Finding the Critical Path Method for Fuzzy Network with Development Ranking Function

Rasha Jalal Mitlif\textsuperscript{a}*, Fatema Ahmad Sadiq\textsuperscript{b}

\textsuperscript{a}Applied Sciences Department, Mathematics and Computer Applications, University of Technology, Baghdad, Iraq, Email: rasha.j.mitlif@uotechnology.edu.iq.

\textsuperscript{b}Applied Sciences Department, Mathematics and Computer Applications, University of Technology, Baghdad, Iraq, Email: fatema.a.sadiq@uotechnology.edu.iq.

Abstract

We propose a development ranking function (RF) to solve project - scheduling problems (PSP) in a foggy environment. The development command works on fuzzy numbers (FN) and this is done by converting the fuzzy parameters to an explicit value and applying the critical path method (CPM) to obtain the solution described in the proposed algorithm. A clear definition of the time limit will aid in the successful implementation of CPM, there is often confusion regarding the length of the process leading to the development of a critical path method (CPM) system. The example and approach strongly suggest that the proposed method is efficient and gives us the critical path (CP) and identifies sensitive activities as well. The results show that the use of the development ranking function (DRF) is better, more efficient, and accurate than the other ranking function (RF) by calculating the hours of the project completion.

MSC. 41A25; 41A35; 41A36.


1. Introduction

In any manufacturing system, the scheduling of activities plays the most important role by controlling the production process. Project scheduling is a problem in decision-making due to goals, limited resources, and difficulties in accurately modeling world problems [1].

*Corresponding author: Rasha Jalal Mitlif
Email addresses: rasha.j.mitlif@uotechnology.edu.iq
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A project network is defined as a set of activities with certain special features, such as durations and the link of precedent. Keeping in mind competition benefits, project executives and other stakeholders of the projects, more information is usually required other of the project characteristics, as the completion of project time, early and late commencement of activities, float periods, and activities important. Accordingly, some methods were developed to meet the needs of these essentials [2]. Soltani and Haji [3], suggest a method of project planning adopted on Fuzzy theory. Many researches introduced the network analysis with Triangular Fuzzy Numbers and Triangular Intuitionistic Fuzzy set [4] and [5]. In [6], study the shortest route in the network can be converted into a crisp value, deciding the best sequence leading to the network’s shortest path.

The Critical Path (CP) approach is developed for preparing and managing projects in the world. To ensure project efficiency, project time and costs must be tracked and the effectiveness of resource allocation optimized according to the dynamic trend. Some projects can be difficult and stressful to deal with in certain situations. There are also occasions where the operation times might not be described correctly. One way to deal with inaccurate data is to use the fuzziness principle through fuzzy activity times that can be represented by fuzzy sets (FS). The fuzzy group theory proposed by Zadeh [7] may play an important role in solving such managerial problem. Several papers are also presented on a critical path method for solution the fuzzy project network [8] and [9]. Narayanamoorthy and Maheswari [10] obtained a modified classification method for assessing the critical path (CP) of the fuzzy project network (FPN), in which the length of each process time is a fuzzy octagonal number (FON), and an updated formula is implemented on the fuzzy numbers in this process. Researches [11-19] have studied the different applications of the ranking function.

The aim of this paper is development ranking function and transforms the Fuzzy Network Analysis formulation to the crisp one, which can be solved by using the critical path method (CPM). We organize this paper as follows. In Section two, we recall fuzzy fundamentals. And in Section three, we introduce the Ranking Trapezoidal Fuzzy Numbers. In Section four, we propose development ranking function by convert fuzzy networking problem (FNP) with trapezoidal fuzzy number (TFN) to crisp networking problem. In section five, shows calculate critical path for solving fuzzy network analysis, solve a numerical example in the Section six. Finally, the results are discussed in the Section seven.

2. Fuzzy Fundamental [20-22]

Some basic definitions are given in this section.

Definition 1:
A fuzzy set (FS) $\tilde{A}$ in $\mathbb{K}$ is a set of two numbers described by, $\tilde{A}(s) = \{(s, \eta\tilde{A}(s)) : s \in \mathbb{K}, \eta\tilde{A}(s) \in [0,1]\}$ where $\eta\tilde{A}(s)$ is a membership function (MF).

Definition 2:
A fuzzy number (FN) $\tilde{A}$ is a fuzzy set (FS) whose membership function (MF) $\eta\tilde{A}(s)$ meets the following condition
(1) $\eta\tilde{A}(s)$ piecewise ongoing
(2) $\eta\tilde{A}(s)$ is convex
(3) $\eta\tilde{A}(s)$ is normal (i.e.) $\eta\tilde{A}(s_0) = 0$. 
**Definition 3:**

A fuzzy number (FN) \( \tilde{g} \) is a trapezoidal fuzzy number (TFN) described by \( \tilde{g} = (q^m, q^n, r_1, r_2) \) where \( q^m, q^n, r_1, r_2 \) are actual points and its \( g(\tilde{g}) \) membership function (MF) is shown below,

\[
g(\tilde{g}) = \begin{cases} 
\frac{\frac{\tilde{g} - (q^n - r_1)}{r_1}}{1}, & q^m - r_1 \leq \tilde{g} \leq q^n \\
\frac{\frac{(q^n + r_2) - \tilde{g}}{r_2}}{0}, & q^n \leq \tilde{g} \leq q^n + r_2 \end{cases}
\]

3. **Ranking Trapezoidal Fuzzy Numbers**

The Yager ranking function proposed in [23] is used for the classification of the fuzzy trapezoidal numbers.

The ranking function \( h: F(\mathbb{H}) \subseteq \mathbb{H} \) where \( F(\mathbb{H}) \) is a set of fuzzy numbers (FN) based on the set of actual points that applies of the fuzzy number to the Actual line where the general pattern exists, that is.

(i) \( T' > C \leftrightarrow h(T') > h(C) \)

(ii) \( T' < C \leftrightarrow h(T') < h(C) \)

(iii) \( T' = C \leftrightarrow h(T') = h(C) \)

4. **Development Ranking Function**

The technique of developing the ranking function with the help of weight so that the weight value between the closed interval is zero and one.

Now let \( W = \frac{9}{10} \), then applying the following Function Ranking:

\[
H(T) = w \int_0^1 (\inf \bar{q}_d + \sup \bar{q}_d) \, d\sigma
\]

Substitute the weight value in the ordinate function as follows:

\[
H(T) = \frac{9}{10} \int_0^1 (\inf \bar{q}_d + \sup \bar{q}_d) \, d\sigma
\]

\[
H(T) = \frac{9}{10} \left[ \int_0^1 (q^m - r_1 (1 - \sigma^4)) \, d\sigma + \int_0^1 q^n + r_2 (1 - \sigma^2) \, d\sigma \right]
\]

and

\[
H(T) = \frac{9}{10} \left[ q^m \sigma - r_1 \sigma + \frac{\sigma^5}{5} r_1 + q^n \sigma + r_2 \sigma - \frac{\sigma^3}{3} r_2 \right]_0^1
\]

\[
H(T) = \frac{9}{10} [q^m - r_1 + \frac{q_1}{5} + q^n + r_2 - \frac{r_2}{3}]
\]
and
\[ H(\mathcal{T}) = \frac{9}{10} \left[ \frac{2}{5} \mathcal{T_1} + \mathcal{T_2} + \frac{2}{3} \mathcal{T_3} \right] \]
\[ H(\mathcal{T}') = \frac{9}{10} \left[ \mathcal{T_1} + \mathcal{T_2} + \frac{2}{15} (5 \mathcal{T_2} - 6 \mathcal{T_1}) \right] \]

The following formula is development ranking function (DRF):
\[ H(\mathcal{T}) = \frac{9}{10} (\mathcal{T_1} + \mathcal{T_2}) + \frac{3}{25} (5 \mathcal{T_2} - 6 \mathcal{T_1}) \]

5. **Finding the Critical Path for Solving Fuzzy Network Analysis**

A project network’s Critical Path (CP) can be obtained by using the following steps:

Step1: Convert trapezoidal fuzzy numbers to the equivalent crisp value by development ranking function.

Step2: Construct the project network.

Step3: Identify Project Activities.

Step4: Set precedent relationships of all activities.

Step5: Estimate the time of activity about each activity.

Step6: Identify all possible production network paths, and consider these paths as alternatives. Then create matrix of decisions based on all possible paths and efficient criteria such as time, expense, quality.

Step7: Calculate network analysis by critical path method.

Step8: Comparison of the development ranking function with the other function to obtain the minimum project period in hours.

6. **Numerical Example:**
The problem arises in the search on the length of the critical path (CP) in the non-rotating project network containing 7 vertices and 9 edges with a trapezoid fuzzy number (TFN).

The following problem can be represented as a trapezoidal fuzzy number in network analysis. And the trapezoidal fuzzy number problem is transformed into a crisp value using the formula of the development ranking function in Table 1:
Table 1: The trapezoidal fuzzy number problem is converted crisp value in example

<table>
<thead>
<tr>
<th>Activity (i,j)</th>
<th>Fuzzy duration</th>
<th>Crisp values using development ranking function</th>
<th>Crisp values using Maleki ranking function [24]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>(3,4,5,6)</td>
<td>6.300</td>
<td>7.500</td>
</tr>
<tr>
<td>1-3</td>
<td>(5,6,7,7)</td>
<td>9.060</td>
<td>11.000</td>
</tr>
<tr>
<td>2-4</td>
<td>(4,5,6,6)</td>
<td>7.380</td>
<td>9.000</td>
</tr>
<tr>
<td>2-6</td>
<td>(8,9,10,11)</td>
<td>14.700</td>
<td>17.500</td>
</tr>
<tr>
<td>3-4</td>
<td>(5,6,7,7)</td>
<td>9.060</td>
<td>11.000</td>
</tr>
<tr>
<td>3-5</td>
<td>(10,11,12,14)</td>
<td>18.660</td>
<td>22.000</td>
</tr>
<tr>
<td>4-7</td>
<td>(14,15,16,18)</td>
<td>25.380</td>
<td>30.000</td>
</tr>
<tr>
<td>5-7</td>
<td>(14,15,15,16)</td>
<td>24.900</td>
<td>29.500</td>
</tr>
<tr>
<td>6-7</td>
<td>(15,16,17,18)</td>
<td>26.460</td>
<td>31.500</td>
</tr>
</tbody>
</table>

Build the project network (PN) with each activity's time period in Fig. 1

![Figure 1: Project network (PN) with time period for each activity based on developed ranking function calculations.](image)

We can calculate and identify the critical path by adopted critical path method as follows in Table 2:
Table 2: The calculate and identify the critical path method

<table>
<thead>
<tr>
<th>Event (i)</th>
<th>ESi</th>
<th>Lf j</th>
<th>Slack(Lfj -ESi)</th>
<th>Critical path</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>6.300</td>
<td>11.460</td>
<td>5.16</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>9.060</td>
<td>9.060</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>18.120</td>
<td>27.240</td>
<td>9.12</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>27.720</td>
<td>27.720</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
<td>26.160</td>
<td>5.16</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>52.620</td>
<td>52.620</td>
<td>0</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Critical path (CP) is 1 → 3 → 5 → 7

The length of the Critical Path (CP) is (52.620) hours, the minimum project period.

Build the project network (PN) with each activity's time period in Fig. 2

Figure 2: Project network (PN) with time period for each activity based on Maleki ranking function [24] calculations.

The calculate and identify the critical path by adopted the critical path method as follows in Table 3:

Table 3: The calculate and identify the critical path method

<table>
<thead>
<tr>
<th>Event (i)</th>
<th>ESi</th>
<th>Lf j</th>
<th>Slack(Lfj -ESi)</th>
<th>Critical path</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>7.500</td>
<td>13.500</td>
<td>6.000</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>11.000</td>
<td>11.000</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>22.000</td>
<td>32.000</td>
<td>10.000</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>33.000</td>
<td>33.000</td>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>25.000</td>
<td>31.000</td>
<td>6.000</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>62.000</td>
<td>62.000</td>
<td>0</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Critical path (CP) is 1 → 3 → 5 → 7
The length of the Critical Path (CP) is (62.000) hours, the minimum project period compared the two ranking functions in the following Table 4 and Fig.3

<table>
<thead>
<tr>
<th>NO.</th>
<th>Ranking Function</th>
<th>The length of the project in hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Development ranking function</td>
<td>52.620</td>
</tr>
<tr>
<td>2</td>
<td>Maleki ranking function</td>
<td>62.000</td>
</tr>
</tbody>
</table>

**Table 4: Compared the two ranking functions**

It appears from the comparison Table 4, Fig.3 that the developed ranking function is the best Maleki ranking function [24].

**Figure3:** Compared the development ranking functions (DRF) and Maleki ranking function (MRF)

It appears from the comparison Table 4, Fig.3 that the developed ranking function is the best Maleki ranking function [24].

7. **Conclusions**

An algorithm is presented and proposed to transform the fuzzy network into a clear network, and achieve the best result in the completion times of the fuzzy network with a trapezoidal fuzzy number. The proposed method is examined by numerical calculations in this paper.
References


