

A Modified Heat Diffusion Based Method for Enhancing Physical Images

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ARTICLE INFO

Article history:

Received: 21 /01/2021

Revised form: 21 /02/2021

Accepted : 23 /03/2021

Available online: 23 /03/2021

Keywords:

MATLAB program, color image, mathematical methods, compressed image, physical image.

ABSTRACT

In this paper, we present a new denoising technique based on a modification on the heat equation. This method is mainly presented to enhance some physical images that were carried out in a physical laboratory during some experiments. A one-dimensional example using a vector and a two-dimensional example using matrices are showed in this paper to clarify how applying the modified way of solution on a heat diffusion equation can remove some existing defects in these images. In addition, we are going to make a model for a heat diffusion in two directions x and y and assemble letters from components in matrices to apply our experiment on some other different colorful and greyscale images. Finally, we calculate PSNR values for our method and compare them with some research methods to conclude that enhancing images by removing undesired defects can be done by the proposed method.

MSC. 41A25; 41A35; 41A36.

DOI : <https://doi.org/10.29304/jqcm.2021.13.1.777>

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Communicated by : Dr. Rana Jumaa Surayh aljanabi

1. Introduction

Image enhancement is the usual goal of the image processing problems. One of the basic image-processing problems is denoising images. In fact, denoising images can be done in many different methods, and by using a variety of different techniques [1-8]. Denoising, as the phrase suggests, can be defined as getting rid of noisy defects from the pixels of images.

Varieties of different research papers have been concentrated in this specific area for a long time, and researchers for achieving good performance for the same reason have proposed many techniques. The new heat diffusion-based method here can be illustrated as using an example from real life to explain the work that we are planning to apply on images for such an enhancement purpose. The proposed method depends mainly on Joseph Fourier idea by considering metal piece that has a length l with two different temperatures on the ends, $0\text{ }C^{\circ}$ and $100\text{ }C^{\circ}$ respectively and as illustrated in Figure 1.



Fig. 1 - A metal rod with a different temperature on ends.

The natural heat transfers gradually from places with high temperature to low temperature, and as shown in the previous figure, the temperature $u(x, t)$ depends totally on this diffusion over time. This idea can be represented as a partial differential equation (P.D.E) as

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u \quad (1)$$

where α represents the thermal diffusivity, $\frac{\partial u}{\partial t}$ is just the partial derivative of u with respect to t , and ∇^2 is the second derivative of the same variable u .

The main purpose of doing such modifications on images is to get some details that are impossible to be seen without modifying the images. Figure 2 shows an example of denoising images, which is the technique that we are going to develop in the next section. In the left side, the original image with some unclear details due to unstable camera handheld or some other reasons that may lead to that and the image in the right side is the result after denoising.

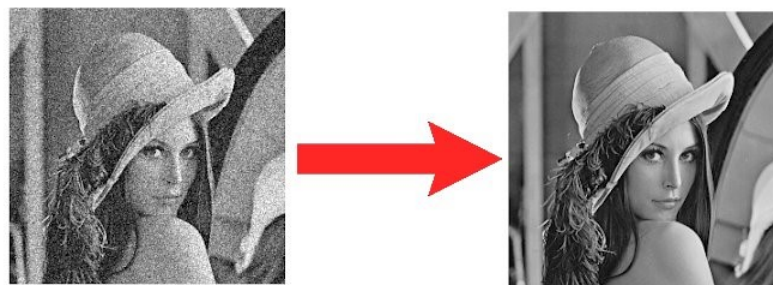


Fig. 2 - An example of denoising an image.

2. Heat Diffusion Equation

Consider Eq. 1 in the previous section, with α representing the thermal diffusivity. The constant α can have different values depending on the material. The high and low values of the heat are comparable to the high and low

concentration of light on an image that results in changing its resolution. More specifically, we are going to take a model of heat transfer in one dimension and two dimensions and apply the explicit method, so we can get an accurate prediction of the diffusion of the heat (where the stability condition is held). In this case, we can noise and de-noise images when the heat is changed. In other words, forward time steps can result in noising images, while the backward time steps can end up with de-noising images. To generalize this to a two-dimensional image, we need to make a model for heat diffusion in two directions x and y . In this case, our function will be in terms of x, y , and t . More precisely, solving the following equation is the idea of image denoising.

$$\partial u / \partial t = \partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 + \alpha(u - u_0) \quad (2)$$

where: $\partial u / \partial t$ is the partial derivative of u with respect to t , $\partial^2 u / \partial x^2$ is the second derivative of the same variable u with respect to x , and $\partial^2 u / \partial y^2$ is the second derivative of the same variable u with respect to y . In fact, α in the right-hand side of Eq. 2 is the actual parameter that can control the similarity of the original image and the modified image (denoised image). In mathematical view, the idea can be described as an optimization problem as

$$k^* = \max_k \{p(k | f)\} \quad (3)$$

where: k represents the solution and $p(k | f)$ is the probability of k occurring given that the observation f occurred, which can be explained as the following:

$$p(k | f) = p(k) \cdot p(f | k) / p(f) \quad (4)$$

Therefore, we can use Eq. 4 to compute the solution, which has been modeled using the same mentioned probability, and a noise reduction can be obtained as the following:

$$p(f | k) = \prod_{(x,y) \in \delta} (1/\sqrt{2\pi\mu}) e^{- (f(x,y)-k(x,y))^2 / 2\mu^2} \quad (5)$$

where: μ^2 is the noise variance, and δ is the set of components that represent pixels in images.

2.1. A vector Example (One Dimension)

In this example, we will create a vector with 100 components and call it U . The vector U has positive numbers on positions 30 to 50 and let this positive number be 10. In addition, there is another positive number on positions 70 to 73 and let this positive number be 15. Finally, the rest of components in U are zeros. Figure 3 shows how the vector looks like.

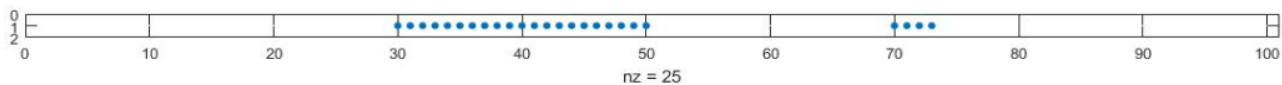


Fig. 3 - U vector with 100 components.

It is important to mention that the vector U here is representing the 1D image. Intuitively, we can consider this vector in MATLAB as an initial temperature within a long thin wire. Later, we can apply the forward and backward finite difference to see how it will affect this vector.

Taking the time steps $k = 10$ and $\Delta t = 0.00001$ and $\Delta x = 0.01$ to satisfy the CFL condition and to give reasonable denoising for the vector. Applying it within the forward time differences to see how the image is denoised in every iteration. Figure 4 show the result which is done by MATLAB.

Applying the forward time steps

$$\text{new}u_{(i)} = u_{(i)} + \frac{dt}{dx^2} (u_{(i-1)} - 2u_{(i)} + u_{(i+1)}) \quad (6)$$

In order to have the forward heat diffusion model of denoising. On the other hand, the negative time steps as shown here

$$\text{new}u_{(i)} = u_{(i)} - \frac{dt}{dx^2} (u_{(i-1)} - 2u_{(i)} + u_{(i+1)}) \quad (7)$$

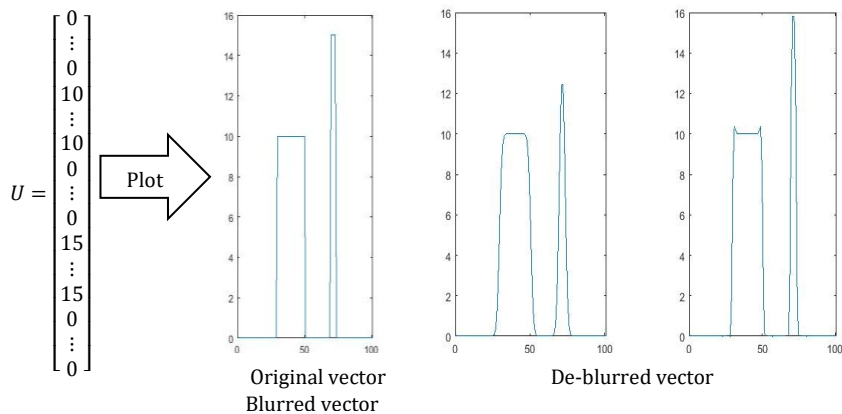


Fig. 4 – The effects by Eqns. 6 and 7 on the vector U.

Eq. 7 will be used once the forward time steps be done in order to noise our 1D image (vector). The backward iterations will give an oscillation eventually in the 1D example. However, it is not easy to get enough information about the result of that oscillation in a physical sense completely since we are dealing with one-dimensional array. Therefore, we are going to have two-dimensional array example to see how it is ended up with these iterations.

2.2. A Matrix Example (Two Dimensions)

In this example, we will simulate the acronym of Journal of Al-Qadisiyah for Computer Science and Mathematics (JQCM) from (m × n) matrices. For example, the letter “J” can be represented by creating (m × n) matrix of zeros, then nonzero components to form the letter. In other words, this will create the letter as in the binary system that is known by 0 for the idle position, and 1 for the busy position. Figure 5 shows the visualize sparsity pattern that can be done by using the command spy(matrix) and a 3D view for the letters in MATLAB.

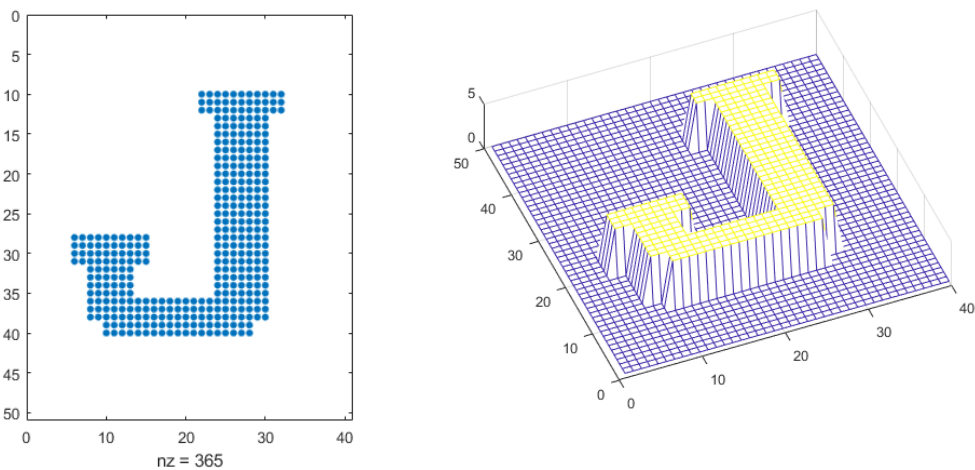


Fig. 5 – The visualize sparsity pattern of the letter "J".

With the same argument, we can create any combination of letters to represent any word or acronym of our choice. Working on these matrices can show us how the heat equation finite difference can affect these letters. More specifically, the matrix is a good example of two dimensions, and using it in MATLAB can clearly display the changes

that are going to happen within the iterations of the forward and backward differences. Figure 6 below shows the final form of the letters JQCM combined.

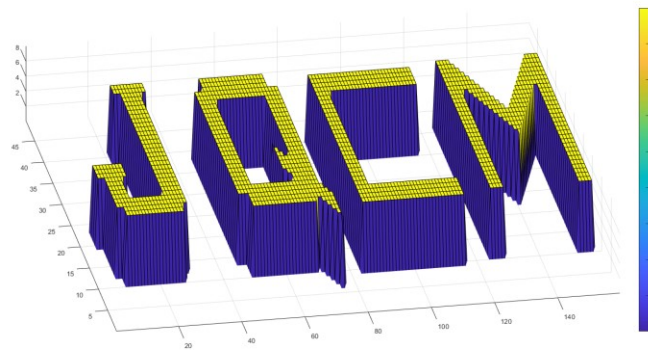


Fig. 6 – A 3D view of the acronym of Journal of Al-Qadisiyan for Computer Science and Mathematics "JQCM".

Applying the forward time steps (FTS) in two dimensions on those matrices (letters) would be in following form

$$\text{new}u_{(i,j)} = u_{(i,j)} + \frac{dt}{dx^2} (u_{(i-1,j)} - 2u_{(i,j)} + u_{(i+1,j)}) + \frac{dt}{dy^2} (u_{(i,j-1)} - 2u_{(i,j)} + u_{(i,j+1)}) \quad (8)$$

Viewing these letters by using the command (mesh) in MATLAB to view it as an initial temperature within a thin plate, after that, we apply the FTS by taking these numbers for the time steps $k = 50$ and $\Delta t = 0.00001$, $\Delta x = 0.01$, and $\Delta y = 0.01$. We can apply different values, but we should satisfy the CFL condition to see clearly by MATLAB how the letters get blurred which is the technique of denoising images. Figure 7 shows the result on the steps $k = 1, k = 10, k = 20, k = 30, k = 40$, and $k = 50$

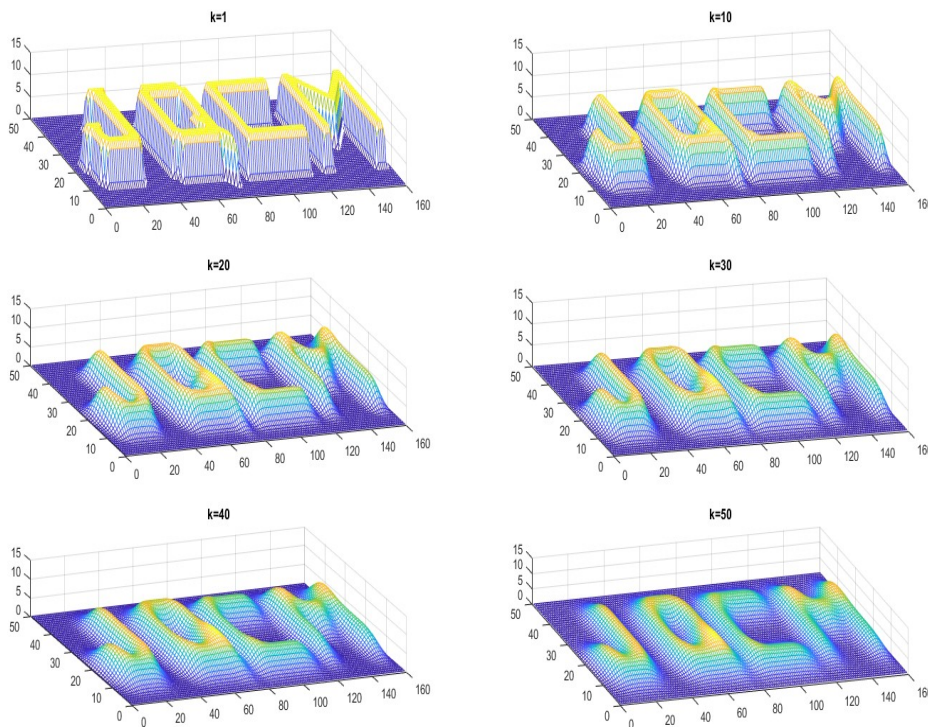


Fig. 7 – T Applying Equation (8) with 50 different time steps k.

To see the real effects on the image of the letters (JQCM), MATLAB has a good option for converting matrices to images. In other words, we can use the command `imwrite(matrix)` to convert those alphabetical matrices to a real image and we can see how far we affected those letters, Figure 8 shows the results as images.

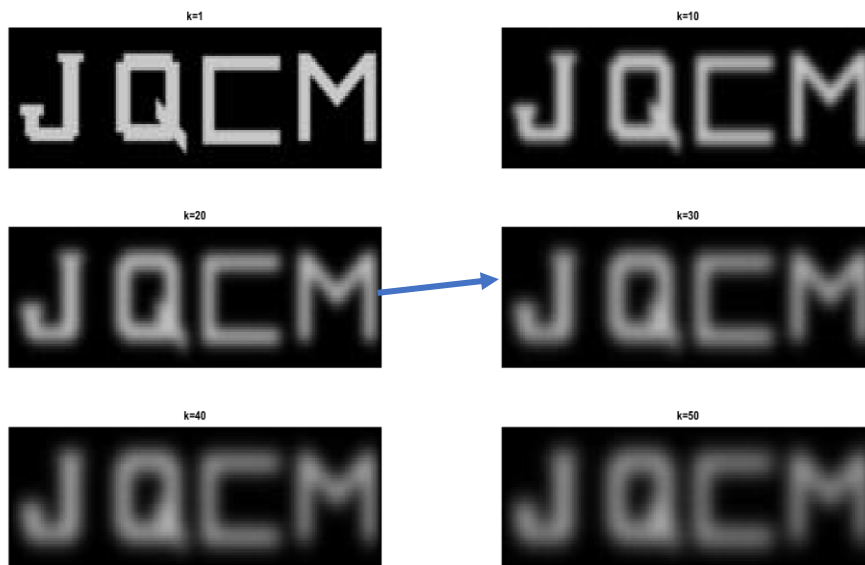


Fig. 8 - The sharpening on the "WSU" letters.

3. Denoising as Blurring

As we can clearly see in the previous section in Figure 7 applying Eq. 8 on the proposed example leads to get these letters blurred. In fact, manipulating the intensity of an image by the parameter α in Eq. 2 and controlling Δx and Δy in Eq. 8 will make us obtain different levels of blurring images. Moreover, to obtain a better spectacle for some objects such as a relief in a painting or details of buildings in Google maps...etc. We can set the parameters in Eq. 8 as $\text{Alpha} = dt/dx^2 = dt/dy^2 = 1$ in the negative time steps, and the intensity = 1.2 to bright the image.

$$\text{new}u_{(i,j)} = 1.2 * u_{(i,j)} - 1 * (u_{(i-1,j)} - 2u_{(i,j)} + u_{(i+1,j)}) - 1 * (u_{(i,j-1)} - 2u_{(i,j)} + u_{(i,j+1)}) \quad (9)$$

Moreover, to apply this method on images, we need to work on three different layers (color images). In other words, we have to deal with three matrices for every image. The reason behind that is the color images are decomposed into three colors, red, green, and blue (RGB). Therefore, we will have a matrix of size of $(m \times n \times 3)$. In Matlab, we need to run the diffusion on each of the 3 RGB bands, so we will deal with the red color by dealing with the matrix $(:, :, 1)$, $(:, :, 2)$ for the green color and finally $(:, :, 3)$ for the blue color. We apply here the denoising technique on some images that were carried out in a physical laboratory in some experiments. In fact, one can deal with these images in the same way of dealing with colorful images by dealing with the matrix $(:, :, 1)$, and $(:, :, 2)$ for the green color and finally $(:, :, 3)$ for the blue color. Figure 9 shows three physical images on the left and the same three images on the right after denoising [1-99].

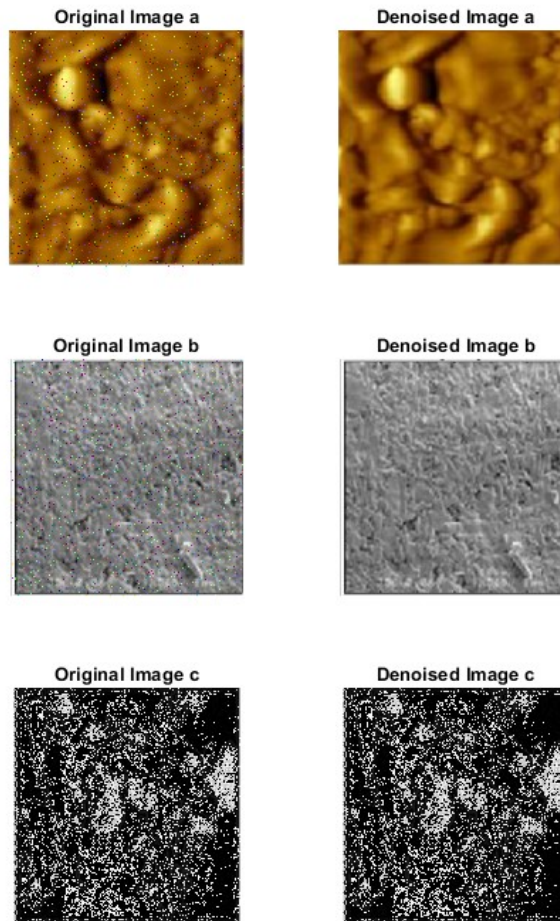


Fig. 9 – Denoising images by Eq. 8 with intensity 1.2 and $\alpha=0.9$.

3.1. PSNR Calculations

The easiest way to define PSNR is via the mean squared error (MSE). Consider a noise-free ($m \times n$) image IM and its noisy approximation K, MSE is defined as:

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [IM(i, j) - K(i, j)]^2 \tag{10}$$

Therefore, PSNR can be defined as

$$PSNR = 10 \cdot \log_{10} \left(\frac{MAX_{IM}^2}{MSE} \right) \tag{11}$$

where MAX_{IM} represents the maximum possible pixel value of the image.

To validate our proposed method, we test our output images and calculate PSNR values for each image and then we compare the results to methods presented in [9-12]. The reason of calculating Peak signal-to-noise ratio for our output images is to validate the proposed algorithm and to prove that it is a competitive method comparing with other methods.

Table 1 - PSNR calculations with two different intensities.

Denoising with intensity 0.9						Denoising with intensity 1.2				
Name of Image	Ref[9]	Ref[10]	Ref[11]	Ref[12]	Our Method	Ref[9]	Ref[10]	Ref[11]	Ref[12]	Our Method
Image a	31.7458	33.3020	31.3640	33.0258	32.8881	27.1586	26.0021	26.3364	27.1808	26.3604
Image b	32.1236	32.3001	32.9958	32.3190	32.8254	27.1975	27.9951	26.9594	27.3331	27.1027
Image c	34.9603	35.1577	34.3148	34.1001	33.9225	28.2105	29.0325	28.0201	28.3232	28.9001
Mean	32.9432	33.5866	32.8915	33.1483	33.2120	27.5222	27.6765	27.1053	27.6123	27.4544

4. Conclusion

In this paper, we presented a heat diffusion-based method for denoising images using the blurring technique. The blurring technique here can be done by using the proposed equation with such modifications to control the intensity of images for the sake of noise removal. We applied our method on images that we got from physical laboratory. We also made a comparison of PSNR values and showed that our suggested method can be used for this purpose. In addition, we illustrated the method by taking example with a variety of dimensions and applied the forward time steps in the discretization of the heat equation to get rid of some undesired details in an image and to reduce the distinction of the image.

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