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Outcomes for the skew-partition (6, 6, 3) / (1, 1, 0)

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ABSTRACT

The aim of this work is to find the complex of characteristic zero (c.c.z.) in the case of skew-partition (6, 6, 3) / (1, 1, 0) as a diagram by utilizing the concepts power of place polarization and Capelli identities.

:الخلاصة

الهدف لهذا العمل هو إيجاد المعقدة للمميز الصفري في حالة الشكل المنحرف (0 ,1 ,1) / (3 ,6 ,6) كمخططات باستخدام المفاهيم قوى مكان الاستقطاب و متطابقات كابلي.

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1. Introduction

Let R be a commutative ring with identity,F be a free R-module and D_bF is the divided power of degree b .The (c.c.z.) with in the below cases, (2,2,2), (3,3,3) and (4,4,3) are studied by the authors in [1], [2] and [3], once the author in [4] display the (c.c.z.) in case of partitions (8,7,3). Other articles [5, 6] found the resolution of Weyl module for (c.c.z.) in the case of the partition (8, 7, 3) by using the mapping Cone as in [7], the partition (7,7,3), (6,5,3)/(t,0,0); where t=1,2 and the skew shape (9,7,3)/(1,0) studied in [8-10] respectively.

In this work we find the complex of skew-partition (6, 6, 3)/ (1, 1, 0) as a diagram after we illustrate the terms of that complex. The map $\partial_{ij}^{(f)}$ means the divided power of the place polarization ∂_{ij} where j must be less than i, with its Capelli identities [8]. So we need the identities below

$$\partial_{21}^{(u)} \circ \partial_{21}^{(V)} = \sum_{e\geq 0}(-1)^e \partial_{32}^{(V-e)} \circ \partial_{21}^{(u-e)} \circ \partial_{31}^{(e)}$$
$$\partial_{32}^{(V)} \circ \partial_{21}^{(u)} = \sum_{e\geq 0} \partial_{21}^{(u-e)} \circ \partial_{32}^{(V-e)} \circ \partial_{31}^{(e)}$$

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2. The skew-partition (6,6,3) / (1,1,0)

To find the terms of (c.c.z.) we used the following:

$$0 \rightarrow \langle (p+|t|+2)|(r-|t|-2)\rangle \xrightarrow{\partial_3} \begin{matrix} \langle (p+|t|+2)|(q-t_1-1)|(r-t_2-1)\rangle \\ \oplus \\ \langle (p+t_1+1)|(q-t_2+1)|(r-|t|-2)\rangle \end{matrix} \xrightarrow{\partial_2} \begin{matrix} \langle (p)|(q+t_2+1)|(r-t_2-1)\rangle \\ \oplus \\ \langle (p+t_1+1)|(q+t_1-1)|(r)\rangle \end{matrix} \xrightarrow{\partial_1} \langle (p)|(q)|(r)\rangle$$

Where $|t| = t_1 + t_2$.
For our case (5, 5, 3;1, 0) we have

$$0 \longrightarrow D_8 \otimes D_5 \otimes D_0 \xrightarrow{\partial_3} \begin{matrix} D_8 \otimes D_4 \otimes D_1 \\ \oplus \\ D_6 \otimes D_7 \otimes D_0 \end{matrix} \xrightarrow{\partial_2} \begin{matrix} D_7 \otimes D_5 \otimes D_1 \\ \oplus \\ D_6 \otimes D_4 \otimes D_3 \end{matrix} \xrightarrow{\partial_1} D_5 \otimes D_5 \otimes D_3$$

Consider the following diagram:

$$\begin{array}{ccccc} D_8F \otimes D_5F \otimes D_0F & \xrightarrow{R_1} & D_8F \otimes D_4F \otimes D_1F & \xrightarrow{R_2} & D_6F \otimes D_4F \otimes D_3F \\ w_1 \downarrow & P & w_2 \downarrow & Q & w_3 \downarrow \\ D_6F \otimes D_7F \otimes D_0F & \xrightarrow{S_1} & D_5F \otimes D_7F \otimes D_1F & \xrightarrow{S_2} & D_5F \otimes D_5F \otimes D_3F \end{array}$$

Where
$R_1(v)$: $D_8F \otimes D_5F \otimes D_0F \longrightarrow D_8F \otimes D_4F \otimes D_1F$, such that
$R_1(v) = \partial_{21}(v)$; $v \in D_8F \otimes D_5F \otimes D_0F$
$w_1(v)$: $D_8F \otimes D_5F \otimes D_0F \longrightarrow D_6F \otimes D_7F \otimes D_0F$, such that
$w_1(v) = \partial_{21}^{(2)}(v)$; $v \in D_8F \otimes D_5F \otimes D_0F$
$S_1(v)$: $D_6F \otimes D_7F \otimes D_0F \longrightarrow D_5F \otimes D_7F \otimes D_3F$, such that
$S_1(v) = \frac{1}{3}\partial_{32} \circ \partial_{21}$- $\partial_{31}^{(1)}$
$S_2(v)$: $D_5F \otimes D_7F \otimes D_1F \longrightarrow D_5F \otimes D_5F \otimes D_3F$, such that
$S_2(v) = \frac{1}{3}\partial_{32}^{(2)}(v)$;$v \in D_5F \otimes D_7F \otimes D_1F$
$R_2(v)$: $D_8F \otimes D_4F \otimes D_1F \longrightarrow D_6F \otimes D_4F \otimes D_3F$, such that
$R_2(v) = \frac{1}{3}\partial_{21}^{(2)} \circ \partial_{32}^{(2)} + \frac{1}{2}\partial_{21} \circ \partial_{32} \circ \partial_{31} + \partial_{31}^{(2)}$
$w_2(v)$: $D_8F \otimes D_4F \otimes D_1F \longrightarrow D_5F \otimes D_7F \otimes D_1F$, such that
$w_2(v) = \partial_{21}^{(3)}(v)$; $v \in D_8F \otimes D_4F \otimes D_1F$
$w_3(v)$: $D_6F \otimes D_4F \otimes D_3F \longrightarrow D_5F \otimes D_5F \otimes D_3F$, such that
$w_3(v) = \partial_{21}(v)$; $v \in D_5F \otimes D_7F \otimes D_1F$

Proposition (2.1): The diagram P is commutative.
Proof: we should proof that $(w_2 \circ R_1)(v) = (S_1 \circ w_1)(v)$
$(w_2 \circ R_1)(v) = \partial_{21}^{(3)}(v) \circ \partial_{32}(v)$, and

$(S_1 \circ w_1)(v) = (\frac{1}{3}\partial_{32} \circ \partial_{21} \circ \partial_{31}) \circ \partial_{21}^{(2)}$

$= (\frac{1}{3}\partial_{32} \circ \partial_{21} \circ \partial_{21}^{(2)}) \circ \partial_{31} \circ \partial_{21}^{(2)}$

$= \partial_{32} \circ \partial_{21}^{(3)} - \partial_{21}^{(2)} \circ \partial_{31}$

$= \partial_{21}^{(3)} \circ \partial_{32}$

$= (w_2 \circ R_1)(v)$

Proposition (2.2):): The diagram Q is commutative.
Proof: we should proof that $(w_3 \circ R_2)(v) = (S_2 \circ w_2)(v)$

$(w_3 \circ R_2)(v) = \partial_{21}(\frac{1}{3}\partial_{21}^{(2)} \circ \partial_{32}^{(2)} + \frac{1}{2}\partial_{21} \circ \partial_{32} \circ \partial_{31} + \partial_{31}^{(2)})$

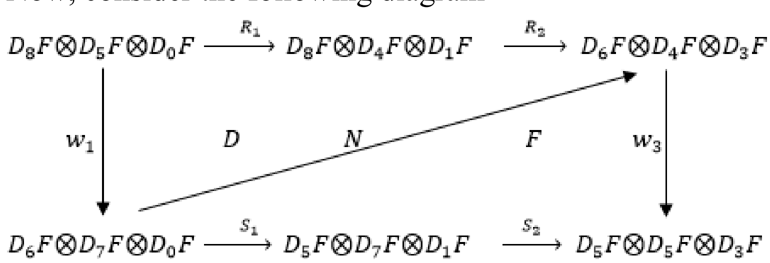
$= \partial_{21}^{(3)} \circ \partial_{32}^{(2)} + \frac{1}{2}\partial_{21}^{(2)} \circ \partial_{32} \circ \partial_{31} + \partial_{21} \circ \partial_{31}^{(2)}$

$(S_2 \circ w_2)(v) = \partial_{32}^{(2)} \circ \partial_{21}^{(3)}$

$= \partial_{21}^{(3)} \circ \partial_{32}^{(2)} + \frac{1}{2}\partial_{21}^{(2)} \circ \partial_{32} \circ \partial_{31} + \partial_{21} \circ \partial_{31}^{(2)}$

$= (w_3 \circ R_2)(v)$

Now, consider the following diagram



Define $N(v): D_6F \otimes D_7F \otimes D_0F \longrightarrow D_6F \otimes D_4F \otimes D_3F$, such that

$N(v) = \partial_{32}^{(3)}(v) \qquad ; v \in D_6F \otimes D_7F \otimes D_0F$

Proposition (2.3): The diagram D is commutative.
Proof: we should proof that $(R_2 \circ R_1)(v) = (N \circ w_1)(v)$

Now, $(R_2 \circ R_1)(v) = (\frac{1}{3}\partial_{21}^{(2)} \circ \partial_{32}^{(2)} + \frac{1}{2}\partial_{21} \circ \partial_{32} \circ \partial_{31}^{(2)}) \circ \partial_{32}$

$= \partial_{21}^{(2)} \circ \partial_{32}^{(3)} + \partial_{21} \circ \partial_{32}^{(2)} \circ \partial_{31} + \partial_{31}^{(2)} \circ \partial_{32}$

And, $(N \circ w_1)(v) = \partial_{32}^{(3)} \circ \partial_{21}^{(2)}$

$= \partial_{21}^{(2)} \circ \partial_{32}^{(3)} + \partial_{21} \circ \partial_{32}^{(2)} \circ \partial_{31} + \partial_{31}^{(2)} \circ \partial_{32}$

Proposition (2.4): The diagram F is commutative.
Proof: we should proof that $(w_2 \circ N)(v) = (S_2 \circ S_1)(v)$

Now, $(w_2 \circ N)(v) = \partial_{21} \circ \partial_{32}^{(3)}$

$= \partial_{32}^{(3)} \circ \partial_{21} - \partial_{32}^{(2)} \circ \partial_{31}$

$= \partial_{32}^{(3)} \circ \partial_{21} - \partial_{31} \circ \partial_{32}^{(2)}$

And $(S_2 \circ S_1)(v) = (\frac{1}{3}\partial_{32} \circ \partial_{21} - \frac{1}{2}\partial_{31}) \circ \partial_{32}^{(2)}$

$= \partial_{32}^{(3)} \circ \partial_{21} - \partial_{31} \circ \partial_{32}^{(2)}$

Lastly, we define the maps ∂_1, ∂_2 and ∂_3 where

$$\partial_3: D_8\otimes D_5\otimes D_0 \xrightarrow{\partial_3} \begin{matrix} D_8\otimes D_4\otimes D_1 \\ \oplus \\ D_6\otimes D_7\otimes D_0 \end{matrix}$$

$\partial_3(x) = (R_1(\varkappa), w_1(\varkappa))$;where $\varkappa \in D_8F \otimes D_5F\otimes D_0F$

$$\partial_2: \begin{matrix} D_8\otimes D_4\otimes D_1 \\ \oplus \\ D_6\otimes D_7\otimes D_0 \end{matrix} \longrightarrow \begin{matrix} D_7\otimes D_5\otimes D_1 \\ \oplus \\ D_6\otimes D_4\otimes D_3 \end{matrix}$$

$\sigma_2((\varkappa_1,\varkappa_2)) = (R_2(\varkappa_1) - N(\varkappa_2)), S_1(\varkappa_2) - w_2(\varkappa_1)$;where $\varkappa \in D_8F \otimes D_4F\otimes D_1F$, $D_6F \otimes D_7F\otimes D_0F$

$$\partial_1: \begin{matrix} D_7\otimes D_5\otimes D_1 \\ \oplus \\ D_6\otimes D_4\otimes D_3 \end{matrix} \longrightarrow D_5\otimes D_5\otimes D_3$$

; where $\varkappa \in D_5F \otimes D_7F\otimes D_1F$, $D_6F \otimes D_4F\otimes D_3F$

Proposition (2.5):

$$0 \longrightarrow D_8\otimes D_5\otimes D_0 \xrightarrow{\partial_3} \begin{matrix} D_8\otimes D_4\otimes D_1 \\ \oplus \\ D_6\otimes D_7\otimes D_0 \end{matrix} \xrightarrow{\partial_2} \begin{matrix} D_7\otimes D_5\otimes D_1 \\ \oplus \\ D_6\otimes D_4\otimes D_3 \end{matrix} \xrightarrow{\partial_1} D_5\otimes D_5\otimes D_3$$

Is complex.

Proof: from definition of place polarization, we have ∂_{21} and ∂_{32} are injective [11-13], and we get ∂_3 is injective, now:

$\sigma_2 \circ \sigma_3(\varkappa) = \sigma_2(R_1(\varkappa), w_1(\varkappa))$

$= \sigma_2(\partial_{32}(\varkappa), \partial_{21}^{(2)}(\varkappa))$

$= (R_2(\varkappa) - N(x_2), S_1((x_2)) - w_2((x_1)))$.

$= (R_2(\partial_{32}(\varkappa)) - N(\partial_{21}^{(2)}(\varkappa)), S_1(\partial_{21}^{(2)}(\varkappa)) - w_1(\partial_{32}(\varkappa)))$.

So,

$R_2(\partial_{32}(\varkappa)) - N(\partial_{21}^{(2)}(\varkappa)) = (\frac{1}{3}\partial_{21}^{(2)} \circ \partial_{32}^{(2)} + \frac{1}{2}\partial_{21} \circ \partial_{32} \circ \partial_{31} + \partial_{31}^{(2)}) \circ \partial_{32}(x) - \partial_{32}^{(3)} \circ \partial_{21}^{(2)}(x)$

$= \partial_{21}^{(2)} \circ \partial_{32}^{(3)} + \partial_{21} \circ \partial_{32}^{(2)} \circ \partial_{31} + \partial_{32}^{(2)} \circ \partial_{31} - \partial_{32}^{(3)} \circ \partial_{21}^{(2)}(x)$

$= (\partial_{21}^{(2)} \circ \partial_{32}^{(3)} + \partial_{21} \circ \partial_{32}^{(2)} \circ \partial_{31} + \partial_{32}^{(2)} \circ \partial_{31} - \partial_{21} \circ \partial_{32}^{(2)} \circ \partial_{31} + \partial_{32}^{(2)} \circ \partial_{31})(x) = 0$

$S_1(\partial_{21}^{(2)}(\varkappa)) - w_1(\partial_{32}(\varkappa)) = (\frac{1}{3}\partial_{32} \circ \partial_{21} - \partial_{31}) \circ \partial_{21}^{(2)}(x) - \partial_{21}^{(3)} \circ \partial_{32}(x)$

$= (\partial_{32} \circ \partial_{21}^{(3)} - \partial_{31} \circ \partial_{21}^{(2)} - \partial_{21}^{(3)} \circ \partial_{32})(x)$

$= (\partial_{21}^{(3)} \circ \partial_{32} - \partial_{21}^{(2)} \circ \partial_{31} - \partial_{31} \circ \partial_{21}^{(2)} - \partial_{21}^{(3)} \circ \partial_{32})(x)$

$= 0$

So, $\sigma_2 \circ \sigma_3(\varkappa) = 0$

And

$\sigma_1 \circ \sigma_2(\varkappa) = (x_1, x_2) = \sigma_1(R_2(\varkappa_1) - N(\varkappa_2), S_1(\varkappa_2) - w_2(\varkappa_1))$

$= \sigma_1((\frac{1}{3}\partial_{21}^{(2)}\partial_{32}^{(2)} + \frac{1}{2}\,\partial_{21} \circ \partial_{32} \circ \partial_{31} + \partial_{31}^{(2)})(\varkappa_1) - \partial_{32}^{(3)})(\varkappa_2), (\frac{1}{3}\,\partial_{32} \circ \partial_{21} \circ \partial_{31})(\varkappa_2) - \partial_{21}^{(3)})(\varkappa_1))$

$= \partial_{21} \circ (\frac{1}{3}\partial_{21}^{(2)} \circ \partial_{32}^{(2)}(\varkappa_1) + \frac{1}{2}\,\partial_{21} \circ \partial_{32} \circ \partial_{31} + \partial_{31}^{(2)}(\varkappa_1) - \partial_{32}^{(3)})(\varkappa_2) + \partial_{32}^{(2)} \circ ((\frac{1}{3}\,\partial_{32} \circ \partial_{21} - \partial_{31}^{(1)})(\varkappa_2) + \partial_{21}^{(3)})(\varkappa_1))$

$= (\partial_{21}^{(3)} \circ \partial_{32}^{(2)} + \partial_{21}^{(2)} \circ \partial_{32} \circ \partial_{31} + \partial_{21} + \partial_{21} \circ \partial_{31}^{(2)} - \partial_{32}^{(2)} \circ \partial_{21}^{(3)})(x_1) + \partial_{32}^{(3)} \circ \partial_{21} - \partial_{32}^{(2)} \circ \partial_{31} - \partial_{21} \circ \partial_{32}^{(3)})(x_2)$

$= \left(\partial_{21}^{(3)} \circ \partial_{32}^{(2)} + \partial_{21}^{(2)} \circ \partial_{32} \circ \partial_{31} + \partial_{21} \circ \partial_{31}^{(2)} - \partial_{21}^{(3)} \circ \partial_{32}^{(2)} - \partial_{21}^{(2)} \circ \partial_{32} \circ \partial_{31} - \partial_{21} \circ \partial_{31}^{(2)}\right)(x_1) + \partial_{21} \circ \partial_{32}^{(3)} + \partial_{32}^{(2)} \circ \partial_{31} - \partial_{21} \circ \partial_{32}^{(3)})(x_2) = 0$

Conclusions

By using a diagram, Capelli identities we find the terms of the complex of characteristic zero for the case (5, 5, 3;1, 0) and we conclusion that the sequence of these terms is complex.

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