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Some Topological Indices and (Hosoya and Schultz) Polynomial of Subgroup intersection graph of a group Z_r

Alaa J.Nawaf [(a)] [*] ***, Akram S.Mohammad*** [(b)]

[a] *Mathematics Department, College of Computing and Mathematics , Tikrit University, Tikrit, Iraq, e-mail: alagamilnawaf@gmail.com .*

[b] *Mathematics Department, College of Computing and Mathematics , Tikrit University, Tikrit, Iraq, e-mail: akr-tel@tu.edu.iq .*

alaa,gamil,nawaf@gmail.com

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ABSTRACT

Let Z_r be any group with identity element (e) . A subgroup intersection graph of a group Z_r is the Graph with V (G) = Z_r- e and two separate a vertices a and b contiguous for a and b if and only if $|\langle a\rangle \cap \langle b\rangle| > 1$, Where $\langle a\rangle$ is a cyclic subgroup of Z_r generated by $a \in Z_r$. We found some topological indices in this paper and (Hosoya and Schultz) Polynomial of $\Gamma_{SI}(Z_r)$, where $r \geq 3$, r is a prime number.

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1.Introduction

The number associated with the graph is the topological indicator of the finite graph, and under automatic morphology this number is constant [1]. We often referred to such a theoretical graph by a descriptor [2,3,4] or descriptor of the molecular structure of the graph [10], topological indicator. In biology and chemistry, many topological indicators have been used to solve problems.

*Corresponding author: ***Alaa J Nawaf***.

Email addresses : alaa_gamil_nawaf@gmail.com .

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In this paper we study some topological indices depending on the degree of the examples Eccentric connectivity index[5] , connectivity index[6] ,Sum connecyivity index[7 − 10], Zagreb index[11 − 16], forgotten index[17 − 20] , The index of geometric-arithmetic [21 − 23], Index of Atom-Bond Connectivity [21], Harmonic index[24 − 27] .

One graphical concept was obtained from the group is the concept of a subgroup intersection graph of a group that was introduced by M. Aschbacher [28].Referring to the definition of subgroup intersection graph of a group by T. Tamizh Chelvam and M. Sattanathan [28], let G be a group the subgroup intersection graph $\Gamma_{SI}(G)$ of G is a graph with $V(\Gamma_{SI}(G)) = G - e$ and two distinct vertices a and b are adjacent in $\Gamma_{SI}(G)$ if $|\langle a\rangle \cap \langle b\rangle| > 1$, Where $\langle a\rangle$ is a cyclic subgroup of G generated from $a \in G$.

In (2020) Mohammed. S [29] studied some properties for topological indices of ring, also in (2019) Abdussakir [30] introduced topological indices about symmetric group graph , also in (2020) G. R. Roshini [31] studied topological indices of transformation graphs ,also in (2020) Chang- Cheng Wei [32] introduced some topological indices of graphs associated.

The search for the topological index was initially related to the graph of biological activity or chemical composition and reactivity, and research in this regard continued. On the other hand, many studies began to examine the topological index of graphs that are not from chemical composition, interaction, or biological. When many researchers introduced new concepts about graphs obtained from an algebraic structure, the search for topological indicators on these graphs began in Emerge.

2. Method and Materials

In the present article, all the graphs are simple, finite, connected, and un directed . For $G = (V(G), E(G))$ graph, the order of G is $p(G) = V(G)$ and the scale of G is $q(G) = E(G)$. Let $deg(u)$ denote the degree of vertex u in G. If $deg(u) = 0$ then u is an isolated vertex. If $deg(u) = 1$ is an end vertex, then u is an end vertex . Let $d(u,v)$ indicate the distance between vertices u and v in G. The eccentricity $e(u)$ of the vertex u is $e(u) = sup\{d(u,v): v \in V(G)\}$.

The following definition refers to a graph $G = (V(G), E(G))$.

Eccentric connectivity index of G is [5] $\xi^C(G))) = \sum_{u\in V(G)} deg(u).e(u)$

The index of connectivity of G is[6] $X(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{deg(u).deg(v)}}$

The summation of the index connectivity of G is [7] $S(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{deg(u)+deg(v)}}$

A first Zagreb index of G is [11] $M_1(G) = \sum_{u\in V(G)}(deg(u))^2$

A second zagreb index of G is [11] $M_2(G) = \sum_{uv\, EG)} deg(u).deg(v)$

The forgotten index of G is [20] $F(G)\,) = \sum_{u\in V(G)}(deg(u))^3$

Atom Bond connectivity index of G is [21] $ABC(G)) = \sum_{uv\in E(G)} \sqrt{\frac{deg(u)+deg(v)-2}{deg(u).deg(v)}}$

Geometric-Arithmetic index of G is [21] $GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{deg(u).deg(v)}}{deg(u)+deg(v)}$

Harmonic index of G is [24] $H(G) = \sum_{uv\in E(G)} \frac{2}{deg(u)+deg(v)}$

3. The main result

For a good outlook $r \geq 3$, a subgroup intersection graph of a group Z_r is $(\Gamma_{SI}(Z_r))$ is a graph with $(\Gamma_{SI}(Z_r)) = Z_r - e$, any two different vertices a and b are adjacent iff $|\langle a\rangle \cap \langle b\rangle| > 1$, where $\langle a\rangle$ is the subgroup generated by $a \in Z_r$

Theorem 3.1

Let Z_r be a group with $r \geq 3$, r *is a prime number* then the Eccentric connectivity index of $\Gamma_{SI}(Z_r)$ is $\xi^c(\Gamma_{SI}(Z_r)) = r^2 - 3r + 2$

Proof:

$deg(u) = r - 2 \ , \forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\dots,r-1$

$e(u) = 1 \ , \forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\dots,r-1$

$$\xi^C(\Gamma_{SI}(Z_r))) = \sum_{u\in V(\Gamma_{SI}(Z_r))} deg(u).e(u)$$

$$= \underbrace{deg(1).e(1) + \cdots + deg(r-1).e(r-1)}_{(r-1)times}$$

$=(r-1)(r-2)$

$= r^2 - 3r + 2$

Theorem 3.2

Let Z_r be a group with $r \geq 3$, r is a prime number then the connectivity index of $\Gamma_{SI}(Z_r)$ is $X(\Gamma_{SI}((Z_r)) = 1 + \frac{\sum_{i=3}^{r-1}(r-i)}{(r-2)}$

Proof:

$deg(u) = r - 2 \ , \forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\dots,r-1$

$$X(\Gamma_{SI}(Z_r)) = \sum_{uv\in E(\Gamma_{SI}(Z_r))} \frac{1}{\sqrt{deg(u).deg(v)}}$$

$$= \underbrace{\frac{1}{\sqrt{deg(1).deg(2)}} + \cdots + \frac{1}{\sqrt{deg(1).deg(r-1)}}}_{(r-2)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(2).deg(3)}} + \cdots + \frac{1}{\sqrt{deg(2).deg(r-1)}}}_{(r-3)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(3).deg(4)}} + \cdots + \frac{1}{\sqrt{deg(3).deg(r-1)}}}_{(r-4)times}$$

$$+ \cdots + \frac{1}{\sqrt{deg(r-2).deg(r-1)}}$$

$$= \frac{(r-2)}{(r-2)} + \frac{(r-3)}{(r-2)} + \frac{(r-4)}{(r-2)} + \cdots + \frac{1}{(r-2)}$$

$$= 1 + \frac{\sum_{i=3}^{r-1}(r-i)}{(r-2)}$$

Theorem 3.3

Let Z_r be a group with $r \geq 3$, r is a prime number then the Summation connectivity index of $\Gamma_{SI}(Z_r)$ is S $(\Gamma_{SI}(Z_r)) = \frac{\sum_{i=2}^{r-1}(r-i)}{\sqrt{2r-4}}$

Proof:

$$deg(u) = r-2 \ , \forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\ldots,r-1$$

$$S(\Gamma_{SI}(Z_r)) = \sum_{uv \in E(\Gamma_{SI}(Z_r))} \frac{1}{\sqrt{deg(u)+deg(v)}}$$

$$= \underbrace{\frac{1}{\sqrt{deg(1)+deg(2)}} + \cdots + \frac{1}{\sqrt{deg(1)+deg(r-1)}}}_{(r-2)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(2)+deg(3)}} + \cdots + \frac{1}{\sqrt{deg(2)+deg(r-1)}}}_{(r-3)times}$$

$$+ \underbrace{\frac{1}{\sqrt{deg(3)+deg(4)}} + \cdots + \frac{1}{\sqrt{deg(3)+deg(r-1)}}}_{(r-4)times}$$

$$+\cdots+\frac{1}{\sqrt{deg(r-2)+deg(r-1)}}$$

$$=\frac{(r-2)}{\sqrt{2r-4}}+\frac{(r-3)}{\sqrt{2r-4}}+\frac{(r-4)}{\sqrt{2r-4}}+\cdots+\frac{1}{\sqrt{2r-4}}$$

$$=\frac{\sum_{i=2}^{r-1}(r-i)}{\sqrt{2r-4}}$$

Theorem 3.4

Let Z_r be a group with $r \geq 3$, r is a prime number then the first zagreb index of $\Gamma_{SI}(Z_r)$ is $M_1(\Gamma_{SI}(Z_r))=(r-1)(r-2)^2$

Proof:

$deg(u) = r-2 \ , \forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\ldots,r-1$

$$M_1(\Gamma_{SI}(Z_r)) = \sum_{u\in V(\Gamma_{SI}(Z_r))}(deg(u))^2$$

$$=\underbrace{(deg(1))^2+\cdots+(deg(r-1))^2}_{(r-1)times}$$

$$=(r-1)(r-2)^2$$

Theorem 3.5

Let Z_r be a group with $r \geq 3$, r is a prime number then the second zagreb index of $\Gamma_{SI}(Z_r)$ is $M_2(\Gamma_{SI}(Z_r)) = (r-2)^3 + (r-2)^2\sum_{i=3}^{r-1}(r-i)$

Proof:

$deg(u) = r-2 \ , \forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\ldots,r-1$

$$M_2(\Gamma_{SI}(Z_r)) = \sum_{uv\ E(\Gamma_{SI}(Z_r))} deg(u).deg(v)$$

$$= \underbrace{deg(1)\,.\deg(2) + \cdots + deg(1)\,.\deg(r-1)}_{(r-2)times} + \underbrace{deg(2)\,.deg(3) + \cdots + deg(2)\,.deg(r-1)}_{(r-3)\text{times}} +$$

$$\underbrace{deg(3)\,.deg(4) + \cdots + deg(3)\,.deg(r-1)}_{(r-4)times} + \cdots + deg(r-2)\,.deg(r-1)$$

$$= (r-2)^3 + (r-3)(r-2)^2 + (r-4)(r-2)^2 + \cdots + (r-2)^2$$

$$= (r-2)^3 + (r-2)^2 \sum_{i=3}^{r-1} (r-i)$$

Theorem 3.6

Let Z_r be a group with $r \geq 3$, r is a prime number then the forgotten index of $\Gamma_{SI}(Z_r)$ is F $(\Gamma_{SI}(Z_r)) = (r-1)(r-2)^3$

Proof:

$deg(u) = r-2$, $\forall u \in V(\Gamma_{SI}(Z_r))$, $u = 1,2,\ldots,r-1$

$$F(\Gamma_{SI}(Z_r)) = \sum_{u \in V(\Gamma_{SI}(Z_r))} (deg(u))^3$$

$$= \underbrace{(deg(1))^3 + \cdots + (deg(r-1))^3}_{(r-1)\ times}$$

$$= (r-1)(r-2)^3$$

Theorem 3.7

Let Z_r be a group with $r \geq 3$, r is a prime number then the Atom Bond connectivity index of $\Gamma_{SI}(Z_r)$ is $ABC(\Gamma_{SI}(Z_r))) = \sqrt{2r-6} + \frac{\sqrt{2r-6}\sum_{i=3}^{r-1}(r-i)}{(r-2)}$

Proof:

$deg(u) = r-2$, $\forall u \in V(\Gamma_{SI}(Z_r))$, $u = 1,2,\ldots,r-1$

$$ABC(\Gamma_{SI}(Z_r)) = \sum_{uv\in E(\Gamma_{SI}(Z_r))} \sqrt{\frac{deg(u)+deg(v)-2}{deg(u)\,.\,deg(v)}}$$

$$= \underbrace{\sqrt{\frac{deg(1)+deg(2)-2}{deg(1)\,.\,deg(2)}} + \cdots + \sqrt{\frac{deg(1)+deg(r-1)-2}{deg(1)\,.\,deg(r-1)}}}_{(r-2)times}$$

$$+ \underbrace{\sqrt{\frac{deg(2)+deg(3)-2}{deg(2)\,.\,deg(3)}} + \cdots + \sqrt{\frac{deg(2)+deg(r-1)-2}{deg(2)\,.\,deg(r-1)}}}_{(r-3)times}$$

$$+ \underbrace{\sqrt{\frac{deg(3)+deg(4)-2}{deg(3)\,.\,deg(4)}} + \cdots + \sqrt{\frac{deg(3)+deg(r-1)-2}{deg(3)\,.\,deg(r-1)}}}_{(r-4)times} + \cdots$$

$$+ \sqrt{\frac{deg(r-2)+deg(r-1)-2}{deg(r-2)\,.\,deg(r-1)}}$$

$$= \sqrt{2r-6} + \frac{(r-3)\sqrt{2r-6}}{(r-2)} + \frac{(r-4)\sqrt{2r-6}}{(r-2)} + \cdots + \frac{\sqrt{2r-6}}{(r-2)}$$

$$= \sqrt{2r-6} + \frac{\sqrt{2r-6}\sum_{i=3}^{r-1}(r-i)}{(r-2)}$$

Theorem 3.8

Let Z_r be a group with $r \geq 3$, r is a prime number then the Geometric –Arithmetic index of $\Gamma_{SI}(Z_r)$ is $GA(\Gamma_{SI}(Z_r)) = (r-2) + \sum_{i=3}^{r-1}(r-i)$

Proof:

$$deg(u) = r-2 \ , \forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\ldots,r-1$$

$$GA(\Gamma_{SI}(Z_r))) = \sum_{uv\in E(\Gamma_{SI}(Z_r))} \frac{2\sqrt{deg(u).deg(v)}}{deg(u)+deg(v)}$$

$$= \underbrace{\frac{2\sqrt{deg(1).deg(2)}}{deg(1)+deg(2)} + \cdots + \frac{2\sqrt{deg(1).deg(r-1)}}{deg(1)+deg(r-1)}}_{(r-2)times}$$

$$+ \underbrace{\frac{2\sqrt{deg(2).deg(3)}}{deg(2)+deg(3)} + \cdots + \frac{2\sqrt{deg(2).deg(r-1)}}{deg(2)+deg(r-1)}}_{(r-3)times}$$

$$+ \underbrace{\frac{2\sqrt{deg(3).deg(4)}}{deg(3)+deg(4)} + \cdots + \frac{2\sqrt{deg(3).deg(r-1)}}{deg(3)+deg(r-1)}}_{(r-4)times} + \cdots$$

$$+ \frac{2\sqrt{deg(r-2).deg(r-1)}}{deg(r-2)+deg(r-1)}$$

$$= \frac{2(r-2)^2}{2r-4} + \frac{2(r-3)(r-2)}{2r-4} + \frac{2(r-4)(r-2)}{2r-4} + \cdots + \frac{2(r-2)}{2r-4}$$

$$= (r-2) + \sum_{i=3}^{r-1}(r-i)$$

Theorem 3.9

Let Z_r be a group with $r \geq 3$, r is a prime number then the Harmonic index of $\Gamma_{SI}(Z_r)$ is $H(\Gamma_{SI}(Z_r))) = 1 + \frac{2}{2r-4}\sum_{i=3}^{r-1}(r-i)$

Proof:

$deg(u) = r-2$, $\forall u \in V(\Gamma_{SI}(Z_r))$, $u = 1,2,\ldots,r-1$

$$H(\Gamma_{SI}(Z_r))) = \sum_{uv\in E(\Gamma_{SI}(Z_r))} \frac{2}{deg(u)+deg(v)}$$

$$= \underbrace{\frac{2}{deg(1)+deg(2)} + \cdots + \frac{2}{deg(1)+deg(r-1)}}_{(r-2)times}$$

$$+ \underbrace{\frac{2}{deg(2)+deg(3)} + \cdots + \frac{2}{deg(2)+deg(r-1)}}_{(r-3)times}$$

$$+ \underbrace{\frac{2}{deg(3)+deg(4)} + \cdots + \frac{2}{deg(3)+deg(r-1)}}_{(r-4)times} + \cdots + \frac{2}{deg(r-2)+deg(r-1)}$$

$$= \frac{2(r-2)}{2r-4} + \frac{2(r-3)}{2r-4} + \frac{2(r-4)}{2r-4} + \cdots + \frac{2}{2r-4}$$

$$= 1 + \frac{2}{2r-4}\sum_{i=3}^{r-1}(r-i)$$

4. (Hosoya and Schultz) Polynomial of $\Gamma_{SI}(Z_r)$

In this section , we found (Hosoya and Schultz) Polynomial of $\Gamma_{SI}(Z_r)$

Definition 4.1(33):

Let G be a connected graph , then a Hosoya Polynomial of graph G is define by $H(G;x) = \sum_{k=0}^{diam(G)} d(G,k)\, x^k$, where $d(G,k)$ is the number of pairs of vertices of a graph G that are at distance k apart , for $k = 0,1,2,\dots, diam(G)$,where $diam(G) = \max_{u,v\in V(G)} d(u,v)$.

Note 4.2(34): 1- $d(G,0) = p(G)$

2- $d(G,1) = q(G)$

Definition 4.3(35):

Let G be an connected graph , then a Schultz Polynomial of a graph G is define by $Sc(G;x) = \sum_{\substack{u,v\in V(G)\\ u\neq v}}(deg(u)+deg(v))x^{d(u,v)}$, where $deg(u)$ is the degree of the vertices u and $deg(v)$ is the degree of vertices v, $d(u,v)$ is the distance between u and v.

Theorem 4.4:

H ($\Gamma_{SI}(Z_r);x) = c_0 + c_1 x$, where $r \geq 3$, r is a prime number and

$c_0 = r-1\,, c_1 = \sum_{i=2}^{r-1} r - i$

Proof:

For every $r \geq 3$, r is a prime number , we noticed that every vertex of graph $\Gamma_{SI}(Z_r)$ is adjacent of all the vertices of graph $\Gamma_{SI}(Z_r)$, then diam($\Gamma_{SI}(Z_r)) = 1$, it is mean H($\Gamma_{SI}(Z_r), x) = c_0 + c_1 x$, where

$c_i = d(\Gamma_{SI}(Z_r), i)$, $\forall\, i = 0,1$

It's clear that $c_0 = d(\Gamma_{SI}(Z_r), 0) = |\Gamma_{SI}(Z_r)| = r-1$

Now to find the size of $\Gamma_{SI}(Z_r)$, we noticed that there exist m_{r-1} of edges s.t

$m_1 = r-2\,, m_2 = r-3, \dots., m_{r-1} = 1$

Then $c_1 = m_1 + m_2 + \cdots + m_{r-1}$

We can write :

$$c_1 = \sum_{i=2}^{r-1} r - i$$

Theorem 4.5:

$Sc(\Gamma_{SI}(Z_r);x) = \sum_{i=2}^{r-1}(r-i)(2r-4)x$, where $r \geq 3$, r is a prime number.

Proof:

$deg(u) = r-2$, $\forall u \in V(\Gamma_{SI}(Z_r)),\ u = 1,2,\ldots,r-1$

$d(u,v) = 1$, $\forall u,v \in V(\Gamma_{SI}(Z_r))$

$$Sc(\Gamma_{SI}(Z_r);x) = \sum_{u,v\in V(\Gamma_{SI}(Z_r))} (deg(u)+deg(v))x^{d(u,v)}$$

$$= \underbrace{(deg(1)+deg(2))\,x + \cdots + (deg(1)+deg(r-1))x}_{(r-2)times} + \underbrace{(deg(2)+deg(3))x + \cdots + (deg(2)+deg(r-1))x}_{(r-3)times} + \cdots + (deg(r-1) + deg(r-2))x$$

$$= \sum_{i=2}^{r-1}(r-i)(2r-4)x$$

5. Conclusions

This article has presented the formulae of some degree-based and eccentric-based topological indices of subgroup intersection graph of a group Z_r , where r is a prime number .For further research, examining on subgroup intersection graph of a group Z_{pq}, where p, q are a prime number

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