh-Pareto distribution with parameters (ρ_2,σ). Now, Bhattacharyya and Johnson established the reliability in the model of the multicomponent stress-strength as [3]:

$R_{(S,K)}$=P (at least S of $Y_1, Y_2, \ldots , Y_k$ exceed X)

$$R_{(S,K)} = \sum_{a=s}^{k} C_a^k \int_{-\infty}^{\infty} [1 - D_y(X)]^a D_y(X)^{k-a}\, dD(X) \tag{3}$$

Here, X and Y are independent random variables follow Rayleigh-Pareto distribution with parameters (ρ_1,σ) and (ρ_2,σ) respectively. By unknown shape parameters ρ_1 , ρ_2 and shared known scale parameter τ. The $R_{(S,K)}$ of Rayleigh-Pareto distribution can be obtained equation (3) by:

$$R_{(S,K)} = \sum_{i=s}^{k}\binom{k}{i} \int_0^{\infty} [e^{-(\frac{x}{\sigma})^{\rho_2}}]^i\, [1 - e^{-(\frac{x}{\sigma})^{\rho_2}}]^{k-i}\, \frac{\rho_1}{\sigma^{\rho_1}} x^{\rho_1-1} e^{-(\frac{x}{\sigma})^{\rho_1}} dx$$

$$= \sum_{i=s}^{k}\binom{k}{i} \int_0^{\infty} [e^{-z^{\frac{\rho_2}{\rho_1}}}]^i\, [1 - e^{-z^{\frac{\rho_2}{\rho_1}}}]^{k-i}\, e^{-z} dz$$

$$= \sum_{i=s}^{k}\binom{k}{i} (-1) \int_0^{1} [u^{\frac{\rho_2}{\rho_1}}]^i\, [1 - u^{\frac{\rho_2}{\rho_1}}]^{k-i}\, du$$

$$= \frac{\rho_1}{\rho_2} \sum_{i=s}^{k}\binom{k}{i} (-1) \int_0^{1} [t]^i\, [1 - t]^{k-i}\, t^{\frac{\rho_1}{\rho_2}-1} dt$$

$$= \frac{\rho_1}{\rho_2} \sum_{i=s}^{k}\binom{k}{i} (-1) \int_0^{1} [1 - t]^{k-i}\, t^{i+\frac{\rho_1}{\rho_2}-1} dt$$

$$= \frac{\rho_1}{\rho_2} \sum_{i=s}^{k}\binom{k}{i}(-1)\, B\left(\left(i + \frac{\rho_1}{\rho_2}\right), (k - i + 1)\right),$$

After the simplification we get

$$R_{(S,K)} = \frac{\rho_1}{\rho_2}\sum_{i=s}^{k}\frac{k!}{i!}\left(\prod_{j=i}^{k}(-1)^j(\frac{\rho_1}{\rho_2}+j)\right)^{-1} \quad (4)$$

where $B = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt$, ands, k, i and j are integers.

3- Estimation methods of $R_{(S,K)}$

3-1- Least squares method (L S) . [4]

The probability density (p. d. f.) and cumulative distribution functions (c. d. f.) of the_proposed distribution are given in equation (1) and (2), respectively. Here, we take logarithm of both sides of Eq.(2) as follows: $\ln(e^{-(\frac{x_i}{\sigma})^{\rho}}) = \ln(1-D(x_i))$, then

$$-(\frac{x_i}{\sigma})^{\rho} = ln(1-D(x_i)) \quad (5)$$

We convert the nonlinear equation (5) into a linear equation by taking logarithm for both sides, so that

$$\ln X_i = \ln\sigma + \frac{1}{\rho}\left[ln\left(-ln(1-D(x_i))\right]. \quad (6)$$

Since equation (6) is a linear equation, it is expressed as follows

$$Y_i = a + bX_i\,, \quad i = 1,2,\dots,n \quad (7)$$

where $\mathrm{Y_i = \ln x_i}$, $\mathrm{a = \ln\sigma}$, $\mathrm{b} = \frac{1}{\rho}$, $\mathrm{X_i = [\ln(-\ln(1-D(x_i)]}$ and n the sample size. We obtain the least square estimates (LS) of b as follows:

$$\hat{\rho}_{LS} = \left(\hat{b}_{LS}\right)^{-1} = \frac{n\sum_{i=1}^{n}X_i^2 - (\sum_{i=1}^{n}X_i)^2}{n\sum_{i=1}^{n}X_iY_i - \sum_{i=1}^{n}X_iY_i}$$

Suggested by, Swain et al. [5] for estimating the parameters of beta distributions. But, it can be used for other cases, too. Let $Y_1, Y_2, \dots\dots Y_n$ be a random sample of size n from a c.d.f. $D(Y_i)$. Let $Y_{(i)}$, $i = 1,2,\dots,n$ indicate denote the order statistics of the $Y_1, Y_2, \dots\dots Y_n$. In the suggested technique procedures the c.d.f. of $Y_{(i)}$ is estimated and replaced by the median rank method as follows:

$$D(Y_i) = \frac{(\mathrm{i}-0.3)}{(\mathrm{n}+0.4)} \qquad ,\mathrm{i} = 1,2,\dots,\mathrm{n} \text{ and } (Y_1 < Y_2 \dots\dots < Y_n\,)$$

Because $D(Y_i)$of the mean rank method

$$E\big(D(Y_i)\big) = \frac{\mathrm{i}}{(\mathrm{n}+1)} = L_i\,,$$

where L_i the plotting position. The LS estimator of ρ_{1LS} and ρ_{2LS} , say $\hat{\rho}_{1LS}$, and $\hat{\rho}_{2LS}$ take the following forms

$$\hat{\rho}_{1\mathrm{LS}} = \frac{n\sum_{i=1}^{n}(ln(-ln(1-D(Y_i))))^2 - (\sum_{i=1}^{n}ln(-ln(1-D(Y_i)))^2}{n\sum_{i=1}^{n}ln(-ln(1-D(Y_i)))\,ln\,Y_i - \sum_{i=1}^{n}ln(-ln(1-D(Y_i)))\,ln\,Y_i}, \quad (8)$$

and

$$\hat{\rho}_{2\mathrm{LS}} = \frac{n\sum_{j=1}^{m}\left(ln(-ln(1-D(X_j)))\right)^2 - (\sum_{j=1}^{m}ln(-ln(1-D(X_j)))^2}{n\sum_{j=1}^{m}ln(-ln(1-D(X_j)))\,ln\,X_j - \sum_{j=1}^{m}ln(-ln(1-D(X_j)))\,ln\,X_j}. \quad (9)$$

For i= 1,2,…,n and j=1,2,…, m. We substitute equation (8) and (9) into equation (4) to find $R_{(S,K)}$, as

$$\hat{\mathrm{R}}_{(\mathrm{S,K})} = \frac{\hat{\rho}_{1\mathrm{LS}}}{\hat{\rho}_{2\mathrm{LS}}}\sum_{i=s}^{k}\frac{k!}{i!}\left(\prod_{j=i}^{k}(-1)^j(\frac{\hat{\rho}_{1\mathrm{LS}}}{\hat{\rho}_{2\mathrm{LS}}}+j)\right)^{-1}, \quad (10)$$

3-2-Relative Least Squares method (RLS) . [12]

The relative least squares estimators of a and b can be obtained by minimizing the sum of squares $\sum_{i=1}^{n}\left(\frac{Y_i - a - bX_i}{Y_i}\right)^2$ of the relative residuals with respect to a and b, Hence we have

$$\hat{\rho}_{RLS} = \left(\hat{b}_{RLS}\right)^{-1} = \frac{(\sum_{i=1}^{n}W_iZ_i\,)^2 - \sum_{i=1}^{n}W_i^2{}_iZ_i}{\sum_{i=1}^{n}W_iZ_i\sum_{i=1}^{n}W_i - \sum_{i=1}^{n}W_i^2Z_i^2}. \quad (11)$$

where $W_i = \frac{1}{\ln X_i}$,and $Z_i = \frac{\ln(-\ln(1-D(X_i)))}{\ln X_i}$. Then the RLS estimator of ρ_{1RLS} and ρ_{2RLS} , say $\hat{\rho}_{1RLS}$, and $\hat{\rho}_{2RLS}$ take the following forms

$$\hat{\rho}_{1RLS}=\frac{\left(\sum_{i=1}^{n}\frac{1}{\ln Y_i}\frac{\ln(-\ln(1-D(Y_i))}{\ln Y_i}\right)^2-\sum_{i=1}^{n}\left(\frac{1}{\ln Y_i}\right)^2\frac{\ln(-\ln(1-D(Y_i))}{\ln Y_i}}{\sum_{i=1}^{n}\frac{1}{\ln Y_i}\frac{\ln(-\ln(1-D(Y_i))}{\ln Y_i}\sum_{i=1}^{n}\frac{1}{\ln Y_i}-\sum_{i=1}^{n}\left(\frac{1}{\ln Y_i}\right)^2\left(\frac{\ln(-\ln(1-D(Y_i))}{\ln Y_i}\right)^2}, \tag{12}$$

and

$$\hat{\rho}_{2RLS}=\frac{\left(\sum_{j=1}^{m}\frac{1}{\ln X_j}\frac{\ln\left(-\ln(1-D(X_j)\right)}{\ln X_j}\right)^2-\sum_{j=1}^{m}\left(\frac{1}{\ln X_j}\right)^2\frac{\ln\left(-\ln(1-D(X_j)\right)}{\ln X_j}}{\sum_{j=1}^{m}\frac{1}{\ln X_j}\frac{\ln\left(-\ln(1-D(X_j)\right)}{\ln X_j}\sum_{j=1}^{m}\frac{1}{\ln X_j}-\sum_{j=1}^{m}\left(\frac{1}{\ln X_j}\right)^2\left(\frac{\ln\left(-\ln(1-D(X_j)\right)}{\ln X_j}\right)^2}, \tag{13}$$

We substitute Equation (12) and (13) into Equation (4), we get

$$\hat{R}_{(S,K)}=\frac{\hat{\rho}_{1RLS}}{\hat{\rho}_{2RLS}}\sum_{i=s}^{k}\frac{k!}{i!}\left(\prod_{j=i}^{k}(-1)^j\left(\frac{\hat{\rho}_{1RLS}}{\hat{\rho}_{2RLS}}+j\right)\right)^{-1} \tag{14}$$

3-3- Weighted Least Square method (WLS). [8]

The weighted least squares estimators can be obtained by minimizing the following equation

$$\sum_{i=1}^{n}W_i[D(Y_i)-E(D(Y_i))]^2, \tag{15}$$

with respect to the unknown parameters .Note that $E\left(D(Y_i)\right)=\frac{i}{n+1}$, and $W_i=\frac{1}{var[D(Y_i)]}=\frac{(n+1)^2(n+2)}{i(n-i+1)}$, $i=1,2,\dots n$. In our case, the WLS estimator of ρ_{1WLS} and ρ_{2WLS} , say $\hat{\rho}_{1WLS}$, and $\hat{\rho}_{2WLS}$ take the following forms (when σ is known) :

$$\hat{\rho}_{WLS}=\left(\hat{b}_{WLS}\right)^{-1}=\frac{\sum_i^n W_i\sum_i^n x_i^2W_i-\sum_i^n X_iW_I}{\sum_i^n X_iY_iW_i\sum_i^n W_i-\sum_i^n X_iW_i\sum_i^n Y_iW_i}. \tag{16}$$

Then,

$$\hat{\rho}_{1WLS}=\frac{\sum_{i=1}^{n}\frac{1}{\ln Y_i}\sum_{i=1}^{n}\left(\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\right)^2\frac{1}{\ln Y_i}-\sum_{i=1}^{n}\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\frac{1}{\ln Y_i}}{\sum_{i=1}^{n}\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\ln Y_i\frac{1}{\ln Y_i}\sum_{i=1}^{n}\frac{1}{\ln Y_i}-\sum_{i=1}^{n}\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\frac{1}{\ln Y_i}\sum_{i=1}^{n}\ln Y_i\frac{1}{\ln Y_i}}. \tag{17}$$

and

$$\hat{\rho}_{2WLS}=\frac{\sum_{j=1}^{m}\frac{1}{\ln X_j}\sum_{j=1}^{m}\left(\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\right)^2\frac{1}{\ln X_j}-\sum_{j=1}^{m}\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\frac{1}{\ln X_j}}{\sum_{j=1}^{m}\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\ln X_j\frac{1}{\ln X_j}\sum_{j=1}^{m}\frac{1}{\ln X_j}-\sum_{j=1}^{m}\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\frac{1}{\ln X_j}\sum_{j=1}^{m}\ln X_j\frac{1}{\ln X_j}} \tag{18}$$

We substitute Equation (17), and (18) into Equation (4) to find $R_{(S,K)}$,

$$\hat{R}_{(S,K)}=\frac{\hat{\rho}_{1WLS}}{\hat{\rho}_{2WLS}}\sum_{i=s}^{k}\frac{k!}{i!}\left(\prod_{j=i}^{k}(-1)^j\left(\frac{\hat{\rho}_{1WLS}}{\hat{\rho}_{2WLS}}+j\right)\right)^{-1} \tag{19}$$

3-4- Ridge Regression method (RRM) . [5]

In 1978, Ronald and Raymond introduced the ridge regression estimators as $\rho_{ridg}=(X'X+GI)^{-1}X'Y$,where (0 < G < 1) is the biased ridge factor, I is the W*W identity matrix and W is the number of parameters

$$X'X=\begin{bmatrix} n & \sum_{i=1}^{n}x_i \\ \sum_{i=1}^{n}x_i & \sum_{i=1}^{n}x_i^2\end{bmatrix} \quad \text{and} \quad X'Y=\begin{bmatrix}\sum_{i=1}^{n}y_i \\ \sum_{i=1}^{n}y_ix_i\end{bmatrix},$$

$$\rho_{RRM}=\begin{bmatrix}a_{RRM}\\ b_{RRM}\end{bmatrix}=\begin{bmatrix} n+K & \sum_{i=1}^{n}x_i \\ \sum_{i=1}^{n}x_i & \sum_{i=1}^{n}x_i^2+K\end{bmatrix}^{-1}\begin{bmatrix}\sum_{i=1}^{n}y_i \\ \sum_{i=1}^{n}y_ix_i\end{bmatrix} \quad \text{for } K>0$$

The estimated value of b_{RRM} after some simplifications can be written as

$$\hat{\rho}_{RRM}=\left(\hat{b}_{RRM}\right)^{-1}=\frac{(n+K)[\sum_{i=1}^{n}x_i^2+K]-\left(\sum_{i=1}^{n}x_i\right)^2}{(n+k)[\sum_{i=1}^{n}y_ix_i]-\sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}. \tag{20}$$

The ridge regression estimator for the parameter unknown strength $\hat{\rho}_{iRRM}$ and the known stress parameter can be formulated as follows ,

$$\hat{\rho}_{1RRM}=\frac{(n+K)\left[\sum_{i=1}^{n}\left(\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\right)^2+K\right]-\left(\sum_{i=1}^{n}\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\right)^2}{(n+k)\left[\sum_{i=1}^{n}\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\ln Y_i\right]-\sum_{i=1}^{n}\frac{\ln(-\ln(1-D(Y_i)}{\ln Y_i}\sum_{i=1}^{n}\ln Y_i},$$

and

$$\hat{\rho}_{2RRM}=\frac{(n+K)\left[\sum_{j=1}^{m}\left(\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\right)^2+K\right]-\left(\sum_{j=1}^{m}\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\right)^2}{(n+k)\left[\sum_{j=1}^{m}\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\ln X_j\right]-\sum_{j=1}^{m}\frac{\ln(-\ln(1-D(X_j)}{\ln X_j}\sum_{j=1}^{m}\ln X_j} \tag{21}$$

with $G=e^{-\left(\frac{n}{m}\right)}$ and n , m are sample sizes. We substitute (21) into Equation (4) to find

$$\widehat{R}_{(S,K)} = \frac{\widehat{\rho}_{1RRM}}{\widehat{\rho}_{2RRM}} \sum_{i=s}^{k} \frac{k!}{i!} \left(\prod_{j=i}^{k} (-1)^{j} \left(\frac{\widehat{\rho}_{1RRM}}{\widehat{\rho}_{2RRM}} + j \right) \right)^{-1}, \tag{22}$$

4- Numerical Results

Here, in order to compare the performance of estimating the reliability of multicomponent stress strength a simulation study is conducted. The samples are generated from Rayleigh – Pareto distribution using inverse transformation method and LS , WLS, RLS, and RRM methods are used to compute the proposed estimators. The assessment of the estimates are completed through the Mean Square Error (MSE) and Mean Absolute Percentage Error (MAPE) criteria. Subject to computer time restrictions, we do comparison by taking sample sizes for n and m as 25, 50, 75 , and 100 with pair of (s, k) = (2, 4), (3,5) for the strength and stress population. The results are based on 7000 replication turns. The true values of $(\sigma, \rho_{1,}\rho_2)$ are =(1.2, 3, 1.8), =(1.2, 1.7, 2.3), =(1.2, 1.5, 1.5) are used for assessment. The true values of s – out – of – k system with (s, k) = (2, 4) are 0.45378 , 0.67719, 0.60000 while, for (s, k) = (3, 5) are for 0.34034, 0.58998, 0.50000 for $R_{(S,K)}$. Results of Mean, MSE and MAPE values of the reliability estimates over 7000 replications where $\mathrm{MSE}(\widehat{R}) = \frac{1}{q}\sum_{i=1}^{q}(\widehat{R}_i - R)^2$ and $\mathrm{MAPE}(\widehat{R}) = \frac{1}{q}\sum_{i=1}^{q}\frac{|(\widehat{R}_i - R)|}{|R|}$, are given in Table[1-3]. From the Tables (1) , (2) and (3) below, it is observed the MSE and MAPE values for estimates shrinkages as the sample size rises in all the estimator methods. Then, when (s, k) = (2, 4), the best MSE and MAPE value is for WLS estimator. However, when (s, k) = (3, 5), the best MSE and MAPE value is for RLS estimator.

Table (1): The Means, MSEs and MAPEs for the estimators of $R_{(S,K)}$. $R_{(2,4)} = 0{,}24828$ and $R_{(3,5)} = 0.21849$ when $\sigma = 1.2,\ \rho_1 = 3, \rho_2 = 1.8$. and q=7000

Method / m,n	(S , K)		LS	WLS	RLs	RRM	Best
(25,25)	(2,4)	Mean	0.2406	0.2399	0.26815	0.24077	-
		MSE	8.4205e-09	1.0039e-08	5.6418e-08	8.0627e-09	RG
		MABE	4.4175e-06	4.8234e-06	1.1434e-05	4.3226e-06	RG
(50,50)	(2,4)	Mean	0.2715	0.28334	0.26057	0.27167	-
		MSE	7.7042e-08	1.7558e-07	2.1564e-08	7.8142e-08	RL
		MABE	1.3362e-05	2.0172e-05	7.0692e-06	1.3457e-05	RL
(75,75)	(2,4)	Mean	0.24923	0.24699	0.20065	0.24934	-
		MSE	1.2982e-10	2.3772e-10	3.2409e-07	1.6014e-10	LS
		MABE	5.485e-07	7.4224e-07	2.7406e-05	6.0919e-07	LS
(100,100)	(2,4)	Mean	0.27445	0.28057	0.24581	0.27448	-
		MSE	9.7859e-08	1.4896e-07	8.7371e-10	9.8068e-08	RL
		MABE	1.5059e-05	1.858e-05	1.423e-06	1.5075e-05	RL
(25,25)	(3,5)	Mean	0.21349	0.22228	0.24259	0.21334	-
		MSE	3.5635e-09	2.0552e-09	8.3019e-08	3.7851e-09	LS
		MABE	3.2656e-06	2.48e-06	1.5762e-05	3.3656e-06	LS
(50,50)	(3,5)	Mean	0.22633	0.21548	0.29896	0.22636	-
		MSE	8.7897e-09	1.2893e-09	9.2511e-07	8.8484e-09	WLS
		MABE	5.1288e-06	1.9643e-06	5.2616e-05	5.1458e-06	WLS
(75,75)	(3,5)	Mean	0.22946	0.22399	0.27095	0.22956	-
		MSE	1.7204e-08	4.3333e-09	3.9321e-07	1.7516e-08	WLS

		MABE	7.1752e-06	3.6011e-06	3.4303e-05	7.2401e-06	WLS
(100,100)	(3,5)	Mean	0.21859	0.22567	0.55106	0.2186	-
		MSE	1.4409e-12	7.375e-09	1.58e-05	1.6641e-12	LS
		MABE	6.5666e-08	4.6979e-06	0.00021745	7.0569e-08	LS

Table (2): The Means, MSEs and MAPEs for the estimators of $R_{(S,K)}$. $R_{(2,4)} = 0.171785$ and $R_{(3,5)} ==$ 0.13827when $\sigma = 1.2,\ \rho_1 = 1.7, \rho_2 = 2.3.$ and q=7000

Method / m,n	(S , K)		LS	WLS	RLS	RRM	Best
(25,25)	(2,4)	Mean	0.14473	0.14542	0.14997	0.14452	-
		MSE	1.0508e-07	9.9829e-08	6.838e-08	1.0675e-07	RL
		MABE	2.2545e-05	2.1975e-05	1.8187e-05	2.2724e-05	RL
(50,50)	(2,4)	Mean	0.17295	0.17414	0.13442	0.17298	-
		MSE	1.7149e-10	7.4674e-10	2.0014e-07	1.8253e-10	LS
		MABE	9.1079e-07	1.9005e-06	3.1114e-05	9.3965e-07	LS
(75,75)	(2,4)	Mean	0.15716	0.15606	0.10205	0.15716	-
		MSE	3.083e-08	3.563e-08	6.9614e-07	3.085e-08	LS
		MABE	1.2212e-05	1.3128e-05	5.8028e-05	1.2216e-05	LS
(100,100)	(2,4)	Mean	0.14357	0.14614	0.17135	0.14354	-
		MSE	1.1431e-07	9.4457e-08	3.6343e-11	1.1453e-07	RL
		MABE	2.3514e-05	2.1375e-05	4.1928e-07	2.3537e-05	RL
(25,25)	(3,5)	Mean	0.19262	0.21233	0.4633	0.19265	-
		MSE	4.2197e-07	7.835e-07	1.5092e-05	4.2251e-07	LS
		MABE	5.6153e-05	7.6516e-05	0.00033582	5.6189e-05	LS
(50,50)	(3,5)	Mean	0.13874	0.13695	0.16086	0.13875	-
		MSE	3.2407e-11	2.4752e-10	7.2929e-08	3.3574e-11	LS
		MABE	4.921e-07	1.36e-06	2.3344e-05	5.0088e-07	LS
(75,75)	(3,5)	Mean	0.13784	0.13333	0.1432	0.13779	-
		MSE	2.5628e-11	3.4775e-09	3.482e-09	3.2764e-11	LS
		MABE	4.3761e-07	5.0976e-06	5.1009e-06	4.948e-07	LS
(100,100)	(3,5)	Mean	0.11825	0.10931	0.27518	0.11824	-
		MSE	5.7247e-08	1.1979e-07	2.678e-06	5.7311e-08	LS
		MABE	2.0683e-05	2.9919e-05	0.00014146	2.0694e-05	LS

. **Table (3)**: The Means, MSEs and MAPEs for the estimators of $R_{(S,K)}$. $R_{(2,4)} = 0.20000,$ and $R_{(3\ ,5)} = 0.16667$when $\sigma = 1.2,\ \rho_1 = 1.5, \rho_2 = 1.5$ and q=7000

Method / m,n	(S , K)		LS	WLS	RLS	RRM	Best
(25,25)	(2,4)	Mean	0.19231	0.19561	0.23748	0.19236	-
		MSE	8.4586e-09	2.7493e-09	2.0063e-07	8.3309e-09	WLS
		MABE	5.4963e-06	3.1335e-06	2.6768e-05	5.4546e-06	WLS
(50,50)	(2,4)	Mean	0.1923	0.1962	0.37693	0.19223	-
		MSE	8.4684e-09	2.0629e-09	4.472e-06	8.6292e-09	WLS
		MABE	5.4995e-06	2.7143e-06	0.0001268	5.5514e-06	WLS
(75,75)	(2,4)	Mean	0.19412	0.18855	0.1134	0.1941	-
		MSE	4.9401e-09	1.8725e-08	1.0713e-06	4.979e-09	LS
		MABE	4.2004e-06	8.1776e-06	6.1855e-05	4.2169e-06	LS

(100,100)	(2,4)	Mean	0.18298	0.17995	0.16405	0.18296	-
		MSE	4.137e-08	5.7419e-08	1.8458e-07	4.1469e-08	LS
		MABE	1.2155e-05	1.432e-05	2.5675e-05	1.217e-05	LS
(25,25)	(3,5)	Mean	0.16093	0.1576	0.21548	0.16077	-
		MSE	4.6993e-09	1.1731e-08	3.4041e-07	4.9739e-09	LS
		MABE	4.9161e-06	7.7674e-06	4.1841e-05	5.0577e-06	LS
(50,50)	(3,5)	Mean	0.15379	0.14549	0.11862	0.15377	-
		MSE	2.3693e-08	6.4082e-08	3.2979e-07	2.3762e-08	LS
		MABE	1.1039e-05	1.8154e-05	4.1183e-05	1.1055e-05	LS
(75,75)	(3,5)	Mean	0.17292	0.16067	0.15697	0.17292	-
		MSE	5.5798e-09	5.141e-09	1.3444e-08	5.5934e-09	WLS
		MABE	5.3569e-06	5.1419e-06	8.3151e-06	5.3634e-06	WLS
(100,100)	(3,5)	Mean	0.16533	0.16585	0.27386	0.16533	-
		MSE	2.554e-10	9.6426e-11	1.6413e-06	2.5543e-10	WLS
		MABE	1.1461e-06	7.042e-07	9.1876e-05	1.1461e-06	WLS

5-Conclusions

The results are recorded in Table (1), (2)and (3). From these tables, we see that the estimated reliability using WLS and LS are better in many cases than others methods based MSE and MAPE criteria in mutually circumstances of (S, K).

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