An Accelerated Algorithm for Finding Roots of Nonlinear Equation

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This paper concerns with a new numerical iterative method for finding the approximate solutions deals with the nonlinear equations of a solar cell (single-diode) formula associated with equivalent electrical circuit. The iterative method suggested here is free from second derivative of the function. The present study proposes to show that this new root-finding algorithm combined with a standard iterative method is capable to give an accurate solution of zero's function than classical method. Numerical experiments of the improved iterative method is analyzed and discussed on nonlinear equations.

Keywords: Dekker's algorithm; Implicit algorithm; numerical experiments; photovoltaic cell; first derivative; absolute error.

MSC: 41A25; 41A35; 41A36

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1. Introduction

Per iterative methods; Let the given equation be \( f(x) = 0 \) and the value of \( x \) to be calculated. Using the Iteration method, one can obtain the zeros of the linear or nonlinear equations. In order to get the zeros of the equation first we have to write the equation in the form of \( x = p(x) \); then Let \( x = x_0 \) be a starting approximation of the required root \( \alpha \), then the first approximation \( x_1 \) is obtained by the formula \( x_1 = p(x_0) \) using the same procedure for second, third and so on. The approximation is expressed as follows \( x_2 = p(x_1), x_3 = p(x_2), x_4 = p(x_3), x_n = p(x_{n-1}) \). Solving nonlinear equations is a case very often performed in variant areas of engineering and physical sciences. In all cases, Newton’s method also called as Newton–Raphson technique is most popularly employed for approximating the numerical solutions of nonlinear equations [1-115].

The suggested algorithm IM requires 6 evaluations of the function while the other technique (DM) needs 5 evaluation of the function. The following steps are investigate the procedure of this work: section two, three and four investigating the modelling and the root finding of IM and DM algorithms respectively while; section five and six indicate the numerical problems, discussion and conclusion results respectively.

2. Introduction and Characteristics of a Nonlinear Equation

KCL Kirchhoff’s law is employed in order to depict the electrical parameters of PV cell scheme [21-31]

\[
I = I_{ph} - I_{Diode}, I_{Diode} = I_0 \left[ \exp \left( \frac{-V_{pv}}{nV_T} \right) - 1 \right]
\]

where:

- \( I_0 \) is diode reverse saturation current measured in (A), \( I_{ph} \) is light current, \( n \) is diode ideality factor (unitless),
- \( k = (1.38 \times 10^{-23})/K \) is Boltzmann constant, \( q = (1.602 \times 10^{-19}) \) is elementary charge, \( V_T \) is thermal voltage given by \( V_T = \frac{kT}{q} \), \( I_{ph} \) is the light generated current in the cell, \( T \) is temperature (p-n junction), \( I_D \) is the voltage dependent current lost to recombination.

The current \( I_{pv} \) and power \( P_{pv} \) of the cell is given by \( I_{pv} = \frac{V_{pv}}{R}, P_{pv} = I_{pv} \times V_{pv} \)

The final equation from the circuit is given by

\[
(I_{source}) - 10^{-12} \left( \frac{-V}{e^{1.2+0.026} - 1} \right) = V / R
\]

3. Implicit Method (IM)

The following points suggestion for solving this method

Step 1: approximate solution \( x_0 = 1 \)

Step 2: compute first derivative of the function \( f \)

\[
\hat{f}(x_{n+1}) = x_n - \frac{f(x_n)}{f'(x_n)} \text{ for } n = 0, 1, 2, ...
\]

Step 3: determine \( x \) from the following expression

\[
x_{n+1} = x_n - \frac{2 \times f(x_n)}{f(x_{n+1}) + f(x_n)}
\]

Step 4: compute

\[
x = x_n - \frac{2\times f(x_n)}{f(x_{n+1}) + f(x)}
\]
4. Dekker’s Algorithm (DM)

This method obtain when we combine the Bisection and Secant Methods achieved by Dekker in 1969.

Step 1: The first one called linear interpolation secant method using the following formula

\[ x_{n+1} = \begin{cases} x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) & \text{if } f(x_{n-1}) \neq f(x_n) \\ m & \text{otherwise} \end{cases} \]  

(4)

Step 2: the second one can be obtained by bisection method

\[ m = \frac{a_n + b_n}{2} \]

where: \( a_n \): the "contrapoint" this means that \( f(x_n) \) and \( f(b_k) \) have opposite signs, so the interval \([a_n, b_n]\) consist of the solution.

For the two algorithms, the tolerance is \( |f(a_n)| \geq |f(b_n)|, |f(x_n)| < \varepsilon, \varepsilon = 10^{-9} \).

5. Results and Discussion

Two numerical iterations is suggested to introduce the performance of the Implicit algorithm (IM) represented in Eq. 3 acquired in the present paper in order to solve non-linear equation with the initial value \( x_0 = 1 \) and we compare it with Dekker’s Algorithm (DM) represented in Eq. 4 with two initial values \( x_0 \) and \( x_1 \). For convergence criteria, the distance between two consecutive iterates is based on Eq. 5, less than \( 10^{-9} \). Five examples in Eq. 2 are used for numerical testing with the R values from 1-5 ohm, represents (load resistance) of the circuit. All determinations are carried out with the algorithm precision introduced in Tables and Figures 1 to 5 and the number of function evaluations needed are extracted from the Eq. 2. The numerical examples and the approximate solutions produced by two techniques for solving Eq. 2.

The following Tables and Figs. indicate that IM algorithm needs 6 iterations while DM technique need 5 iterations to reach to the convergence which proves that IM is faster than DM.

**Table 1 - Study numerical techniques of PV model.**

<table>
<thead>
<tr>
<th>Iterations</th>
<th>( V_{pv} - \text{IM} )</th>
<th>( I_{pv} - \text{IM} )</th>
<th>( P_{pv} - \text{IM} )</th>
<th>( V_{pv} - \text{DM} )</th>
<th>( I_{pv} - \text{DM} )</th>
<th>( P_{pv} - \text{DM} )</th>
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Table 2 - Study numerical techniques of PV model.

<table>
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<tr>
<th>Iterations</th>
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<th>$P_{pv}$-IM</th>
<th>$V_{pv}$-DM</th>
<th>$I_{pv}$-DM</th>
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Fig. 1 – Numerical algorithms for Examples based on Eq. 2.

Fig. 2 – Numerical algorithms for Examples based on Eq. 2.

Table 3 - Study numerical techniques of PV model.

<table>
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<tr>
<th>Iterations</th>
<th>$V_{pv}$-IM</th>
<th>$I_{pv}$-IM</th>
<th>$P_{pv}$-IM</th>
<th>$V_{pv}$-DM</th>
<th>$I_{pv}$-DM</th>
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Fig. 3 – Numerical algorithms for Examples based on Eq. 2.

Table 4 - Study numerical techniques of PV model.

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<tr>
<th>Iterations</th>
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6. Conclusion

A new iterative numerical method for approximating the solutions of nonlinear equation of PV cell is investigated in this paper. Based on an iterative procedure improved in a study, we suggest here a type of the numerical algorithm involving the employ of an iterative procedure to solve a nonlinear equation of a solar cell for electronic applications. A detective numerical analysis deals with this suggested method for producing a more accurate approximate solution for nonlinear equations is examined, estimated and explained specific experiments.

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