A Class of Methods for Solving Nonlinear Equation

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ABSTRACT

We propose a new two-step iterative technique for solving nonlinear equations for PV cell on single diode based on the equivalent circuit of a solar cell. Some numerical examples show that the new proposed technique is comparable with the standard, classical methods and compete with Two-Point Bracketing technique. A new iterative method is used to compute the zeros of nonlinear equations of the cell.

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1. Introduction

One of the most important subjects in numerical methods is solving nonlinear equations of the functions. More attention has offered to improve many iterative methods for solving nonlinear equations of the functions for example PV cell on single diode. Rasheed et al. have suggested and studied many techniques for solving nonlinear equations with order of convergence using decomposition technique of the Two-Point Bracketing technique and other standard and classical methods. They used one of the first order derivatives of the functions, which is not a serious drawback. Therefore, we propose and study an algorithm for solving nonlinear equations which does not been included the derivative with the higher order of the function [1-115].

TPBM requires 8 evaluations of the function while the other technique (DM) needs 6 evaluation of the function. The following steps are investigate the procedure of this work: section two, three and four investigating the modelling and the root finding of TPBM and DM algorithms respectively while; section five and six indicate the numerical problems, discussion and conclusion results respectively.

2. PV Model’s Formula

KCL Kirchhoff’s law is employed in order to depict the electrical parameters of PV cell scheme [30-50]

\[ I = I_{ph} - I_{Diode}, I_{Diode} = I_0 \left[ \exp \left( \frac{-V_{pv}}{nVT} \right) - 1 \right] \]  \( (1) \)

where:

\[ I_0 \] is diode reverse saturation current measured in (A), \( I_{ph} \) is light current, \( n \) is diode ideality factor (unitless), \( k = (1.38 \times 10^{-23}) / K \) is Boltzmann constant, \( q = (1.602 \times 10^{-19}) \) is elementary charge, \( V_T \) is thermal voltage given by \( V_T = \frac{kT}{q} \), \( I_{ph} \) is the light generated current in the cell, \( T \) is temperature (p-n junction), \( I_0 \) is the voltage dependent current lost to recombination.

The current \( I_{pv} \) and power \( P_{pv} \) of the cell is given by \( I_{pv} = \frac{V_{pv}}{R} \), \( P_{pv} = I_{pv} \times V_{pv} \)

The final equation from the circuit is given by

\[ (I_{source} - 10^{-12}) \left( \frac{V}{6.12+0.026} - 1 \right) = V / R \] \( (2) \)

3. Two-Point Bracketing Method (TPBM)

Step 1: for a given \([a_k, b_k]\]

Step 2: compute \( c_k \) as follows \( c_k = \frac{a_k + b_k}{2} \), \( c_k \) is between \( a_k \) and \( b_k \). \( (3) \)

Step 3: If \( |x_{n+1} - x_n| < \varepsilon, |f(x_n)| < \varepsilon, \varepsilon = 10^{-9} \) as a tolerance; stop else go to Step 2.

4. Dekker’s Algorithm (DM)

This method obtain when we combine the Bisection and Secant Methods achieved by Dekker in 1969.

Step 1: The first one called linear interpolation secant method using the following formula

\[ x_{n+1} = \begin{cases} x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) & \text{if } f(x_{n-1}) \neq f(x_n) \\ m & \text{otherwise} \end{cases} \] \( (4) \)

Step 2: the second one can be obtained by bisection method

\[ m = \frac{a_n + b_n}{2} \]
Step 3: If\(|f(a_n)|\geq|f(b_n)|, |f(x_n)| < \epsilon, \epsilon = 10^{-9}\) as a tolerance; stop else go to Step 1. 

where: \(a_n\): the "contrapoint" this means that \(f(x_n)\) and \(f(b_k)\) have opposite signs, so the interval \([a_n, b_n]\) consist of the solution.

5. Results and Discussion

The estimate for the root of Eq. 2, two numerical iterations is applied to introduce the performance of the algebraic expression (TPBM)-Eq. 3 acquired in the present paper in order to solve non-linear equation with the initial value \(x_0 = 1\) and we compare it with other algebraic expression (DM)-Eq. 4 with two initial values \(x_0\) and \(x_1\). For convergence criteria, the distance between two consecutive iterates is based on Eq. 5, less than \(10^{-9}\).

Five examples have been applied in Eq. 2 for numerical testing, R-values (1-5) ohm-(load resistance) of the circuit. All determinations are carried out with the algorithm precision introduced in Tables and Figures 1 to 5 and the number of function evaluations needed are extracted from the Eq. 2. The numerical examples and the approximate solutions produced by two techniques for solving Eq. 2.

The following Tables and Figs. indicate that TPBM algorithm needs 8 iterations while DM technique need 6 iterations to reach to the convergence which proves that DM is faster than TPBM.

<table>
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<th>Iterations</th>
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<th>(I_p)-TPBM</th>
<th>(P_p)-TPBM</th>
<th>(V_p)-DM</th>
<th>(I_p)-DM</th>
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Fig. 1 – Analysis problems / examples of non-linear equation based on (TPBM and DM) models.
### Table 2 - Analysis of two iterative methods (TPBM and DM).

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<th>Iterations</th>
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<th>$I_{p^*}$-TPBM</th>
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### Fig. 2 – Analysis problems / examples of non-linear equation based on (TPBM and DM) models.

### Table 3 - Analysis of two iterative methods (TPBM and DM).

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<th>Iterations</th>
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<th>$I_{p^*}$-TPBM</th>
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<th>$V_{p^*}$-DM</th>
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### Fig. 3 – Analysis problems / examples of non-linear equation based on (TPBM and DM) models.
Table 4 - Analysis of two iterative methods (TPBM and DM).

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Fig. 4 – Analysis problems / examples of non-linear equation based on (TPBM and DM) models.

Table 5 - Analysis of two iterative methods (TPBM and DM).

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Fig. 5 – Analysis problems / examples of non-linear equation based on (TPBM and DM) models.

6. Conclusion

We suggested and analyzed new two-step iterative methods for solving nonlinear equations in the science and engineering field such as PV cell based on equivalent circuit of a single diode model. The new proposed iterative
method DM has good efficient, some numerical experiments clarify the accuracy and performance of the proposed method is better than Two-Point Bracketing algorithm which has sixth iterations.

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[61] S Gharbi, R Dhahti, M Rasheed, E Dhahti, R Barille, M Rguiti, A Toztri, Mohamed R Berber, "Effect of Bi substitution on nanostructural, morphologic, and electrical behavior of nanocrystalline La1-xBixNi0. 5Ti0. 5O3 (x= 0 and x= 0.2) for the electrical devices", Materials Science and Engineering: B, 270, 115191, (2021).


