University Of AL-Qadisiyah

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

On The λ-Statistically Convergent for Quadruple Sequence Spaces Characterized by The Triple Orlicz Functions by Using Matrix Transformation

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ARTICLE INFO

Article history:
Received: 10 /07/2021
Rrevised form: 28 /07/2021
Accepted : 22 /08/2021
Available online: 03 /09/2021

Keywords: Matrix transformation, quadruple sequence , triple Orlicz function

ABSTRACT

In this paper , we present the λ-statistically convergent for Quadruple sequence spaces characterized by the triple Orlicz functions by using matrix transformation and investigated the qualities that are similar to those of spaces $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$, $(\mathbb{W}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$, $(\mathbb{W}_\infty^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$ are linear spaces, and the space $\mathbb{W}_\infty^4(\mathbb{M},\mathbb{A},\mathbb{p})$ is a paranormed space, and the spaces $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$, $(\mathbb{W}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$ are normal and monotone

MSC. 41A25; 41A35; 41A36

DOI : https://doi.org/10.29304/jqcm.2021.13.3.840

1.Introduction

Schoenberg [15] and Fast [7] independently proposed the concept of statistical convergence. Statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, and number theory over the years and under various names by Buck [1], Esi and Et [6].

The various types of Orlicz sequence spaces have been introduced and researched by Parasar and Choudhury [13], Esi and Et [6] , Tripathy and Hazarika [22] and many others .

Nakano [9] has proposed the concept of paranormed sequences.Tripathy et al.[19,20,27] has looked into it further and [5,21,23,25,26,28] provide additional information on λ-convergence.

In the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces, statistical convergence generalizations have appeared in recent years.

The term λ-convergence was coined by Kostyrko et al.[10] to describe a new generalization of statistical convergence.

In this work ,we introduced the quadruple sequence spaces of complex numbers characterized by double Orlicz functions $(\mathbb{W}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$,$(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$,$(\mathbb{W}_\infty^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathbb{p})$. We discussed several of these spaces are examined in terms of their topological and algebraic properties .

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Communicated by*: Dr. Rana Jumaa Surayh aljanabi.*

2. Definitions and Preliminaries

Definition 2.1[1]:

A triple Orlicz function is a function that has three parts $\mathbb{M}:[0,\infty)\times[0,\infty)\times[0,\infty) \to [0,\infty)\times[0,\infty)\times[0,\infty)$ as a result $\mathbb{M}(\mathfrak{N},\mathfrak{K},\mathfrak{R}) = (\mathbb{M}_1(\mathfrak{N}),\mathbb{M}_2(\mathfrak{K}),\mathbb{M}_3(\mathfrak{R}))$, in which $\mathbb{M}_1:[0,\infty)\to[0,\infty)$ and $\mathbb{M}_2:[0,\infty)\to[0,\infty)$ and $\mathbb{M}_3:[0,\infty)\to[0,\infty)$, These functions are non-decreasing, continuous, even, convex functions that meet the following criteria:

i) $\mathbb{M}_1(0)=0, \mathbb{M}_2(0)=0, \mathbb{M}_3(0)=0 \Longrightarrow \mathbb{M}(\mathfrak{N},\mathfrak{K},\mathfrak{R}) = (\mathbb{M}_1(0),\mathbb{M}_2(0),\mathbb{M}_3(0)) = (0,0,0)$.

ii) $\mathbb{M}_1(\mathfrak{N}) \succ 0, \mathbb{M}_2(\mathfrak{K}) \succ 0, \mathbb{M}_3(\mathfrak{R}) \succ 0 \Longrightarrow \mathbb{M}(\mathfrak{N},\mathfrak{K},\mathfrak{R}) = (\mathbb{M}_1(\mathfrak{N}),\mathbb{M}_2(\mathfrak{K}),\mathbb{M}_3(\mathfrak{R})) \succ (0,0,0)$, for $\mathfrak{N} \succ 0, \mathfrak{K} \succ 0, \mathfrak{R} \succ 0$,by which we say $(\mathfrak{N},\mathfrak{K},\mathfrak{R}) \succ (0,0,0)$,this $\mathbb{M}_1(\mathfrak{N}) \succ 0, \mathbb{M}_2(\mathfrak{K}) \succ 0, \mathbb{M}_3(\mathfrak{R}) \succ 0$.

iii) $\mathbb{M}_1(\mathfrak{N}) \to \infty, \mathbb{M}_2(\mathfrak{K}) \to \infty, \mathbb{M}_3(\mathfrak{R}) \to \infty$ as $\mathfrak{N} \to \infty, \mathfrak{K} \to \infty, \mathfrak{R} \to \infty$, after that $\mathbb{M}(\mathfrak{N},\mathfrak{K},\mathfrak{R}) = (\mathbb{M}_1(\mathfrak{N}),\mathbb{M}_2(\mathfrak{K}),\mathbb{M}_3(\mathfrak{R})) \to (\infty,\infty,\infty)$ as $(\mathfrak{N},\mathfrak{K},\mathfrak{R}) \to (\infty,\infty,\infty)$,by which we say $\mathbb{M}(\mathfrak{N},\mathfrak{K},\mathfrak{R}) \to (\infty,\infty,\infty)$, as $\mathbb{M}_1(\mathfrak{N}) \to \infty, \mathbb{M}_2(\mathfrak{K}) \to \infty, \mathbb{M}_3(\mathfrak{R}) \to \infty$.

Definition 2.2[15] :

If each and every $\varepsilon \succ 0$,$\left\{\mathfrak{n} \in \mathbb{N}: \frac{1}{\mathfrak{n}}|\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq \mathfrak{n}: \|\mathfrak{Q}_{\mathfrak{srec}} - \mathbb{L}\| \succcurlyeq \varepsilon| \succcurlyeq \mathcal{S}\right\} \in \lambda$ then a quadruple sequence $\mathfrak{Q} = (\mathfrak{Q}_{\mathfrak{srec}})$ be a λ-statistically convergent to a number $\mathbb{L} \in \mathbb{R}$.

Definition 2.3[17] :

If $(\mathfrak{Q}_{\mathfrak{srec}}) \in \mathbb{E}^4$ whenever $(\mathfrak{P}_{\mathfrak{srec}}) \in \mathbb{E}^4$ and $|\mathfrak{Q}_{\mathfrak{srec}}| \preccurlyeq |\mathfrak{P}_{\mathfrak{srec}}|$ for everybody $\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \in \mathbb{N}$ then a quadruple sequence space $\mathbb{E}^4$ be a solid.

Lemma 2.4[9] :

A quadruple sequence space $\mathbb{E}^4$ be a solid suggests that it is monotone.

Lemma 2.5[18] :

If $\lambda \subset 2^{\mathbb{N}}$ is a maximal ideal, then there's either $v \in \lambda$ or $\mathbb{N}- v \in \lambda$ each and every $v \subset \mathbb{N}$.

Let's assume $\mathbb{M} = (\mathbb{M}_1,\mathbb{M}_2,\mathbb{M}_3) = ((\mathbb{M}_1)_{\mathfrak{srec}}, (\mathbb{M}_2)_{\mathfrak{srec}}, (\mathbb{M}_3)_{\mathfrak{srec}})$ be a triple Orlicz functions, $\mathbb{A} = (\mathbb{A}_1,\mathbb{A}_2,\mathbb{A}_3) = ((\mathbb{A}_1)_{\mathfrak{srec}}, (\mathbb{A}_2)_{\mathfrak{srec}}, (\mathbb{A}_3)_{\mathfrak{srec}})$ be an infinite double matrix, and $\mathbb{Y} = ((\mathbb{Y}_1)_{\mathfrak{srec}}, (\mathbb{Y}_2)_{\mathfrak{srec}}, (\mathbb{Y}_3)_{\mathfrak{srec}})$ be a quadruple sequence of complex numbers.

In this paper, The quadruple sequence spaces defined as follows as :

$$(\mathbb{W}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p}) = \Big\{((\mathbb{Y}_1)_{\mathfrak{srec}}, (\mathbb{Y}_2)_{\mathfrak{srec}}, (\mathbb{Y}_3)_{\mathfrak{srec}}) \in \mathbb{W}^4 : \Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathbb{Y}_1)-\mathbb{L}_1\|}{\mathfrak{p}}\Big) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathbb{Y}_2)-\mathbb{L}_2\|}{\mathfrak{p}}\Big) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathbb{Y}_3)-\mathbb{L}_3\|}{\mathfrak{p}}\Big)\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \varepsilon\Big\}\Big| \succcurlyeq \mathcal{S}\Big\} \in \lambda \text{ for some } \mathfrak{p} \succ 0 \text{ and } \mathbb{L}_1,\mathbb{L}_2,\mathbb{L}_3 \in \mathbb{R}\Big\}.$$

$$(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p}) = \Big\{((\mathbb{Y}_1)_{\mathfrak{srec}}, (\mathbb{Y}_2)_{\mathfrak{srec}}, (\mathbb{Y}_3)_{\mathfrak{srec}}) \in \mathbb{W}^4 : \Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq$$

$n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\left[(\mathbb{M}_1)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathbb{Y}_1)\|}{\mathfrak{p}}\right)\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathbb{Y}_2)\|}{\mathfrak{p}}\right)\curlyvee(\mathbb{M}_3)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathbb{Y}_3)\|}{\mathfrak{p}}\right)\right]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \varepsilon\Big\}\Big| \succcurlyeq$

$\mathcal{S}\Big\} \in \lambda$ for some $\mathfrak{p} \succ 0\Big\}$.

$(\mathbb{W}_{\infty}^{4})^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p}) =$

$\Big\{((\mathbb{Y}_1)_{\mathfrak{srec}}, (\mathbb{Y}_2)_{\mathfrak{srec}}, (\mathbb{Y}_3)_{\mathfrak{srec}}) \in \mathbb{W}^4 : \Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq$

$n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\left[(\mathbb{M}_1)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathbb{Y}_1)\|}{\mathfrak{p}}\right)\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathbb{Y}_2)\|}{\mathfrak{p}}\right)\curlyvee(\mathbb{M}_3)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathbb{Y}_3)\|}{\mathfrak{p}}\right)\right]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \mathrm{M}\Big\}\Big| \succcurlyeq$

$\mathcal{S}\Big\} \in \lambda$ for some $\mathrm{M} \succ 0\Big\}$.

$\mathbb{W}_{\infty}^{4}(\mathbb{M},\mathbb{A},\mathfrak{p}) = \Big\{((\mathbb{Y}_1)_{\mathfrak{srec}}, (\mathbb{Y}_2)_{\mathfrak{srec}}, (\mathbb{Y}_3)_{\mathfrak{srec}}) \in \mathbb{W}^4: \Big\{n \in \mathbb{N}: \sup\frac{1}{n}\sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathbb{Y}_1)\|}{\mathfrak{p}}\right)\curlyvee$

$(\mathbb{M}_2)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathbb{Y}_2)\|}{\mathfrak{p}}\right)\curlyvee(\mathbb{M}_3)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathbb{Y}_3)\|}{\mathfrak{p}}\right)\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \prec \infty\Big\}\Big\}$.

It is clear from the above description that $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p}) \subset (\mathbb{W}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p}) \subset (\mathbb{W}_{\infty}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$.

3. Main Result :

Theorem 3.1 :

The spaces $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$, $(\mathbb{W}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$, $(\mathbb{W}_{\infty}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$ are linear space and $\mathbb{M} = (\mathbb{M}_{\mathfrak{srec}}) = ((\mathbb{M}_1)_{\mathfrak{srec}}, (\mathbb{M}_2)_{\mathfrak{srec}}), \mathbb{A} = (\mathbb{A}_{\mathfrak{srec}}) = ((\mathbb{A}_1)_{\mathfrak{srec}}, (\mathbb{A}_2)_{\mathfrak{srec}})$.

Proof :

We demonstrate that the solution for the space $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$. Let's assume $\mathfrak{P}_{\mathfrak{srec}} = ((\mathfrak{P}_1)_{\mathfrak{srec}}, (\mathfrak{P}_2)_{\mathfrak{srec}}, (\mathfrak{P}_3)_{\mathfrak{srec}})$ and $\mathfrak{Q}_{\mathfrak{srec}} = ((\mathfrak{Q}_1)_{\mathfrak{srec}}, (\mathfrak{Q}_2)_{\mathfrak{srec}}, (\mathfrak{Q}_3)_{\mathfrak{srec}})$ be any two elements in $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$ .Then there are those who exist $\mathfrak{p}_1 \succ 0$ and $\mathfrak{p}_2 \succ 0$ as a result

$\mathcal{B} =$

$\Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq$

$n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\left[(\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_1})\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_1})\curlyvee(\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_1})\right]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \frac{\varepsilon}{2}\Big\}\Big| \succcurlyeq \mathcal{S}\Big\} \in \lambda.$

$\mathcal{C} =$

$\Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq$

$n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\left[(\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_2})\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_2})\curlyvee(\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_2})\right]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \frac{\varepsilon}{2}\Big\}\Big| \succcurlyeq \mathcal{S}\Big\} \in \lambda.$

Let's assume $\mathfrak{m}, \mathfrak{mm}$ be any scalars. Because of the double sequence's continuity $\mathbb{M}_{\mathfrak{sr}} = ((\mathbb{M}_1)_{\mathfrak{sr}}, (\mathbb{M}_2)_{\mathfrak{sr}})$ the following inequality holds:

$\sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{m}\mathfrak{P}_1+\mathfrak{mm}\mathfrak{Q}_1)\|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{m}\mathfrak{P}_2+\mathfrak{mm}\mathfrak{Q}_2)\|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{m}\mathfrak{P}_3+\mathfrak{mm}\mathfrak{Q}_3)\|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2})\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \preccurlyeq$

$\mathbb{DJ}\ \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_1})\Big]^{\mathfrak{p}_{\mathfrak{srec}}} + \mathbb{DJ}$

$\sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_2}) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_2}) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_2})\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \preccurlyeq$

$\mathbb{D}\ \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[\frac{|\mathfrak{m}|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}\Big((\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_1})\Big)\Big]^{\mathfrak{p}_{\mathfrak{srec}}} +$

$\mathbb{D}\sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[\frac{|\mathfrak{m}|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}\Big((\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{Q}_1)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{Q}_2)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{Q}_3)\|}{\mathfrak{p}_1})\Big)\Big]^{\mathfrak{p}_{\mathfrak{srec}}}$, in which $\mathbb{J} = \max\Big\{1, \frac{|\mathfrak{m}|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}, \frac{|\mathfrak{mm}|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}\Big\}$.

We can deduce the following from the above relation :

$\Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{m}\mathfrak{P}_1+\mathfrak{mm}\mathfrak{Q}_1)\|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}\Big) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{m}\mathfrak{P}_2+\mathfrak{mm}\mathfrak{Q}_2)\|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}\Big) \curlyvee$
$(\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{m}\mathfrak{P}_3+\mathfrak{mm}\mathfrak{Q}_3)\|}{|\mathfrak{m}|\mathfrak{p}_1+|\mathfrak{mm}|\mathfrak{p}_2}\Big)\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \frac{\varepsilon}{2}\Big\}\Big| \succcurlyeq \mathcal{S}\Big\} \subseteq$

$\Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq$
$n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\mathbb{DJ}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_1}\Big) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_1}\Big) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_1}\Big)\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \frac{\varepsilon}{2}\Big\}\Big| \succcurlyeq \mathcal{S}\Big\} \cup$

$\Big\{n \in \mathbb{N}: \frac{1}{n}\Big|\Big\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq$
$n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\mathbb{DJ}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{Q}_1)\|}{\mathfrak{p}_2}\Big) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{Q}_2)\|}{\mathfrak{p}_2}\Big) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{Q}_3)\|}{\mathfrak{p}_2}\Big)\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \frac{\varepsilon}{2}\Big\}\Big| \succcurlyeq \mathcal{S}\Big\}$.

Theorem 3.2 :

The space $\mathbb{W}_{\infty}^{4}(\mathbb{M},\mathbb{A},\mathfrak{p})$ be a paranormed space with paranorm g defined by:

$\hbar(\mathfrak{P}) = \inf\Big\{\mathfrak{p}^{\frac{\mathfrak{p}_{\mathfrak{srec}}}{\mathbb{H}}} :\ \sup_{\mathfrak{srec}}\ \Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_1}\Big) \curlyvee (\mathbb{M}_2)_{\mathfrak{sr}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_1}\Big) \curlyvee (\mathbb{M}_3)_{\mathfrak{sr}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_1}\Big)\Big] \preccurlyeq 1, \text{for } \mathfrak{p} \succ 0\Big\}$, in which $\mathbb{H} = \max\{1, \sup_{\mathfrak{srec}} \mathfrak{p}_{\mathfrak{srec}}\}$ and $\mathbb{M} = (\mathbb{M}_{\mathfrak{srec}}) = ((\mathbb{M}_1)_{\mathfrak{srec}}, (\mathbb{M}_2)_{\mathfrak{srec}}, (\mathbb{M}_3)_{\mathfrak{srec}})$,
$\mathbb{A} = (\mathbb{A}_{\mathfrak{srec}}) = ((\mathbb{A}_1)_{\mathfrak{srec}}, (\mathbb{A}_2)_{\mathfrak{srec}}, (\mathbb{A}_3)_{\mathfrak{srec}})$.

Proof :

It is obvious that $\hbar(\theta) = 0$, $\hbar(-\mathfrak{B}) = \hbar(\mathfrak{B})$ and at can be easily shown that $\hbar(\mathfrak{P}+\mathfrak{Q}) \preccurlyeq \hbar(\mathfrak{P}) + \hbar(\mathfrak{Q})$. In which $\mathfrak{P} = (\mathfrak{P}_1, \mathfrak{P}_2, \mathfrak{P}_3)$ and $\mathfrak{Q} = (\mathfrak{Q}_1, \mathfrak{Q}_2, \mathfrak{Q}_3)$

Let's $\mathcal{T}_{\mathfrak{nmij}} \to \mathcal{T}$, in which $\mathcal{T}_{\mathfrak{nmij}} = ((\mathcal{T}_1)_{\mathfrak{nmij}}, (\mathcal{T}_2)_{\mathfrak{nmij}}, (\mathcal{T}_3)_{\mathfrak{nmij}})$, $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) \in \mathbb{C}$ and let's

$\hbar\Big(((\mathfrak{P}_1)_{\mathfrak{nmij}} - \mathfrak{P}_1), ((\mathfrak{P}_2)_{\mathfrak{nmij}} - \mathfrak{P}_2)((\mathfrak{P}_3)_{\mathfrak{nmij}} - \mathfrak{P}_3)\Big) \to 0$, as $\mathfrak{n} \to \infty$, $\mathfrak{m} \to \infty$, $\mathfrak{i} \to \infty$, $\mathfrak{j} \to \infty$.

To demonstrate that $\hbar\Big(((\mathcal{T}_1)_{\mathfrak{nmij}}(\mathfrak{P}_1)_{\mathfrak{nmij}} - \mathcal{T}_1\mathfrak{P}_1), ((\mathcal{T}_2)_{\mathfrak{nmij}}(\mathfrak{P}_2)_{\mathfrak{nmij}} - \mathcal{T}_2\mathfrak{P}_2), ((\mathcal{T}_3)_{\mathfrak{nmij}}(\mathfrak{P}_3)_{\mathfrak{nmij}} - \mathcal{T}_3\mathfrak{P}_3)\Big) \to 0$, as $\mathfrak{n} \to \infty$, $\mathfrak{m} \to \infty$, $\mathfrak{i} \to \infty$, $\mathfrak{j} \to \infty$. We positioned ,

$\mathbb{Q} = \Big\{\mathfrak{p}_1 \succ 0 :\ \sup_{\mathfrak{srec}}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_1}) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_1})\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \preccurlyeq 1\Big\}$

and

$\mathbb{Z} = \Big\{\mathfrak{p}_2 \succ 0 : \sup_{\mathfrak{srec}}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}_2}) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}_2}) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}_2})\Big]^{\mathfrak{p}_{\mathfrak{srec}}} \preccurlyeq 1\Big\}$.

By of the quadruple sequence's continuity $\mathbb{M} = (\mathbb{M}_1, \mathbb{M}_2) = ((\mathbb{M}_1)_{\mathfrak{srec}}, (\mathbb{M}_2)_{\mathfrak{srec}})$, we observe

$$\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}((\mathcal{T}_1)_{\mathfrak{nmij}}(\mathfrak{P}_1)_{\mathfrak{nmij}}-\mathcal{T}_1\mathfrak{P}_1)\|}{|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1+|\mathcal{T}_1|\mathfrak{p}_2}\Big)\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}((\mathcal{T}_2)_{\mathfrak{nmij}}(\mathfrak{P}_2)_{\mathfrak{nmij}}-\mathcal{T}_2\mathfrak{P}_2)\|}{|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_2}\Big)\curlyvee$$

$$(\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}((\mathcal{T}_3)_{\mathfrak{nmij}}(\mathfrak{P}_3)_{\mathfrak{nmij}}-\mathcal{T}_3\mathfrak{P}_3)\|}{|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_3}\Big)\Big]\preccurlyeq$$

$$\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}((\mathcal{T}_1)_{\mathfrak{nmij}}(\mathfrak{P}_1)_{\mathfrak{nmij}}-\mathcal{T}_1(\mathfrak{P}_1)_{\mathfrak{nmij}})\|}{|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1+|\mathcal{T}_1|\mathfrak{p}_2}\Big)\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}((\mathcal{T}_2)_{\mathfrak{nmij}}(\mathfrak{P}_2)_{\mathfrak{nmij}}-\mathcal{T}_2(\mathfrak{P}_2)_{\mathfrak{nmij}})\|}{|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_2}\Big)\curlyvee$$

$$(\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}((\mathcal{T}_3)_{\mathfrak{nmij}}(\mathfrak{P}_3)_{\mathfrak{nmij}}-\mathcal{T}_2(\mathfrak{P}_3)_{\mathfrak{nmij}})\|}{|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1+|\mathcal{T}_3|\mathfrak{p}_2}\Big)\Big]+\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathcal{T}_1(\mathfrak{P}_1)_{\mathfrak{nmij}}-\mathcal{T}_1\mathfrak{P}_1)\|}{|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1+|\mathcal{T}_1|\mathfrak{p}_2}\Big)\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathcal{T}_2(\mathfrak{P}_2)_{\mathfrak{nmij}}-\mathcal{T}_2\mathfrak{P}_2)\|}{|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_2}\Big)\curlyvee$$

$$(\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathcal{T}_2(\mathfrak{P}_3)_{\mathfrak{nmij}}-\mathcal{T}_3\mathfrak{P}_3)\|}{|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1+|\mathcal{T}_3|\mathfrak{p}_2}\Big)\Big]\preccurlyeq$$

$$\Big[\frac{|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1}{|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1+|\mathcal{T}_1|\mathfrak{p}_2}(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}((\mathfrak{P}_1)_{\mathfrak{nmij}})\|}{\mathfrak{p}_1}\Big)\curlyvee\frac{|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1}{|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_2}(\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}((\mathfrak{P}_2)_{\mathfrak{nmij}})\|}{\mathfrak{p}_1}\Big)\curlyvee$$

$$\frac{|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1}{|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1+|\mathcal{T}_3|\mathfrak{p}_2}(\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}((\mathfrak{P}_3)_{\mathfrak{nmij}})\|}{\mathfrak{p}_1}\Big)\Big]+$$

$$\Big[\frac{|\mathcal{T}_1|\mathfrak{p}_2}{|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1+|\mathcal{T}_1|\mathfrak{p}_2}(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}((\mathfrak{P}_1)_{\mathfrak{nmij}}-(\mathfrak{P}_1))\|}{\mathfrak{p}_2}\Big)\curlyvee\frac{|\mathcal{T}_2|\mathfrak{p}_2}{|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_2}(\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}((\mathfrak{P}_2)_{\mathfrak{nmij}}-(\mathfrak{P}_2))\|}{\mathfrak{p}_2}\Big)\curlyvee$$

$$\frac{|\mathcal{T}_3|\mathfrak{p}_2}{|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1+|\mathcal{T}_3|\mathfrak{p}_2}(\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}((\mathfrak{P}_3)_{\mathfrak{nmij}}-(\mathfrak{P}_3))\|}{\mathfrak{p}_2}\Big)\Big]$$

. As a result of the preceding inequity, it follows that

$$\sup_{\mathfrak{srec}}\Big[(\mathbb{M}_1)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_1)_{\mathfrak{src}}((\mathcal{T}_1)_{\mathfrak{nmij}}(\mathfrak{P}_1)_{\mathfrak{nmij}}-\mathcal{T}_1\mathfrak{P}_1)\|}{|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1+|\mathcal{T}_1|\mathfrak{p}_2}\Big)\curlyvee(\mathbb{M}_2)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}((\mathcal{T}_2)_{\mathfrak{nmij}}(\mathfrak{P}_2)_{\mathfrak{nmij}}-\mathcal{T}_2\mathfrak{P}_2)\|}{|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_2}\Big)\curlyvee$$

$$(\mathbb{M}_3)_{\mathfrak{srec}}\Big(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}((\mathcal{T}_3)_{\mathfrak{nmij}}(\mathfrak{P}_3)_{\mathfrak{nmij}}-\mathcal{T}_3\mathfrak{P}_3)\|}{|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_3}\Big)\Big]^{\mathfrak{p}_{\mathfrak{srec}}}\preccurlyeq 1$$

and consequently

$$\mathcal{k}\Big(\big((\mathcal{T}_1)_{\mathfrak{nmij}}(\mathfrak{P}_1)_{\mathfrak{nmij}}-\mathcal{T}_1\mathfrak{P}_1\big),\big((\mathcal{T}_2)_{\mathfrak{nmij}}(\mathfrak{P}_2)_{\mathfrak{nmij}}-\mathcal{T}_2\mathfrak{P}_2\big),\big((\mathcal{T}_3)_{\mathfrak{nmij}}(\mathfrak{P}_3)_{\mathfrak{nmij}}-\mathcal{T}_3\mathfrak{P}_3\big)\Big)=$$

$$\inf\Big\{\Big(\big(|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|\mathfrak{p}_1+|\mathcal{T}_1|\mathfrak{p}_2\big),\big(|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|\mathfrak{p}_1+|\mathcal{T}_2|\mathfrak{p}_2\big)\big(|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\mathfrak{p}_1+|\mathcal{T}_3|\mathfrak{p}_2\big)\Big)^{\frac{\mathfrak{p}_{\mathfrak{srec}}}{\mathbb{H}}}:\mathfrak{p}_1\in\mathbb{Q},\mathfrak{p}_2\in\mathbb{Z}\Big\}\preccurlyeq$$

$$\big(|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|,|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|,|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\big)^{\frac{\mathfrak{p}_{\mathfrak{srec}}}{\mathbb{H}}}\inf\Big\{\mathfrak{p}_1^{\frac{\mathfrak{p}_{\mathfrak{src}}}{\mathbb{H}}}:\mathfrak{p}_1\in\mathbb{Q}\Big\}+\big(|\mathcal{T}_1|,|\mathcal{T}_2|,|\mathcal{T}_3|\big)^{\frac{\mathfrak{p}_{\mathfrak{srec}}}{\mathbb{H}}}\inf\Big\{\mathfrak{p}_2^{\frac{\mathfrak{p}_{\mathfrak{srec}}}{\mathbb{H}}}:\mathfrak{p}_2\in\mathbb{Z}\Big\}\preccurlyeq\max$$

$$\Big\{\big(|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|,|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|,|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\big),\big(|(\mathcal{T}_1)_{\mathfrak{nmij}}-\mathcal{T}_1|,|(\mathcal{T}_2)_{\mathfrak{nmij}}-\mathcal{T}_2|,|(\mathcal{T}_3)_{\mathfrak{nmij}}-\mathcal{T}_3|\big)^{\frac{\mathfrak{p}_{\mathfrak{srec}}}{\mathbb{H}}}\Big\}\mathcal{k}\Big(\big((\mathfrak{P}_1)_{\mathfrak{nmij}}\big),\big((\mathfrak{P}_2)_{\mathfrak{nmij}}\big),\big((\mathfrak{P}_3)_{\mathfrak{nmij}}\big)\Big)+$$

$$\max\Big\{\big(|\mathcal{T}_1|,|\mathcal{T}_2|,|\mathcal{T}_3|\big),\big(|\mathcal{T}_1|,|\mathcal{T}_2|,|\mathcal{T}_3|\big)^{\frac{\mathfrak{p}_{\mathfrak{srec}}}{\mathbb{H}}}\Big\}\mathcal{k}\Big(\big((\mathfrak{P}_1)_{\mathfrak{nmij}}-\mathfrak{P}_1\big),\big((\mathfrak{P}_2)_{\mathfrak{nmij}}-\mathfrak{P}_2\big),\big((\mathfrak{P}_3)_{\mathfrak{nmij}}-\mathfrak{P}_3\big)\Big)$$

As $\mathcal{k}\Big(\big((\mathfrak{P}_1)_{\mathfrak{nmij}}\big),\big((\mathfrak{P}_2)_{\mathfrak{nmij}}\big),\big((\mathfrak{P}_3)_{\mathfrak{nmij}}\big)\Big)\preccurlyeq\mathcal{k}\big((\mathfrak{P}_1),(\mathfrak{P}_2),(\mathfrak{P}_3)\big)+\mathcal{k}\Big(\big((\mathfrak{P}_1)_{\mathfrak{nmij}}-\mathfrak{P}_1\big),\big((\mathfrak{P}_2)_{\mathfrak{nmj}}-\mathfrak{P}_2\big),\big((\mathfrak{P}_3)_{\mathfrak{nmj}}-\mathfrak{P}_3\big)\Big)$ for everybody $\mathfrak{n},\mathfrak{m},\mathfrak{i},\mathfrak{j}\in\mathbb{N}$, consequently The right-hand side of the above relation converges on zero as $\mathfrak{n}\to\infty$, $\mathfrak{m}\to\infty$, $\mathfrak{i}\to\infty$, $\mathfrak{j}\to\infty$.

Theorem 3.3 :

The spaces $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$, $(\mathbb{W}^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$ are normal and monotone as a result $\mathbb{M}=(\mathbb{M}_{\mathfrak{srec}})=((\mathbb{M}_1)_{\mathfrak{srec}},(\mathbb{M}_2)_{\mathfrak{srec}},(\mathbb{M}_3)_{\mathfrak{srec}})$, $\mathbb{A}=(\mathbb{A}_{\mathfrak{srec}})=((\mathbb{A}_1)_{\mathfrak{srec}},(\mathbb{A}_2)_{\mathfrak{srec}},(\mathbb{A}_3)_{\mathfrak{srec}})$.

Proof :

Let's assume $\mathfrak{P}=(\mathfrak{B}_{\mathfrak{srec}})=((\mathfrak{B}_1)_{\mathfrak{srec}},(\mathfrak{B}_2)_{\mathfrak{srec}},(\mathfrak{B}_3)_{\mathfrak{srec}})$ and $\mathfrak{Q}=(\mathfrak{Q}_{\mathfrak{srec}})=((\mathfrak{Q}_1)_{\mathfrak{srec}},(\mathfrak{Q}_2)_{\mathfrak{srec}},(\mathfrak{Q}_3)_{\mathfrak{srec}})\in(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$ be as a result $|\mathfrak{Q}_{\mathfrak{srec}}|\preccurlyeq|\mathfrak{P}_{\mathfrak{srec}}|$.

Then for $\varepsilon \succ 0$,

$$\left\{n \in \mathbb{N}: \frac{1}{n}\left|\left\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\left[(\mathbb{M}_1)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{P}_1)\|}{\mathfrak{p}}\right) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{P}_2)\|}{\mathfrak{p}}\right) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{P}_3)\|}{\mathfrak{p}}\right)\right]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \varepsilon\right\}\right| \succcurlyeq \mathcal{S}\right\} \supseteq$$

$$\left\{n \in \mathbb{N}: \frac{1}{n}\left|\left\{\mathfrak{s},\mathfrak{r},\mathfrak{e},\mathfrak{c} \preccurlyeq n: \sum_{\mathfrak{s}=1}^{n}\sum_{\mathfrak{r}=1}^{n}\sum_{\mathfrak{e}=1}^{n}\sum_{\mathfrak{c}=1}^{n}\left[(\mathbb{M}_1)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_1)_{\mathfrak{srec}}(\mathfrak{Q}_1)\|}{\mathfrak{p}}\right) \curlyvee (\mathbb{M}_2)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_2)_{\mathfrak{srec}}(\mathfrak{Q}_2)\|}{\mathfrak{p}}\right) \curlyvee (\mathbb{M}_3)_{\mathfrak{srec}}\left(\frac{\|(\mathbb{A}_3)_{\mathfrak{srec}}(\mathfrak{Q}_3)\|}{\mathfrak{p}}\right)\right]^{\mathfrak{p}_{\mathfrak{srec}}} \succcurlyeq \varepsilon\right\}\right| \succcurlyeq \mathcal{S}\right\}$$

. The space $(\mathbb{W}_0^4)^{\lambda(\mathcal{S})}(\mathbb{M},\mathbb{A},\mathfrak{p})$ be a normal , and thus monotone.

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