Efficient Numerical Algorithms for Solving Nonlinear Equation

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ABSTRACT

Some iterative numerical methods of common family of iterative method for solving nonlinear equations depending on single diode model of a solar cell with are presented in this paper. Several numerical experiments have been introduced. The results obtained show that the new method is more accurate, easy to use, and efficient than other numerical methods with lesser computations than the other methods.

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1. Introduction

In numerical analysis, the iterative method is a mathematical formula, which employs starting values in order to produce a sequence of developing approximate solutions for a family of examples or experiments. A special application of an iterative numerical technique including terminus criteria is an algorithm of the iterative method. There are several algorithms such as two-step method; three step method; multi step method; Bisection method; secant method; two point bracketing method; Newton’s and modified Newton’s methods; implicit method; Halley’s method; Aitken’s extrapolation and modified Aitken’s algorithms are used by many researchers for solving nonlinear equations of the function for a solar cell application [1-115].

The suggested algorithm MAEM requires 5 evaluations of the function while the other technique (DM) needs 3 evaluation of the function. The following steps are investigating the procedure of this work: section two, three and four investigating the modelling and the root finding of MAEM and DM algorithms respectively while; section five and six indicate the numerical problems, discussion and conclusion results respectively.

2. Fundamentals of a Non-Linear PV Equation

KCL Kirchhoff’s law is employed in order to depict the electrical parameters of PV cell scheme [21-31]

\[ I = I_{ph} - I_{Diode} \]

\[ I_{Diode} = I_0 \left( \exp \left( \frac{-V}{nV_T} \right) - 1 \right) \]

where:

- \( I_0 \) is diode reverse saturation current measured in (A), \( I_{ph} \) is light current, \( n \) is diode ideality factor (unitless),
- \( k = (1.38 \times 10^{-23})/K \) is Boltzmann constant, \( q = (1.602 \times 10^{-19}) \) is elementary charge, \( V_T \) is thermal voltage given by \( V_T = \frac{kT}{q} \), \( I_{ph} \) is the light generated current in the cell, \( T \) is temperature (p-n junction), \( I_D \) is the voltage dependent current lost to recombination.

The current \( I_{pv} \) and power \( P_{pv} \) of the cell is given by \( I_{pv} = \frac{V_{pv}}{R} \) \( P_{pv} = I_{pv} \times V_{pv} \)

The final equation from the circuit is given by

\[ (I_{source}) - 10^{-12} \left( e^{\frac{-V}{1.2+0.26}} - 1 \right) = V / R \]

3. Modified Aitken’s Algorithm (MAEM)

The following steps is indicating Aitken’s extrapolation

Step 1: Given: \( x_0, \varepsilon = 10^{-9}, N, f, df, \) and For \( i = 1 \) to \( 2 \)

Step 2: Calculate \( E_n = E_{n+2} - \frac{(E_{n+2} - E_{n+1})^2}{E_{n+2} - 2E_{n+1} + E_n} \) for \( n = 0, 1, 2, \ldots \)

Step 3: If \( f(x_i) = 0 \) or \( f(x_i) < \varepsilon \), then go to Step 6

Step 4: Set \( E_{n+1} = E_n \)

Step 5: \( n = n + 1, i = i + 1, \) go back to Step 2.

Step 6: OUTPUT \( x_{n+1} \) and stop iteration.

4. Dekker’s Algorithm (DM)

This method obtain when we combine the Bisection and Secant Methods achieved by Dekker in 1969.
Step 1: The first one called linear interpolation secant method using the following formula

\[
x_{n+1} = \begin{cases} 
  x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) & \text{if } f(x_{n-1}) \neq f(x_n) \\
  m & \text{otherwise}
\end{cases}
\]

(4)

Step 2: the second one can be obtained by bisection method

\[ m = \frac{a_n + b_n}{2} \]

where: \( a_n \): the ”contrapoint” this means that \( f(x_n) \) and \( f(b_n) \) have opposite signs, so the interval \([a_n, b_n]\) consist of the solution.

For the two algorithms, the tolerance is \(|f(a_n)| \geq |f(b_n)|, |f(x_n)| < \varepsilon, \varepsilon = 10^{-9}\).

5. Results and Discussion

Two numerical iterations is suggested to introduce the performance of the Modified Aitken's Algorithm (MAEM) represented in Eq. 3 acquired in the present paper in order to solve non-linear equation with the initial value \( x_0 = 1 \) and we compare it with Dekker's Algorithm (DM) represented in Eq. 4 with two initial values \( x_0 \) and \( x_1 \). For convergence criteria, the distance between two consecutive iterates is based on Eq. 5, less than \( 10^{-9} \). Five examples in Eq. 2 are used for numerical testing with the R values from 1-5 ohm, represents (load resistance) of the circuit. All determinations are carried out with the algorithm precision introduced in Tables and Figures 1 to 5 and the number of function evaluations needed are extracted from the Eq. 2. The numerical examples and the approximate solutions produced by two techniques for solving Eq. 2.

The following Tables and Figs. indicate that MAEM algorithm needs 5 iterations while DM technique need 3 iterations to reach to the convergence which proves that DM is faster than MAEM.

<table>
<thead>
<tr>
<th>Table 1 - Solved: Eq. 2 – numerical algorithms MAEM and DM.</th>
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<tbody>
<tr>
<td>Iterations</td>
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<tr>
<td>-----</td>
</tr>
<tr>
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</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
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</table>

Fig. 1 – Existence results and numerical solutions for Eq. 2: PV equation.
Table 2 - Solved: Eq. 2 – numerical algorithms MAEM and DM.

<table>
<thead>
<tr>
<th>Iterations</th>
<th>$V_p$-MAEM</th>
<th>$I_p$-MAEM</th>
<th>$P_p$-MAEM</th>
<th>$V_p$-DM</th>
<th>$I_p$-DM</th>
<th>$P_p$-DM</th>
<th>$\varepsilon$-MAEM</th>
<th>$\varepsilon$-DM</th>
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<td>0.421834870</td>
<td>0.917036183</td>
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Fig. 2 – Existence results and numerical solutions for Eq. 2: PV equation.

Table 3 - Solved: Eq. 2 – numerical algorithms MAEM and DM.

<table>
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<tr>
<th>Iterations</th>
<th>$V_p$-MAEM</th>
<th>$I_p$-MAEM</th>
<th>$P_p$-MAEM</th>
<th>$V_p$-DM</th>
<th>$I_p$-DM</th>
<th>$P_p$-DM</th>
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Fig. 3 – Existence results and numerical solutions for Eq. 2: PV equation.

Table 4 - Solved: Eq. 2 – numerical algorithms MAEM and DM.

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<th>Iterations</th>
<th>$V_p$-MAEM</th>
<th>$I_p$-MAEM</th>
<th>$P_p$-MAEM</th>
<th>$V_p$-DM</th>
<th>$I_p$-DM</th>
<th>$P_p$-DM</th>
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6. Conclusion

This study reports many formulas and families of three step iterative algorithms for solving nonlinear equations of PV cell based on electrical circuit diagram of single diode mode. This circuit analyzed using Kirchhoff's current law. The proposed method do not need evaluation of the second derivative of the function $f(x)$. The obtained results from the numerical examples verify the assertiveness. Thus; the new method is more accurate, efficient and easy to use with lesser iterations compared with other methods and realizes better than common and classical ones.

References


