Further Acceleration of Two-Point Bracketing Method for Determining the Voltages of Nonlinear Equation

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ABSTRACT

A general class of three point iterative algorithms for solving nonlinear equations based on single diode model of a PV cell is constructed in the present work. Three step iterative methods for solving nonlinear equation of solar cell, Illinois Algorithm and Two-Point Bracketing algorithm are presented and analyzed. In addition, the absolute error values for all the algorithms have been discussed and compared. The new proposed method has the lesser number of iterations than the other ones has been found. Numerical examples are contained to test and investigate the performance, accuracy and efficiency of these methods.

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1. Introduction

During the last decade, iterative methods for solving nonlinear equations of engineering and science fields have been introduced in many articles. The main aim of these articles were the construction of the iterative methods based on computational efficiency as high as possible, which supposes the structure or style of iterative algorithms having the faster convergence based on the number of evaluations of the functions per iteration. Some iterative algorithms of nonlinear equations using free of second derivatives have been presented and analyzed, and there are many methods improved on the advancement the convergence of the standard iterative method, in order to attain lesser iterations than it [1-115].

The suggested algorithm TPBM requires 6 evaluations of the function while the other technique (IRFM) needs 7 evaluation of the function. The following steps are investigate the procedure of this work: section two, three and four investigating the modeling and the root finding of IRFM and TPBM algorithms respectively while; section five and six indicate the numerical problems, discussion and conclusion results respectively.

2. Equation : Non-Linear Formula

KCL Kirchhoff’s law is employed in order to depict the electrical parameters of PV cell scheme [30-50]

\[ I = I_{ph} - I_{Diode}, \quad I_{Diode} = I_0 \left[ \exp\left(\frac{-V_{pv}}{nV_T}\right) - 1 \right] \]  

where:

- \( I_0 \) is diode reverse saturation current measured in (A),
- \( I_{ph} \) is light current,
- \( n \) is diode ideality factor (unitless),
- \( k = (1.38 \times 10^{-23} \text{J/K}) \) is Boltzmann constant,
- \( q = (1.602 \times 10^{-19} \text{C}) \) is elementary charge,
- \( V_T \) is thermal voltage given by \( V_T = \frac{kT}{q} \),
- \( I_{ph} \) is the light generated current in the cell,
- \( T \) is temperature (p-n junction),
- \( I_0 \) is the voltage dependent current lost to recombination.

The current \( I_{pv} \) and power \( P_{pv} \) of the cell is given by \( I_{pv} = \frac{V_{pv}}{R}, \quad P_{pv} = I_{pv} \times V_{pv} \)

The final equation from the circuit is given by

\[ (I_{source}) - 10^{-12} \left( e^{\frac{-V}{1.2+0.026}} - 1 \right) = V / R \]  

3. Illinois Algorithm (IRFM)

\( x_0 \) is the Initial value, \( x_{n+1} \) is the approximation value

Step 1: Suppose \( f(x_1) = b_1, \quad f(x_2) = b_2 \)

Step 2: \( f(x) = ax + b; \)

Step 3: \( x = \frac{b_1x_2 - b_2x_1}{(b_1 - b_2)} \)  

4. Two Point Bracketing Method (TPBM)

Step 1: for a given \([a_k, b_k]\)

Step 2: compute \( c_k \) as follows \( c_k = \frac{a_k+b_k}{2}, \) \( c_k \) is between \( a_k \) and \( b_k \)

Step 3: If \( |x_{n+1} - x_n| < \varepsilon, \quad |f(x_n)| < \varepsilon, \quad \varepsilon = 10^{-9} \) as a tolerance; stop else go to Step 2.
5. Results and Discussion

Two numerical iterations is suggested to introduce the performance of the Illinois Algorithm (IRFM) represented in Eq. 3 acquired in the present paper in order to solve non-linear equation with the initial value \( x_0 = 1 \) and we compare it with Two-Point Bracketing Method (TPBM) represented in Eq. 4 with two initial values \( x_0 \) and \( x_1 \). For convergence criteria, the distance between two consecutive iterates is based on Eq. 5, less than \( 10^{-9} \). Five examples in Eq. 2 are used for numerical testing with the \( R \) values from 1-5 ohm, represents (load resistance) of the circuit. All determinations are carried out with the algorithm precision introduced in Tables and Figures 1 to 5 and the number of function evaluations needed are extracted from the Eq. 2. The numerical examples and the approximate solutions produced by two techniques for solving Eq. 2.

The following Tables and Figs. indicate that TPBM algorithm needs 5 iterations while IRFM technique need 6 iterations to reach to the convergence which proves that TPBM is faster than IRFM.

Table 1 - From Iterative techniques IRFM and TPBM.

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<thead>
<tr>
<th>Iterations</th>
<th>( V_{ps-IRFM} )</th>
<th>( I_{ps-IRFM} )</th>
<th>( P_{ps-IRFM} )</th>
<th>( V_{ps-TPBM} )</th>
<th>( I_{ps-TPBM} )</th>
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Fig. 1 – Results from IRFM and TPBM techniques.

Table 2 - From Iterative techniques IRFM and TPBM.

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<th>( P_{ps-IRFM} )</th>
<th>( V_{ps-TPBM} )</th>
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Fig. 2 – Results from IRFM and TPBM techniques.

Table 3 - From Iterative techniques IRFM and TPBM.

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Fig. 3 – Results from IRFM and TPBM techniques.

Table 4 - From Iterative techniques IRFM and TPBM.

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<th>Iterations</th>
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### Table 5 - From Iterative techniques IRFM and TPBM.

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### Fig. 5 – Results from IRFM and TPBM techniques.

### 6. Conclusion

The obtained results from the proposed method is comparable with the other methods Newton’s and Two-Point Bracketing algorithms have been noticed in all cases in this paper. Several problems prove that the new proposed method is more accurate, efficient and easy to use with lesser iterations compared with other methods and realizes better than common and classical Newton’s algorithm.

### References


[61] S Gharbi, R Dhahri, M Rasheed, E Dhahri, R Barillé, M Rguiti, A Tozri, Mohamed R Berber, "Effect of Bi substitution on nanostructural, morphologic, and electrical behavior of nanocrystalline La1-xBixNi0.5Ti0.5O3 (x= 0 and x= 0.2) for the electrical devices", Materials Science and Engineering: B, 270, 115191, (2021).


