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Results for Skew-Shape$(9,7,3)/(1,0)$

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ABSTRACT

In this paper, we find the conditions for the sequence of characteristic zero in the case of the skew-shape (9,7,3)/(1,0). We also study this sequence as a diagram and prove that this sequence is complex using mapping Cone and other concepts.

الخلاصة

في هذا البحث نجد شروط تسلسل الخاصية صفر في حالة الشكل المنحرف (9,7,3)/(1,0) . ندرس أيضًا هذا التسلسل كرسم تخطيطي ونثبت أن هذا التسلسل معقد باستخدام رسم الخرائط المخروطيه ومفاهيم أخرى.

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Introduction

Suppose $\mathcal{F}$ is a free R-module over a commutative ring $\mathcal{R}$ with identity 1 and $Đ_{r}\mathcal{F}$ is divided power algebra of degrees r.
The authors in [1], [2] and [3] clarify the partitions (2,2,2), (3,3,3) and (4,4,3) for the same result, while the authors in [4], [5], [6] , [7] and [8] survey the itself idea for the partitions (3,3,2), (6,6,3), (6,5,3), (7,6,3) and (8,7,3). The authors in [9] study the case (6,6,4;0,0) by using the mapping Cone, [10].

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The case (9,7,3)/(1,0) is illustrated in this work. The map ($\partial_{ij}^{(k)}$) means the divided power of the place polarization(∂_{ij}) where(j) must be less than(i) , with its Capelli identities [11]. So we need

- $\partial_{21}^{(g)} \circ \partial_{32}^{(f)} = \sum_{a\geq 0}(-1)^a \circ \partial_{32}^{(f-a)} \circ \partial_{21}^{(g-a)} \circ \partial_{31}^{(a)}$ ……….. (1.1)
- $\partial_{32}^{(f)} \circ \partial_{21}^{(g)} = \sum_{a\geq 0} \partial_{21}^{(g-a)} \circ \partial_{32}^{(f-a)} \circ \partial_{31}^{(a)}$ … ……..(1.2)

1. The terms in the case $(9,7,3)/(1,0)$

In order to find the conditions in our case $(p_1, p_2, p_3; t_1, t_2)$, we use the following [10]:

$$0 \longrightarrow \big((p_1+|t|+2)(p_2)(p_3-|t|-2)\big) \xrightarrow{\sigma_3} \begin{matrix}(p_1+|t|+2)(p_2-t_1-1)(p_3-t_2-1)\\ \oplus \\ (p_1+t_1+1)(p_2+t_2+1)(p_3-|t|-2)\end{matrix}$$
$$\xrightarrow{\sigma_2} \begin{matrix}(p_1)(p_2+t_2+1)(p_3-t_2-1)\\ \oplus \\ (p_1+t_1+1)(p_2-t_2-1)(p_3)\end{matrix} \xrightarrow{\sigma_1} (p_1)\,(p_2)(p_3) \longrightarrow 0$$

Where $|t| = t_1 + t_2$.

Then for the case of the partition (9,7,3)/(1,0) we have the formula:-

$$\mathbf{0} \to Đ_{11}\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_0\mathcal{F} \to \begin{matrix} Đ_{11}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_2\mathcal{F} \\ \oplus \\ Đ_{10}\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_0\mathcal{F}\end{matrix}$$
$$\to \begin{matrix} Đ_8\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_2\mathcal{F} \\ \oplus \\ Đ_{10}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_3\mathcal{F}\end{matrix} \to Đ_8\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_3\mathcal{F} \to \mathbf{0}$$

2. The diagram in the case $(9,7,3)/(1,0)$

Consider the following diagram:

$$\begin{array}{ccccc} Đ_{11}\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_0\mathcal{F} & \xrightarrow{r_1} & Đ_{11}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_2\mathcal{F} & \xrightarrow{r_2} & Đ_{10}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_3\mathcal{F} \\ \ell_1 \downarrow & \mathcal{H} & \ell_2 \downarrow & \mathcal{L} & \downarrow \ell_3 \\ Đ_{10}\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_0\mathcal{F} & \xrightarrow[e_1]{} & Đ_8\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_2\mathcal{F} & \xrightarrow[e_2]{} & Đ_8\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_3\mathcal{F} \end{array}$$

Where

r_1: $Đ_{11}\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_0\mathcal{F} \longrightarrow Đ_{11}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_2\mathcal{F}$ as

- $r_1(\mathrm{v}) = \partial_{32}^{(2)}(\mathrm{v})$; where $v \in Đ_{11}\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_0\mathcal{F}$

ℓ_1: $Đ_{11}\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_0\mathcal{F} \longrightarrow Đ_{10}\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_0\mathcal{F}$

- $\ell_1(\mathrm{v}) = \partial_{21}(\mathrm{v})$; where $v \in Đ_{10}\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_0\mathcal{F}$

And

ℓ_2: $Đ_{11}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_2\mathcal{F} \longrightarrow Đ_8\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_2\mathcal{F}$

- $\ell_2(\mathrm{v}) = \partial_{21}^{(3)}(\mathrm{v})$; *where* $v \in Đ_{11}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_2\mathcal{F}$

Now, judge to know the map

$\mathrm{e}_1 : Đ_{10}\mathcal{F} \otimes \;\; Đ_8\,\mathcal{F} \otimes \;\; Đ_0\mathcal{F} \longrightarrow Đ_8\mathcal{F} \;\otimes\;\; Đ_8\mathcal{F} \;\otimes\;\; Đ_2\mathcal{F}$

Which feign the diagram $\mathcal{H}$ commutative, (i.e)

$$\mathrm{e}_1 \circ \ell_1 = \ell_2 \circ r_1$$

So $\mathrm{e}_1 \circ \partial_{21} = \partial_{21}^{(3)} \circ \partial_{32}^{(2)}$

$$\begin{aligned}\partial_{21}^{(3)} \circ \partial_{32}^{(2)} &= \partial_{32}^{(2)}\,\partial_{21}^{(3)} - \partial_{32}\,\partial_{21}^{(2)}\,\partial_{31} + \partial_{21}\,\partial_{31}^{(2)}\\ &= \tfrac{1}{3}\partial_{32}^{(2)}\,\partial_{21}^{(2)}\,\partial_{21} - \tfrac{1}{2}\partial_{32}\,\partial_{21}\,\partial_{21}\,\partial_{31} + \partial_{31}^{(2)}\,\partial_{21}\\ &= \left(\tfrac{1}{3}\partial_{32}^{(2)}\,\partial_{21}^{(2)} - \tfrac{1}{2}\partial_{32}\,\partial_{21}\,\partial_{31} + \partial_{31}^{(2)}\right) \circ \partial_{21}\end{aligned}$$

Thus , $\mathrm{e}_1 = \frac{1}{3}\partial_{32}^{(2)}\,\partial_{21}^{(2)} - \frac{1}{2}\partial_{32}\,\partial_{21}\,\partial_{31} + \partial_{31}^{(2)}$

If we acquaint the map

$\mathrm{e}_2 : Đ_8\mathcal{F} \;\otimes\;\; Đ_8\mathcal{F} \;\otimes\;\; Đ_2\mathcal{F} \longrightarrow Đ_8\mathcal{F} \otimes \;\; Đ_7\,\mathcal{F} \otimes \;\; Đ_3\mathcal{F}$ as

- $\mathrm{e}_2\,(\mathrm{v}) = \partial_{32}(\mathrm{v})$; where $v \in Đ_8\mathcal{F} \;\otimes\;\; Đ_8\mathcal{F} \;\otimes\;\; Đ_2\mathcal{F}$

And

$\ell_3 : Đ_{10}\,\mathcal{F} \otimes \;\; Đ_5\,\mathcal{F} \otimes \;\; Đ_3\mathcal{F} \longrightarrow Đ_8\mathcal{F} \otimes \;\; Đ_7\,\mathcal{F} \otimes \;\; Đ_3\mathcal{F}$

- $\ell_3(\mathrm{v}) = \partial_{21}^{(2)}(\mathrm{v})$; where $v \in Đ_{10}\mathcal{F} \;\otimes\;\; Đ_5\mathcal{F} \;\otimes\;\; Đ_3\mathcal{F}$

We require acquainting r_2 to feign the diagram $\mathcal{L}$ commutative:

$r_2 : Đ_{11}\mathcal{F} \otimes \;\; Đ_5\,\mathcal{F} \otimes \;\; Đ_2\mathcal{F} \longrightarrow Đ_{10}\mathcal{F} \;\otimes\;\; Đ_5\mathcal{F} \;\otimes\;\; Đ_3\mathcal{F}$

Such that

$$\ell_3 \circ r_2 = \mathrm{e}_2 \circ \ell_2$$

$\partial_{21}^{(2)} \circ r_2 = \partial_{32} \circ \partial_{21}^{(3)}$

$$\begin{aligned}\partial_{32} \circ \partial_{21}^{(3)} &= \partial_{21}^{(3)}\,\partial_{32} + \partial_{21}^{(2)}\,\partial_{31}\\ &= \tfrac{1}{3}\partial_{21}^{(2)}\,\partial_{21}\,\partial_{32} + \partial_{21}^{(2)}\,\partial_{31}\\ &= \left(\tfrac{1}{3}\partial_{21}\,\partial_{32} + \partial_{31}\right) \circ \partial_{21}^{(2)}\end{aligned}$$

Thus , $r_2 = \frac{1}{3}\partial_{21}\,\partial_{32} + \partial_{31}$

Consider the following graph:

$$\begin{array}{ccccc} Đ_{11}\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_0\mathcal{F} & \xrightarrow{r_1} & Đ_{11}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_2 & \xrightarrow{r_2} & Đ_{10}\mathcal{F} \otimes Đ_5\mathcal{F} \otimes Đ_3\mathcal{F} \\ \ell_1 \downarrow & \mathcal{X} & \mathcal{G} & \mathcal{Y} & \downarrow \ell_3 \\ Đ_{10}\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_0\mathcal{F} & \xrightarrow[\mathrm{e}_1]{} & Đ_8\mathcal{F} \otimes Đ_8\mathcal{F} \otimes Đ_2\mathcal{F} & \xrightarrow[\mathrm{e}_2]{} & Đ_8\mathcal{F} \otimes Đ_7\mathcal{F} \otimes Đ_3\mathcal{F} \end{array}$$

Define:

$\mathcal{G} : \;\; Đ_{10}\mathcal{F} \;\otimes\;\; Đ_8\mathcal{F} \;\otimes\;\; Đ_0\,\mathcal{F} \longrightarrow Đ_{10}\,\mathcal{F} \otimes \;\; Đ_5\,\mathcal{F} \otimes \;\; Đ_3\mathcal{F}$

by $\mathcal{R}(\mathrm{v}) = \partial_{32}^{(2)}\,(\mathrm{v})$; where $v \in Đ_{10}\mathcal{F} \otimes \;\; Đ_8\,\mathcal{F} \otimes \;\; Đ_0\mathcal{F}$

Proposition (3.1):The graph $\mathcal{X}$ is commutative.
Proof: we need to prove it $r_2 \circ r_1 = \mathcal{G} \circ \ell_1$

$$So\ (r_2 \circ r_1) = \left(\frac{1}{3}\partial_{21}\partial_{32} + \partial_{31}\right) \circ \partial_{32}^{(2)}$$
$$= \partial_{21}\partial_{32}^{(3)} + \partial_{31}\partial_{32}^{(2)}$$
$$= \partial_{32}^{(3)}\partial_{21} - \partial_{32}^{(2)}\partial_{31} + \partial_{31}\partial_{32}^{(2)}$$
$$= \partial_{32}^{(3)}\partial_{21}$$
$$= \mathcal{G} \circ \ell_1 . \quad \square$$

***Proposition* (3.2):**The graph $\mathcal{Y}$ is commutative.
Proof: we need to prove it $e_2 \circ e_1 = \ell_3 \circ \mathcal{G}$

$$So\ (e_2\ e_1) = \partial_{32} \circ \left(\frac{1}{3}\partial_{32}^{(2)}\partial_{21}^{(2)} - \frac{1}{2}\partial_{32}\partial_{21}\partial_{31} + \partial_{31}^{(2)}\right)$$
$$= \partial_{32}^{(3)}\partial_{21}^{(2)} - \partial_{32}^{(2)}\partial_{21}\partial_{31} + \partial_{31}^{(2)}\partial_{32}$$
$$= \partial_{21}^{(2)}\partial_{32}^{(3)} - \partial_{32}\partial_{32}\partial_{21}\partial_{31} + \partial_{31}^{(2)}\partial_{32}$$
$$= \partial_{21}^{(2)}\partial_{32}^{(3)} - \partial_{32}\partial_{31}^{(2)} + \partial_{31}^{(2)}\partial_{32}$$
$$= \partial_{21}^{(2)}\partial_{32}^{(3)}$$
$$= \ell_3 \circ \mathcal{G} . \quad \square$$

Finally , we look at the maps σ_1, σ_2 *and* σ_3 where :

- $\sigma_3(\varkappa) = \left(r_1(\varkappa), \ell_1(\varkappa)\right); \forall\, \varkappa \in\ Đ_{11}F \otimes Đ_7 F \otimes Đ_0 F$
- $\sigma_3 : Đ_{11}F \otimes Đ_7 F \otimes Đ_0 F \longrightarrow \begin{matrix} Đ_{11}F \otimes Đ_5 F \otimes Đ_2 F \\ \oplus \\ Đ_{10}F \otimes Đ_8 F \otimes Đ_0 F \end{matrix}$
- $\sigma_2(\varkappa_1, \varkappa_2) = \left(r_2(\varkappa_1) - \mathcal{G}(\varkappa_2), e_1(\varkappa_2) - \ell_2(\varkappa_1)\right); \forall\, \varkappa \in$ $Đ_{11}F \otimes Đ_5 F \otimes Đ_2 F \oplus Đ_{10}F \otimes Đ_8 F \otimes Đ_0 F$
- $\sigma_2 : \begin{matrix} Đ_{11}F \otimes Đ_5 F \otimes Đ_2 F \\ \oplus \\ Đ_{10}F \otimes Đ_8 F \otimes Đ_0 F \end{matrix} \longrightarrow \begin{matrix} Đ_{10}F \otimes Đ_5 F \otimes Đ_3 F \\ \oplus \\ Đ_8 F \otimes Đ_8 F \otimes Đ_2 F \end{matrix}$
- $\sigma_1(\varkappa_1, \varkappa_2) =$ $(\ell_3(\varkappa_1), e_2(\varkappa_2)); \forall\, \varkappa Đ_{10} F \otimes Đ_5 F \otimes Đ_3 F \oplus Đ_8 F \otimes Đ_8 F \otimes Đ_2 F$

And

- $\sigma_1 : \begin{matrix} Đ_{10}F \otimes Đ_5 F \otimes Đ_3 F \\ \oplus \\ Đ_8 F \otimes Đ_8 F \otimes Đ_2 F \end{matrix} \longrightarrow Đ_8 F \otimes Đ_7 F \otimes Đ_3 F$

Proposition (3.3):- The graph

$$0 \to Đ_{11}F \otimes Đ_7 F \otimes Đ_0 F \xrightarrow{\sigma_3} \begin{matrix} Đ_{11}F \otimes Đ_5 F \otimes Đ_2 F \\ \oplus \\ Đ_{10}F \otimes Đ_8 F \otimes Đ_0 F \end{matrix}$$

$$\xrightarrow{\sigma_2} \begin{matrix} Đ_8 F \otimes Đ_8 F \otimes Đ_2 F \\ \oplus \\ Đ_{10} F \otimes Đ_5 F \otimes Đ_3 F \end{matrix} \xrightarrow{\sigma_1} Đ_8 F \otimes Đ_7 F \otimes Đ_3 F \to 0$$

is complex.

Proof: from acquaintance ,we have a place of polarization ∂_{21} *and* ∂_{32}an injection [12] , then we get it by injection with capelli identities

Now

$$(\sigma_2 \circ \sigma_3)(\varkappa) = \sigma_2 \circ (r_1(\varkappa), \ell_1(\varkappa))$$

$$= \sigma_2(\partial_{32}^{(2)}(\varkappa), \partial_{21}(\varkappa))$$

$$= \left(r_2 \left(\partial_{32}^{(2)}(\varkappa)\right) - \mathcal{G}(\partial_{21}(\varkappa)), e_1(\partial_{21}(\varkappa)) - \ell_2\left(\partial_{32}^{(2)}(\varkappa)\right)\right)$$

$$r_2 \left(\partial_{32}^{(2)}(\varkappa)\right) - \mathcal{G}(\partial_{21}(\varkappa))$$

$$= \left(\tfrac{1}{3}\partial_{21}\partial_{32} + \partial_{31}\right) \circ \partial_{32}^{(2)}(\varkappa) - \partial_{32}^{(3)} \circ \partial_{21}(\varkappa)$$

$$= (\partial_{21}\partial_{32}^{(3)} + \partial_{31}\partial_{32}^{(2)} - \partial_{32}^{(3)}\partial_{21})(\varkappa)$$

$$= (\partial_{32}^{(3)}\partial_{21} - \partial_{32}^{(2)}\partial_{31} + \partial_{31}\partial_{32}^{(2)} - \partial_{32}^{(3)}\partial_{21})(\varkappa)$$

$$= 0$$

$$e_1(\partial_{21}(\varkappa)) - \ell_2\left(\partial_{32}^{(2)}(\varkappa)\right)$$

$$= \left(\tfrac{1}{3}\partial_{32}^{(2)}\partial_{21}^{(2)} - \tfrac{1}{2}\partial_{32}\partial_{21}\partial_{31} + \partial_{31}^{(2)}\right) \circ \partial_{21}(\varkappa) - \partial_{21}^{(3)} \circ \partial_{32}^{(2)}(\varkappa)$$

$$= \left(\tfrac{1}{3}\partial_{32}^{(2)}\partial_{21}^{(2)}\partial_{21} - \tfrac{1}{2}\partial_{32}\partial_{21}\partial_{21}\partial_{31} + \partial_{31}^{(2)}\partial_{21} - \partial_{21}^{(3)}\partial_{32}^{(2)}\right)(\varkappa)$$

$$= \left(\partial_{32}^{(2)}\partial_{21}^{(3)} - \partial_{32}\partial_{21}^{(2)}\partial_{31} + \partial_{21}\partial_{31}^{(2)} - \partial_{21}^{(3)}\partial_{32}^{(2)} + \partial_{32}\partial_{21}^{(2)}\partial_{31} - \partial_{21}\partial_{31}^{(2)}\right)(\varkappa)$$

$$= 0 .$$

Thus , $(\sigma_2 \circ \sigma_3)(\varkappa) = 0$

And

$$(\sigma_1 \circ \sigma_2)(\varkappa_1, \varkappa_2) = \sigma_1 \circ \left(r_2(\varkappa_1) - \mathcal{G}(\varkappa_2), e_1(\varkappa_2) - \ell_2(\varkappa_1) \right)$$

$$= \sigma_1 \circ \left(\left(\frac{1}{3} \partial_{21}\partial_{32} + \partial_{31} \right)(\varkappa_1) - \partial_{32}^{(3)}(\varkappa_2), \left(\frac{1}{3} \partial_{32}^{(2)}\partial_{21}^{(2)} - \frac{1}{2} \partial_{32}\partial_{21}\partial_{31} + \partial_{31}^{(2)} \right)(\varkappa_2) - \partial_{21}^{(3)}(\varkappa_1) \right)$$

$$= \partial_{21}^{(2)} \circ \left(\left(\frac{1}{3} \partial_{21}\partial_{32} + \partial_{31} \right)(\varkappa_1) - \partial_{32}^{(3)}(\varkappa_2) + \partial_{32} \circ \left(\frac{1}{3} \partial_{32}^{(2)}\partial_{21}^{(2)} - \frac{1}{2} \partial_{32}\partial_{21}\partial_{31} + \partial_{31}^{(2)} \right)(\varkappa_2) - \partial_{21}^{(3)}(\varkappa_1) \right)$$

$$= \left(\partial_{21}^{(3)}\partial_{32} + \partial_{21}^{(2)}\partial_{31} - \partial_{32}\partial_{21}^{(3)} \right)(\varkappa_1) + \left(\partial_{32}^{(3)}\partial_{21}^{(2)} - \partial_{32}^{(2)}\partial_{21}\partial_{31} + \partial_{32}\partial_{31}^{(2)} - \partial_{21}^{(2)}\partial_{32}^{(3)} \right)(\varkappa_2)$$

$$= \left(\partial_{32}\partial_{21}^{(3)} - \partial_{21}^{(2)}\partial_{31} + \partial_{21}^{(2)}\partial_{31} - \partial_{32}\partial_{21}^{(3)} \right)(\varkappa_1) + \left(\partial_{32}^{(3)}\partial_{21}^{(2)} - \partial_{32}\partial_{31}^{(2)} + \partial_{32}\partial_{31}^{(2)} - \partial_{21}^{(2)}\partial_{32}^{(3)} \right)(\varkappa_2)$$

$$= 0 . \quad \square$$

Conclusion

When we studied the conditions of the zero property sequence in the case of a skew. (9,7,3) / (1,0) and the sequence as a schematic diagram. We proved that this sequence is complex by using cone mapping, and this we want to reach it.

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