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Direct Restarted Pell Algorithm for Solving Calculus of Variation Problems

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ABSTRACT

The Pell Polynomial series is employed to solve problems in calculus of variations. The solution can be considered as linear combinations of Pell polynomials with unknown coefficients. Then a calculus of variational problem transforms into an optimization problem and will be solved using mathematical programming techniques. The proposed method converges rapidly to the exact solution and gives accurate results using small number of Pell polynomials basis polynomials. Some illustrative examples are included and the approximate results obtained by restarted Pell series are compared with the exact solutions.

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1. Introduction

In the modeling of a large class of problems in science and engineering, the minimization of a functional is needed. Functional minimization problems known as variational problems. For example, the battery discharging schedules were optimized by utilizing calculus of variations elements and optimal control theory. The energy allocation policy was constructed by solving an optimization problem based on Lagrange multipliers [1, 2]. Several methods have been used to solve variational problems such as Sumudu transform method used in [3] to solve four variational problems and calculate their numerical solutions. In [4], the calculus of variation technique of Hamilton's problem was illustrated by utilizing a case in geotechnics.

The Chebyshev non-uniform finite difference scheme and Bernoulli polynomials approximation with collocation method were used in [5] and [6] respectively to obtain the solution of the two point boundary value problem which arise from problems of calculus of variations by reducing the problem to a set of algebraic equations. In [7], a direct method for solving variational problems using nonclassical parameterization was presented depending on nonclassical orthogonal polynomials to transform a variational problem to a nonlinear mathematical programming problem [1-125].

In this paper, the application of the Pell polynomials for approximates solution of calculus of variational problems is presented. The Pell polynomials were proposed in [8-10] to solve singular differential equations and partial differential equations approximately.

The paper is organized as follows: the statement of the varaitional problem is given in section 2. Section 3 is about the definition of Pell polynomials and their important properties. The proposed direct method is described in section 4 while four numerical examples are included in section 5. Finally, some concluded remarks are listed in section 6.

2. Statement of the Problem

In a large number of problems in analysis, geometry and mechanics, it is necessary to find the minimal and maximal of a certain functional. Such problems are called variational problems and they have an important role in science and engineering.

Let us consider the variational problem

$$\dot{J}(u(t)) = \int_0^1 F(t, u(t), \dot{u}(t))dt \quad (1)$$

Where j is the functional that it's extremum must be determined. In order to find the extreme value of J, the boundary points of the admissible curves are given by

$$u(0) = 0, u(1) = \alpha \quad (2)$$

3. Pell Polynomials: Definition and Properties

Pell polynomials sequence $p_n(t)$ can be defined as below

$$p_n(t) = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} e_{n,k} t^{n-2k}, n > 0$$

where $e_{n,k} = 2^{n-2k} \frac{(n-k)!}{k!(n-2k)!}$

From the definition of Pell polynomials, the first few polynomials are given as

$p_0(t) = 0, p_1(t) = 1, p_2(t) = 2t, p_3(t) = 4t^2 + 1, p_4(t) = 8t^3 + 4t,$

$p_5(t) = 16t^4 + 12t^2 + 1$

Note that $p_n(t)$ that correspond to Pell polynomials can be defined by

$$p_n(t) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ tp_{n-1}(t) + p_{n-2}(t) & \text{if } n > 1 \end{cases}$$

The analytic expression for first Pell derivative is given by [10]

$$\dot{p}_n(t) = 2\sum_{k=0}^{n-1}(-1)^{k+1}p_{k+1}(t) \tag{3}$$

$(k + n)$ even $n \geq 1$

and the relationship between the coefficients expansions and the derivative in Eq. 3 can be constructed as follows

Assume a function $u(t)$ can be expanded in Pell polynomials and it's derivative $\dot{u}(t)$ are respectively

$$u(t) = \sum_{i=1}^{n} ap_i(t) \text{ and } \dot{u}(t) = \sum_{i=1}^{n-1} c_i p_i(t)$$

Then, the relationship between the unknown coefficients a_i and c_i are given as

$$c_n = 0$$

$$c_{n-1} = 2(n-1)a_n$$

$$c_{n-2} = 2(n-1)a_{n-1}$$

$$c_r = 2ra_{r+1} - \frac{1}{r+2}c_{r+2}, r = n-3, n-4, \dots 1$$

4. Solution of the Problem Using Direct Restarted Pell Method

Restarted method is presented in this section by using Pell polynomials basis to solve problems in calculus of variational, Eqns. (1-2) as illustrated in the following steps

Step 1: Approximate $u(t)$ as follows

$$u_1(t, a_1, a_2, a_3) = \sum_{i=0}^{z} a_i P_i(t) \tag{4}$$

Step 2: use the boundary conditions, yields

$$a_1 = -a_3 \tag{5}$$

$$a_2 = \frac{1}{2}(\alpha - 4a_3) \tag{6}$$

Substitute (5) and (6) into (4) to obtain

$$u_1(t, a_3) = -a_3P_1(t) + \frac{1}{2}(\alpha - 4a_3)P_2(t) + a_3P_3(t)$$

Step 3: the variable $\dot{u}_1(t)$ is determined by differentiate Eq. 4 and get

$$\dot{u}_1(t) = a_2\dot{P}_2(t) + a_3\dot{P}_3(t) = \frac{1}{2}(\alpha - 4a_3)\dot{P}_2(t) + a_3\dot{P}_3(t)$$

Step 4: substitute the approximations $u_1(t)$ and $\dot{u}_1(t)$ into the performance index Eq. 1, yields

$J(u_1(t)) = \int_0^1 F(t, u_1(t), \dot{u}_1(t))dt$

If fact the new problem can be started as $\min_{a_3 \epsilon \Re} J(a_3)$

Now, the variables u(t) and $\dot{u}(t)$ can be approximated as a function of time in the next step $u_2(t)$ is approximated for a given m

$a_2(t, a_2, a_3, a_4) = u_1(t) + \sum_{i=2}^{4} a_i P_i(t)$

Using the boundary condition to obtain $a_3 = 0, a_2 = -6a_4$

Now, $u_2(t, a_4) = u_1(t) - 12a_4 P_2(t) + a_4 P_4(t)$

Continue the process to obtain a favorable accuracy

$u_{n+1}(t) = u_n(t) + a_{n+1}P_{n+1}(t) + a_{n+2}P_{n+2}(t) + a_{n+3}P_{n+3}(t)$

As in the previous steps, using the boundary conditions to obtain

$u_{n+1}(0) = u_{n+3}P_{n+3}(0) + a_{n+2}P_{n+2}(0) + a_{n+1}P_{n+1}(0) = 0$

and

$u_{n+1}(1) = u_{n+1}P_{n+3}(1) + a_{n+2}P_{n+2}(1) + a_{n+1}P_{n+1}(1) = 0$

Calculate the unknown coefficients a_{n+1}, a_{n+2} as a function of a_{n+3} as below

$$a_{n+1} = \frac{P_{n+2}(0)+P_{n+3}(1)-P_{n+2}(1)P_{n+3}(0)}{P_{n+1}(0)+P_{n+2}(1)-P_{n+1}(1)P_{n+3}(0)} a_{n+3} \quad (7)$$

and

$$a_{n+2} = \frac{P_{n+1}(0)+P_{n+3}(1)-P_{n+1}(1)P_{n+3}(0)}{P_{n+1}(1)+P_{n+2}(0)-P_{n+1}(0)P_{n+3}(1)} a_{n+3} \quad (8)$$

Rewrite Eqns. (7) and (8) as

$a_{n+1} = \alpha_1 a_{n+3}, a_{n+2} = \alpha_2 a_{n+3}$

where $\alpha_1 = \frac{P_{n+2}(0)+P_{n+3}(1)-P_{n+2}(1)P_{n+3}(0)}{P_{n+1}(0)+P_{n+2}(1)-P_{n+1}(1)P_{n+3}(0)} a_{n+3}$

$$\alpha_2 = \frac{P_{n+1}(0) + P_{n+3}(1) - P_{n+1}(1)P_{n+3}(0)}{P_{n+1}(1) + P_{n+2}(0) - P_{n+1}(0)P_{n+3}(1)} a_{n+3}$$

Then,

$$u_{n+1}(t) = u_n(t) + \alpha_1 a_{n+3} P_{n+1}(1) + \alpha_2 a_{n+3} P_{n+2}(t) + a_{n+3} P_{n+3}(t) \quad (9)$$

and then $\dot{u}_{n+1}(t)$ can be obtained from Eq. 9.

Now, the functional $J(a_{n+3})$ is obtained by determining

$$J(u_{n+1}(t)) = \int_0^1 F(t, u_{n+1}(t), \dot{u}_{n+1}(t))dt$$

5. Numerical Examples

Some examples are considered in this section to illustrate the efficiency of the restarted suggested method. to allow validation of the presented method, comparison with results of the exact solutions are included in this section.

Example 1

This problem is concerned with minimization of

$$J(u(t)) = \int_0^1 [\dot{u}^2(t) + t\,\dot{u}(t)]dt \tag{10}$$

Together with the boundary conditions

$$u(0) = 0\,,\, u(1) = 0.25 \tag{11}$$

To start with the direct restarted Pell algorithm, an approximation $u_1(t)$ is considered as below

$$u_1(t) = -a_3 + \left(\frac{1}{4} - 4a_3\right)t + \alpha_3(4t^2 + 1) \tag{12}$$

and then

$$\dot{u}_1(t) = \frac{1}{4} - 4a_3 + 8ta_3 \tag{13}$$

By substituting (12) and (13) into (10), yields

$$J(a_3) = \frac{64}{3}a_3 + \frac{2}{3}a_3 + 8\left(\frac{1}{4} - 4a_3\right) + \frac{1}{8} + \left(\frac{1}{4} - 4a_3\right)^2 \tag{14}$$

Eq. 14 gives $a_3 = -\frac{1}{16}$ which represents the value for minimizing J. That is $J(a_3) = \frac{1}{6}$ represents the solution of the variational problem (10-11). The variable u(t) can be calculated approximately as

$$u_1(t) = \frac{1}{2}t - \frac{1}{4}t^2$$

Fig. 1 plots the exact solution and the obtained approximate solution. Note that both solutions are identical and equal to $u(t) = 0.5t - 0.25t^2$.

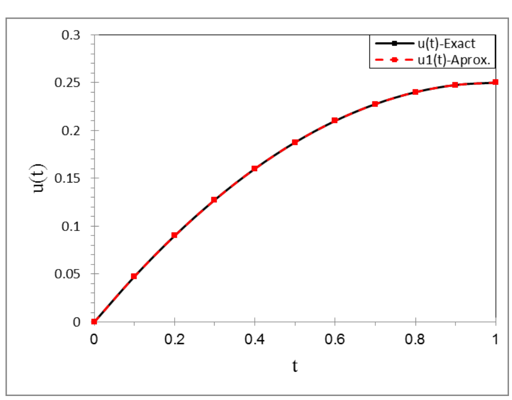

Fig. 1 – The exact solution against the obtained approximate solution for Example 1.

Example 2

Consider the following variational problem of finding the extrumum of the integral

$$J(u(t)) = \int_0^1 [\dot{u}^2(t) + t\,\dot{u}(t) + u^2(t)]dt \quad (15)$$

In addition, the variation problem is subject to the following boundary conditions $u(0) = 0$, $u(1) = 0.25$ (16)

In the following, the approximate solution is performed by using the proposed direct algorithm. So, an approximate solution $u_1(t)$ is considered as below

$$u_1(t) = -a_3 + \left(\frac{1}{4} - 4a_3\right)t + \alpha_3(4t^2 + 1) \quad (17)$$

and then

$$\dot{u}_1(t) = \frac{1}{4} - 4a_3 + 8a_3 t \quad (18)$$

By substituting (17) and (18) into (15) gives

$$J(a_3) = \frac{368}{15}a_3^2 + 10\left(\frac{1}{4} - 4a_3\right)a_3 + \frac{2}{3}a_3 + \frac{4}{3}\left(\frac{1}{4} - 4a_3\right)^2 + \frac{1}{8}$$

The value that minimize J is $a_3 = -\frac{15}{352}$ then $J(a_3) = 0.1977$ is the solution of the variational problem (15). Substituting the value of a_3 into Eq. (17) to determine the variable $u_1(t)$ approximately as

$$u_1(t) = \frac{37}{88}t - \frac{15}{88}t^2$$

In the next step, $u_2(t)$ is given by

$$u_2(t) = \frac{13539}{32384}t - \frac{15}{88}t^2 + \frac{7}{2944}t^3$$

The resulted solution and the analytical solution are illustrated in Fig. 2

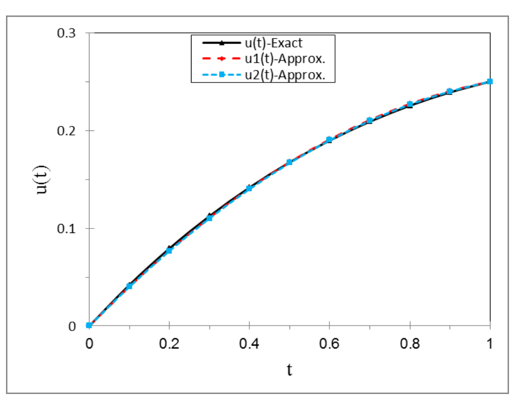

Fig. 2 – The exact solution against the obtained approximate solutions for Example 2.

Example 3

Consider the following test example

$$J(u(t)) = \int_0^1 \left(\dot{u}^2(t) - u^2(t)\right)dt \tag{19}$$

With the boundary conditions $u(0) = 0, u(1) = 1$ (20)

With the exact solution $u(t) = \frac{\sin t}{\sin 1}$.

By using the steps of the suggested Pell technique, one can obtain the following two approximation

$u_1(t) = \frac{23}{18}x - \frac{5}{18}x^2$ (in this case $a_3 = \frac{-5}{72}$)

$u_2(t) = \frac{7055}{5472}x - \frac{5}{18}x^2 - \frac{7}{608}x^3$ (in this case $a_4 = \frac{-7}{4864}$)

The resulted solution and the analytical solution are plotted in Fig. 3

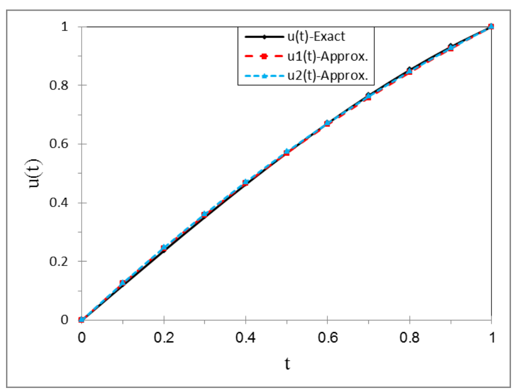

Fig. 3 – The exact solution against the obtained approximate solutions for Example 3.

Example 4

Consider the following problem

$$J(u(t)) = \int_0^1 \left(\dot{u}^2(t) - u^2(t)\right)dt, \tag{21}$$

with the boundary conditions $u(0) = 0, u(1) = 1$ (22)

The exact solution in this case is $u(t) = \frac{e^t - e^{-t}}{e^1 - e^{-1}}$.

Following the steps of the suggested Pell technique, leads to the following two approximate solutions

$u_1(t) = \frac{17}{22}x + \frac{5}{22}x^2$ (in this case $a_3 = \frac{5}{88}$)

$$u_2(t) = \frac{6179}{8096}x + \frac{5}{22}x^2 + \frac{7}{736}x^3$$

Fig. 4 plots the resulted solution and the analytical solution.

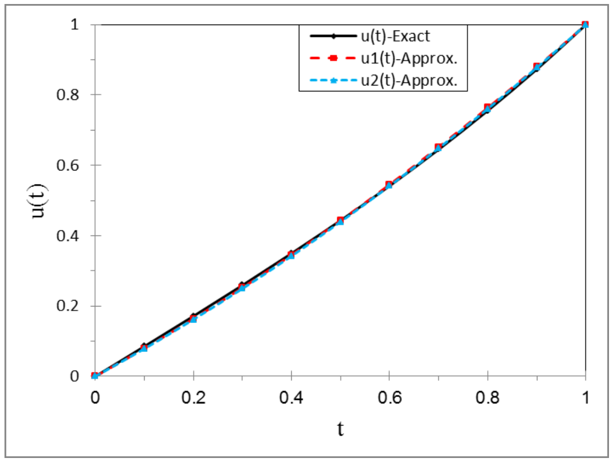

Fig. 4 – The exact solution against the obtained approximate solutions for Example 4.

6. Conclusion

An iterative direct algorithm was proposed based on Pell polynomials to find approximate solution for calculus of variation problem, which can easily applied with a simple computations and fast convergence.

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