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# The Resolution of Three- Mowed Weyl Module in the case of (7, 6, 3) / (1, 0, 0)

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ABSTRACT

The object of this work is investigation a presentation of the characteristic-free resolution and Lascoux resolution of (7, 6, 3)/(1, 0, 0) . A joining between the resolution of Weyl module is also included in this work

**الخلاصة:**

الهدف لهذا العمل هو دراسة تطبيق المميز الحر وتحلل لاسكو في حالة الشكل المنحرف (7, 6, 3) / (t, 0, 0) عندما t = 1 وايضا نقوم بدراسة العلاقة بين تحلل مقاس وايل في هذه الحالة لكل وضع لنفس الشكل المنحرف.

MSC. 41A25; 41A35; 41A36

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## 1. Introduction

let F be a free module over a commutative ring with identity and $D_r F$ be divided power algebra of degree r motivated the free module, [1] the three generators, $Z_{21}$, $Z_{32}$ and $Z_{31}$are the formal polarization operators defined in [2].

Shaymaa and Haytham [3] presented some results for the skew shape (8,6)/(2,0) and (8,6)/(2,1). The authors in [4] and [5] discussed some outcomes for the skew partition (8,6,3)/(u,1) where u=1,2 and the partition (8, 7, 3) . Also Shaymaa and Haytham [6] illustrated the connection between the resolution of Weyl module (8, 6, 3)/(u,1) where u=1,2 for the same skew partition.

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Inthis work, we study an application of the resolution of three-rowed Weyl module for the skew-shape (7, 6, 3) /(1, 0, 0) . First, the terms of characteristic-free resolution are found, then the complex of Lascoux and relationship between character-free resolution and Lascoux is investigated.

## 1. The Results for the skew-shape (7, 6, 3)/(1, 0, 0)

We enforce the following formula for the case of skew-partion $[p, q, r, t_1, t_2]$ to obtain the terms of the resolution for the skew-shape (7, 6, 3)/(1, 0, 0).

$$\mathrm{Res}([p,q;t_1]) \otimes \mathcal{D}_r \oplus \sum_{l\geq 0} \underline{Z}_{32}^{(t_2+1)}\, y \odot \underline{Z}_{32}^{(l)}\, Res\,([p,q+t_2+1+l;t_1+t_2+1+l]) \otimes \mathcal{D}_{(r-(t_2+1+l)} \oplus \underline{Z}_{32}^{(l_2+1)} y \underline{Z}_{31}^{(t_1+1)} z\mathrm{Res}([p+t_1+1, q+l_2+1; t_2])$$

where $\underline{Z}_{ab}^{(n)}$ is the pursue Bar complex

$$0 \longrightarrow Z_{ab} u Z_{ab} u \ldots Z_{ab} \longrightarrow \sum_{k_i\geq 1}, \sum_{k_i=n} Z_{ab}^{(k_1)} u Z_{ab}^{(k_2)} u \ldots Z_{ab}^{(k_n-1)} \longrightarrow \cdots \longrightarrow Z_{ab}^{(n)} \longrightarrow 0$$

Hence

$$\mathrm{Res}([6,6;1]) \otimes \mathcal{D}_3 \oplus \sum_{l\geq 0} \underline{Z}_{32}^{(1)}\, y \underline{Z}_{32}^{(l)}\, \mathrm{Res}([6,7+l;2+l]) \otimes D_{2-l} \quad \oplus$$

$$\underline{Z}_{32}^{(1)} y \underline{Z}_{31}^{(2)} z\mathrm{Res}([8,7;0]) \otimes D_0$$

So

$$\sum_{l\geq 0} Z_{32}^{(1)}\, y \odot \underline{Z}_{32}^{(l)}\, y\, Res([6,7+l;2+l]) \otimes D_{2-l}$$

$$= Z_{32}^{(1)} y Res([6,7;2]) \otimes D_2 \oplus Z_{32}^{(1)} y Z_{32} y Res([6,8;3]) \otimes D_1$$

where $\underline{Z}_{32} y$

$$0 \longrightarrow Z_{32} y \xrightarrow{\partial_y} Z_{32} \longrightarrow 0$$

$\underline{Z}_{32}^{(2)} y$ is the bar complex

$$0 \longrightarrow Z_{32} y Z_{32} y \xrightarrow{\partial_y} Z_{32}^{(2)} y \xrightarrow{\partial_y} Z_{32}^{(2)} \longrightarrow 0$$

$\underline{Z}_{32}^{(3)} y$ is the bar complex

$$0 \longrightarrow Z_{32} y Z_{32} y Z_{32} y \xrightarrow{\partial_y} \begin{matrix} Z_{32}^{(2)} y Z_{32} y \\ \oplus \\ Z_{32} y Z_{32}^{(2)} y \end{matrix} \xrightarrow{\partial_y} Z_{32}^{(3)} y \xrightarrow{\partial_y} Z_{32}^{(3)} \longrightarrow 0$$

and $\underline{Z}_{31}z$ is the bar complex

$$0 \longrightarrow Z_{31}z \xrightarrow{\partial_z} Z_{31} \longrightarrow 0$$

The terms of the characteristic-free resolution for the skew-shape (7, 6, 3)/(1, 0, 0) where $b, b_1, b_2, b_3, b_4, b_5, b_6 \in Z^+$ are:

$(M_0) = D_6 \otimes D_6 \otimes D_3$

$(M_1)$ equal to the total of:

•$Z_{21}^{(b)} \varkappa D_{6+b} \otimes D_{6-b} \otimes D_3$ ; with $2 \leq b \leq 6$

•$Z_{32} y D_6 \otimes D_7 \otimes D_2$

•$Z_{32}^{(2)} y D_6 \otimes D_8 \otimes D_1$

$(M_2)$ equal to the total of:

•$Z_{21}^{(b_1)} \varkappa Z_{21}^{(b_2)} \varkappa D_{6+|b|} \otimes D_{6-b} \otimes D_3$ ;with $3 \leq |b| = b_1 + b_2 \leq 6$

•$Z_{32} y Z_{21}^{(b)} \varkappa D_{6+b} \otimes D_{6-b} \otimes D_2$ ; with $3 \leq b \leq 7$

•$Z_{32} y Z_{32} y D_6 \otimes D_8 \otimes D_1$

•$Z_{32} y Z_{31}^{(2)} z D_8 \otimes D_7 \otimes D_0$

$(M_3)$ equal to the total of:

•$Z_{21}^{(b_1)} \varkappa Z_{21}^{(b_2)} \varkappa Z_{21}^{(b_3)} \varkappa D_{6+|b|} \otimes D_{6-|b|} \otimes D_3$ ;where $4 \leq |b| = \sum_{i=1}^{3} b_i \leq 6$ and $b_1 \geq 2$

•$Z_{32} y Z_{21}^{(b_1)} \varkappa Z_{21}^{(b_2)} \varkappa D_{6+|b|} \otimes D_{6-|b|} \otimes D_2$ ; where $4 \leq |b| = b_1 + b_2 \leq 7$ and $b_1 \geq 3$

•$Z_{32} y Z_{32} y Z_{21}^{(b)} \varkappa D_{6+b} \otimes D_{8-b} \otimes D_1$ ; where $4 \leq b \leq 8$

•$Z_{32} y Z_{31}^{(2)} z Z_{21}^{(b)} \varkappa D_{8+b} \otimes D_{7-b} \otimes D_0$ ; where $1 \leq b \leq 7$

$(M_4)$ equal to the total of:

•$Z_{21}^{(b_1)} \varkappa Z_{21}^{(b_2)} \varkappa Z_{21}^{(b_3)} \varkappa Z_{21}^{(b_4)} \varkappa D_{6+|b|} \otimes D_{6-|b|} \otimes D_3$

;where $5 \leq |b| = \sum_{i=1}^{4} b_i \leq 6$ and $b_1 \geq 2$

•$Z_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa D_{7+|b|}\otimes D_{7-|b|}\otimes D_2$

; where $5 \le |b| = \sum_{i=1}^{4} b_i \le 8$ and $b_1 \ge 3$

•$Z_{32}yZ_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa\ D_{6+|b|}\otimes D_{8-|b|}\otimes D_1$

; where $5 \le |b| = b_1 + b_2 \le 8$ and $b_1 \ge 4$

•$Z_{32}yZ_{31}^{(2)}zZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa D_{8+|b|}\otimes D_{7-|b|}\otimes D_0$

; where $2 \le |b| = b_1 + b_2 \le 7$ and $b_1 \ge 1$

$(M_5)$ equal to the total of:

•$Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{12}\otimes D_0\otimes D_3$

•$Z_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa Z_{21}^{(b_4)}\varkappa D_{6+|b|}\otimes D_{7-|b|}\otimes D_2$

; where $6 \le |b| = \sum_{i=1}^{4} b_i \le 7$ and $b_1 \ge 3$

•$Z_{32}yZ_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa D_{6+|b|}\otimes D_{8-|b|}\otimes D_1$

; where $6 \le |b| = \sum_{i=1}^{3} b_i \le 8$ and $b_1 \ge 4$

•$Z_{32}yZ_{31}^{(2)}zZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa D_{8+|b|}\otimes D_{7-|b|}\otimes D_0$

; where $3 \le |b| = \sum_{i=1}^{3} b_i \le 7$ and $b_1 \ge 1$

$(M_6)$ equal to the total of:

•$Z_{32}yZ_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{13}\otimes D_0\otimes D_2$

•$Z_{32}yZ_{32}yZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa Z_{21}^{(b_4)}\varkappa D_{6+|b|}\otimes D_{8-|b|}\otimes D_1$

;where $7 \le |b| = \sum_{i=1}^{4} b_i \le 8$ and $b_1 \ge 4$

•$Z_{32}yZ_{31}^{(2)}zZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa Z_{21}^{(b_4)}\varkappa D_{8+|b|}\otimes D_{7-|b|}\otimes D_0$

;where $4 \le |b| = \sum_{i=1}^{4} b_i \le 7$ and $b_1 \ge 1$

$(M_7)$ equal to the total of:

•$Z_{32}yZ_{32}yZ_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{14}\otimes D_0\otimes D_1$

$\bullet Z_{32}yZ_{31}^{(2)}zZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa Z_{21}^{(b_4)}\varkappa Z_{21}^{(b_5)}\varkappa D_{8+|b|}\otimes D_{7-|b|}\otimes D_0$

;where $5 \leq |b| = \sum_{i=1}^{5} b_i \leq 7$ and $b_1 \geq 1$

$(M_8)$ equal to the total of

$\bullet Z_{32}yZ_{31}^{(2)}zZ_{21}^{(b_1)}\varkappa Z_{21}^{(b_2)}\varkappa Z_{21}^{(b_3)}\varkappa Z_{21}^{(b_4)}\varkappa Z_{21}^{(b_5)}\varkappa Z_{21}^{(b_6)}\varkappa D_{8+|b|}\otimes D_{7-|b|}\otimes D_0$

;where $6 \leq |b| = \sum_{i=1}^{6} b_i \leq 7$ and $b_1 \geq 1$

$(M_9)$ equal to the total of

$\bullet Z_{32}yZ_{31}^{(2)}zZ_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa Z_{21}\varkappa D_{15}\otimes D_0\otimes D_0$

As in [7], the Lascoux resolution of the Weyl module associated to the Skew-shape (7, 6, 3)/(1, 0, 0) is

$$0 \longrightarrow D_9F\otimes D_6F\otimes D_0F \longrightarrow \begin{matrix} D_9F\otimes D_4F\otimes D_2F \\ \oplus \\ D_8F\otimes D_7F\otimes D_0F \end{matrix} \longrightarrow \begin{matrix} D_6F\otimes D_7F\otimes D_2F \\ \oplus \\ D_8F\otimes D_4F\otimes D_3F \end{matrix} \longrightarrow D_6F\otimes D_6F\otimes D_3F$$

As in [4], the terms can be exhibit as pursue

$M_0 = \mathcal{L}_0 = E_0$

$M_1 = \mathcal{L}_1 \oplus E_1$

$M_2 = \mathcal{L}_2 \oplus E_2$

$M_3 = \mathcal{L}_3 \oplus E_3$

$M_j = E_j$ ; for $j = 4, 5, \ldots 9$.

where $\mathcal{L}_e$ are the total of the Lascoux terms, and the $E_e$ are the sums of the others.

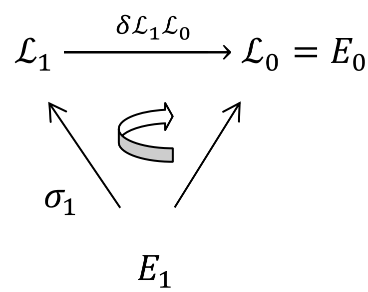
Then we define$\sigma_1 : E_1 \to \mathcal{L}_1$ as follows

- $Z_{21}^{(3)}\varkappa(v) \longmapsto \frac{1}{3} Z_{21}^{(2)}\ \varkappa\partial_{21}(v)$ ;where $v \in D_9\otimes D_3\otimes D_3$
- $Z_{21}^{(4)}\varkappa(v) \longmapsto \frac{1}{6} Z_{21}^{(2)}\ \varkappa\partial_{21}^{(2)}(v)$ ;where $v \in D_{10}\otimes D_2\otimes D_3$
- $Z_{21}^{(5)}\varkappa(v) \longmapsto \frac{1}{10} Z_{21}^{(2)}\ \varkappa\partial_{21}^{(3)}(v)$ ;where $v \in D_{11}\otimes D_1\otimes D_3$
- $Z_{21}^{(6)}\varkappa(v) \longmapsto \frac{1}{15} Z_{21}^{(2)}\ \varkappa\partial_{21}^{(4)}(v)$ ;where $v \in D_{12}\otimes D_0\otimes D_3$

- $Ⱬ_{32}^{(2)}y(v) \longmapsto \frac{1}{2}Ⱬ_{32}y\partial_{32}(v)$ ;where $v \in D_6 \otimes D_8 \otimes D_1$

We ought to indicate that the map $\sigma_1$ satisfied the identity:

$\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1 = \delta_{E_1E_0}$ ... 1




Where $\delta_{\mathcal{L}_1\mathcal{L}_0}$ the component of the boundary of the fat complex which conveys $\mathcal{L}_1$ to $\mathcal{L}_0$.

We will use notation $\delta_{\mathcal{L}_{t+1}\mathcal{L}_t}, \delta_{\mathcal{L}_{t+1}}E_1$ ... $etc.$

Thus we can define $\partial_1 : \mathcal{L}_1 \longrightarrow \mathcal{L}_0$ as $\partial_1 = \delta_{\mathcal{L}_1\mathcal{L}_0}$

It's easy to show that $\partial_1$ implement 1, for example:

$$(\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1)\left(Ⱬ_{21}^{(5)}\varkappa(v)\right) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1\left(\frac{1}{10}Ⱬ_{21}^{(2)}\varkappa\partial_{21}^{(3)}(v)\right)$$

$$= \frac{1}{10}\left(\partial_{21}^{(2)}\partial_{21}^{(3)}(v)\right)$$

$$= \partial_{21}^{(5)}(v)$$

$$= \delta_{E_1E_0}\left(Ⱬ_{21}^{(5)}\varkappa(v)\right)$$

As long as $\partial_2 : \mathcal{L}_2 \longrightarrow \mathcal{L}_1$ as $\partial_2 = \delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2E_1}$

**Proposition 2.1**: The composition $\partial_1 \circ \partial_2 = 0$.

**Proof:**

$$\partial_1 \circ \partial_2(g) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \left(\delta_{\mathcal{L}_2\mathcal{L}_1}(g) + \sigma_1\delta_{\mathcal{L}_2E_1}(g)\right)$$

$$= \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2\mathcal{L}_1}(g) + \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1\delta_{\mathcal{L}_2E_1}(g)$$

$\delta_{\mathcal{L}_1\mathcal{L}_0} \circ \sigma_1 = \delta_{E_1E_0}$, then we get

$\partial_1 \circ \partial_2(g) = \delta_{\mathcal{L}_1\mathcal{L}_0} \circ \delta_{\mathcal{L}_2E_1}(g) + \delta_{E_1E_0}\delta_{\mathcal{L}_2E_1}(g) = 0$

Define $\sigma_2: E_2 \longrightarrow \mathcal{L}_2$ such that:

$\delta_{E_2\mathcal{L}_1} + \sigma_1 \circ \delta_{E_2E_1} = (\delta_{\mathcal{L}_2\mathcal{L}_1} + \sigma_1 \circ \delta_{\mathcal{L}_2E_1}) \circ \sigma_2 \ldots 2$

We define

- $Z_{21}^{(2)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_9 \otimes D_3 \otimes D_3$
- $Z_{21}^{(3)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(4)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(5)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}^{(4)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{32}yZ_{21}^{(4)}\varkappa(v) \longmapsto \frac{1}{4} Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}(v)$ ;where $v \in \mathrm{D}_{10} \otimes \mathrm{D}_3 \otimes \mathrm{D}_2$
- $Z_{32}yZ_{21}^{(5)}\varkappa(v) \longmapsto \frac{1}{10} Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}^{(2)}(v)$ ;where $v \in \mathrm{D}_{11} \otimes \mathrm{D}_2 \otimes \mathrm{D}_2$
- $Z_{32}yZ_{21}^{(6)}\varkappa(v) \longmapsto \frac{1}{20} Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}^{(3)}(v)$ ;where $v \in \mathrm{D}_{12} \otimes \mathrm{D}_1 \otimes \mathrm{D}_2$
- $Z_{32}yZ_{21}^{(7)}\varkappa(v) \longmapsto \frac{1}{35} Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}^{(4)}(v)$ ;where $v \in \mathrm{D}_{13} \otimes \mathrm{D}_0 \otimes \mathrm{D}_2$
- $Z_{32}yZ_{32}y(v) \longmapsto 0$ ;where $v \in \mathrm{D}_6 \otimes \mathrm{D}_8 \otimes \mathrm{D}_1$

Its plainsman to exhibit that $\sigma_2$ which is acquainting above implement 2, we adopt where $v \in \mathrm{D}_{12} \otimes \mathrm{D}_1 \otimes \mathrm{D}_2$

$(\delta_{E_2\mathcal{L}_1} + \sigma_1 \circ \delta_{E_2E_1})\left(Z_{32}yZ_{21}^{(6)}\varkappa(v)\right)$

$= \sigma_1\left(Z_{21}^{(6)}\varkappa\partial_{32}(v) + Z_{21}^{(5)}\varkappa\partial_{31}(v)\right) - Z_{32}y\partial_{21}^{(6)}(v)$

$= \frac{1}{6} Z_{21}\varkappa\partial_{21}^{(5)}\partial_{32}(v) + \frac{1}{5} Z_{21}\varkappa\partial_{21}^{(4)}\partial_{31}(v) - Z_{32}y\partial_{21}^{(6)}(v)$

and

$$(\delta_{\mathcal{L}_2\mathcal{L}_1}+\sigma_1\circ\delta_{\mathcal{L}_2E_1})\left(\frac{1}{20}\,Z_{32}yZ_{21}^{(3)}\varkappa\partial_{21}^{(3)}(v)\right)$$

$$=\sigma_1\left(\frac{1}{20}Z_{21}^{(3)}\varkappa\partial_{21}^{(3)}\partial_{32}(v)\;+\frac{1}{20}\,Z_{21}^{(3)}\varkappa\partial_{21}^{(2)}\partial_{31}(v)+\frac{1}{20}Z_{21}^{(2)}\varkappa\partial_{21}^{(3)}\partial_{31}(v)\right)-$$

$$\frac{1}{20}Z_{32}y\partial_{21}^{(3)}\partial_{21}^{(3)}(v)$$

$$=\frac{1}{6}Z_{21}\varkappa\partial_{21}^{(5)}\partial_{32}(v)+\frac{1}{10}Z_{21}\varkappa\partial_{21}^{(4)}\partial_{31}(v)+\frac{1}{10}Z_{21}\varkappa\partial_{21}^{(4)}\partial_{31}(v)-Z_{32}y\partial_{21}^{(6)}(v)$$

$$=\frac{1}{6}Z_{21}\varkappa\partial_{21}^{(5)}\partial_{32}(v)+\frac{1}{5}Z_{21}\varkappa\partial_{21}^{(4)}\partial_{31}(v)-Z_{32}y\partial_{21}^{(6)}(v)$$

**Proposition (2.2):** we have exactness at $\mathcal{L}_i$.

**Proof:** see [9].

Now by using $\sigma_2$ we can also acquaint $\partial_3$:$\mathcal{L}_3\longrightarrow\mathcal{L}_2$ as $\partial_3=\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3E_2}$.

**Proposition (2.3):**

The composition $\partial_2\circ\partial_3=0$.

**Proof:** the same way employ in proposition (2.1).

**Remark (2.4)**

In our case $t_1=1$ and $t_2=0$, this leads to an inevitable problem of where we want to polarize the $y$ to 1 in the terms of form

$$Z_{32}^{(t_2+1)}yZ_{31}^{(t_1+1)}zRes([p_1+t_1+1,p_2+t_2+1;t_2])$$

Since we have no terms of the form $Z_{32}^{(1)}Z_{31}^{(2)}z$.

It is clear that by remark (2.4) all the terms in dimension three will be reduction to zero, we can show that:

Firstly we need the definition of map

$\sigma_3$: $E_3\longrightarrow\mathcal{L}_3$ such that $\delta_{E_3\mathcal{L}_2}+\sigma_2\circ\delta_{E_3E_2}=(\delta_{\mathcal{L}_3\mathcal{L}_2}+\sigma_2\circ\delta_{\mathcal{L}_3E_2})\circ\sigma_3$ (3)

As follows:

- $Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{10} \otimes D_2 \otimes D_3$
- $Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_1 \otimes D_3$
- $Z_{21}^{(4)}\varkappa Z_{21}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}^{(3)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(3)}\varkappa Z_{21}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{21}^{(2)}\varkappa Z_{21}\varkappa Z_{21}^{(3)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_0 \otimes D_3$
- $Z_{32}yZ_{21}^{(3)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{10} \otimes D_3 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_2 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}\varkappa Z_{21}^{(3)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_1 \otimes D_2$
- $Z_{32}yZ_{21}^{(6)}\varkappa Z_{21}\varkappa(v) \longmapsto 0$ ;where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(5)}\varkappa Z_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(4)}\varkappa Z_{21}^{(3)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{21}^{(3)}\varkappa Z_{21}^{(4)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{13} \otimes D_0 \otimes D_2$
- $Z_{32}yZ_{32}yZ_{21}^{(4)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{10} \otimes D_4 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(5)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_3 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(6)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_2 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(7)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{13} \otimes D_1 \otimes D_1$
- $Z_{32}yZ_{32}yZ_{21}^{(8)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{14} \otimes D_0 \otimes D_1$
- $Z_{32}yZ_{31}^{(2)}zZ_{21}^{(2)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{10} \otimes D_5 \otimes D_0$
- $Z_{32}yZ_{31}^{(2)}zZ_{21}^{(3)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{11} \otimes D_4 \otimes D_0$
- $Z_{32}yZ_{31}^{(2)}zZ_{21}^{(4)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{12} \otimes D_3 \otimes D_0$
- $Z_{32}yZ_{31}^{(2)}zZ_{21}^{(5)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{13} \otimes D_2 \otimes D_0$
- $Z_{32}yZ_{31}^{(2)}zZ_{21}^{(6)}\varkappa(v) \longmapsto 0$ ;where $v \in D_{14} \otimes D_1 \otimes D_0$

- $Ꝣ_{32}yꝢ_{31}^{(2)}zꝢ_{21}^{(7)}\varkappa(v) \longmapsto 0$ ;where $v \in Ɗ_{15}\otimes Ɗ_0\otimes Ɗ_0$

It is easy to show that (3) is satisfy.

Now by remark (2.4) all the terms in $M_3$ reduction to zero.

Eventually, define the boundary map in the complex:

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0$$

Where $\partial_1$ and $\partial_2$ defined as follows:

- $\partial_1\left(Ꝣ_{21}^{(2)}\varkappa\ (v)\right) = \partial_{21}^{(2)}(v)$ ; where $v \in Ɗ_8\otimes Ɗ_4\otimes Ɗ_3$
- $\partial_1(Ꝣ_{32}y\ (v)) = \partial_{32}(v)$ ; where $v \in Ɗ_6\otimes Ɗ_7\otimes Ɗ_2$
- $\partial_2\left(Ꝣ_{32}yꝢ_{21}^{(3)}\varkappa\ (v)\right) = \frac{1}{3}\ Ꝣ_{21}\varkappa\partial_{21}^{(2)}\partial_{32}(v) + \frac{1}{2}\ Ꝣ_{21}\varkappa\partial_{21}\partial_{31}(v) - Ꝣ_{32}y\partial_{21}(v)$

  ; where $v \in Ɗ_9\otimes Ɗ_4\otimes Ɗ_2$

**Proposition (2.5)**

The complex

$$0 \longrightarrow \mathcal{L}_3 \xrightarrow{\partial_3} \mathcal{L}_2 \xrightarrow{\partial_2} \mathcal{L}_1 \xrightarrow{\partial_1} \mathcal{L}_0 \longrightarrow K_{(7,6,3)/(1,0)}$$

is exact.

**Proof**: see [8] and [9].

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**Conclusions**

From this paper we deduced these results:

1-The terms of characteristic-free resolution are found

2- The complex of Lascoux and relationship between character-free resolution and Lascoux is investigated.

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