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Finding the Critical Path Method for Fuzzy Network with Development Ranking Function

Rasha Jalal Mitlif [a*], ***Fatema Ahmad Sadiq*** [b]

[a]*Applied Sciences Department, Mathematics and Computer Applications, University of Technology, Baghdad, Iraq,*

Email: rasha.j.mitlif@uotechnology.edu.iq.

[b]*Applied Sciences Department, Mathematics and Computer Applications, University of Technology, Baghdad, Iraq,*

Email: fatema.a.sadiq@uotechnology.edu.iq.

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ABSTRACT

We propose a development ranking function (RF) to solve project - scheduling problems (PSP) in a foggy environment. The development command works on fuzzy numbers (FN) and this is done by converting the fuzzy parameters to an explicit value and applying the critical path method (CPM) to obtain the solution described in the proposed algorithm. A clear definition of the time limit will aid in the successful implementation of CPM, there is often confusion regarding the length of the process leading to the development of a critical path method (CPM) system. The example and approach strongly suggest that the proposed method is efficient and gives us the critical path (CP) and identifies sensitive activities as well. The results show that the use of the development ranking function (DRF) is better, more efficient, and accurate than the other ranking function (RF) by calculating the hours of the project completion.

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1. Introduction

In any manufacturing system, the scheduling of activities plays the most important role by controlling the production process. Project scheduling is a problem in decision-making due to goals, limited resources, and difficulties in accurately modeling world problems [1].

∗Corresponding author : *Rasha Jalal Mitlif*
Email addresses: rasha.j.mitlif@uotechnology.edu.iq
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A project network is defined as a set of activities with certain special features, such as durations and the link of precedent. Keeping in mind competition benefits, project executives and other stakeholders of the projects, more information is usually required other of the project characteristics, as the completion of project time, early and late commencement of activities, float periods, and activities important. Accordingly, some methods were developed to meet the needs of those essentials [2]. Soltani and Haji [3], suggest a method of project planning adopted on Fuzzy theory. Many researches introduced the network analysis with Triangular Fuzzy Numbers and Triangular Intuitionistic Fuzzy set [4] and [5]. In [6], study the shortest route in the network can be converted into a crisp value, deciding the best sequence leading to the network's shortest path.

The Critical Path (CP) approach is developed for preparing and managing projects in the world. To ensure project efficiency, project time and costs must be tracked and the effectiveness of resource allocation optimized according to the dynamic trend. Some projects can be difficult and stressful to deal with in certain situations. There are also occasions where the operation times might not be described correctly. One way to deal with inaccurate data is to use the fuzziness principle through fuzzy activity times that can be represented by fuzzy sets (FS). The fuzzy group theory proposed by Zadeh [7] may play an important role in solving such managerial problem. Several papers are also presented on a critical path method for solution the fuzzy project network [8] and [9]. Narayanamoorthy and Maheswari [10] obtained a modified classification method for assessing the critical path (CP) of the fuzzy project network (FPN), in which the length of each process time is a fuzzy octagonal number (FON), and an updated formula is implemented on the fuzzy numbers in this process. Researches [11-19] have studied the different applications of the ranking function.

The aim of this paper is development ranking function and transforms the Fuzzy Network Analysis formulation to the crisp one, which can be solved by using the critical path method (CPM). We organize this paper as follows. In Section two, we recall fuzzy fundamentals. And in Section three, we introduce the Ranking Trapezoidal Fuzzy Numbers. . In Section four, we propose development ranking function by convert fuzzy networking problem (FNP) with trapezoidal fuzzy number (TFN) to crisp networking problem. In section five, shows calculate critical path for solving fuzzy network analysis, solve a numerical example in the Section six. Finally, the results are discussed in the Section seven.

2. Fuzzy Fundamental [20- 22]

Some basic definitions are given in this section.

Definition 1:

A fuzzy set (FS) $\tilde{Ţ}$ in $Ķ$ is a set of two numbers described by, $\tilde{ş}(ş) = \{(ş, ɳ\,\tilde{ş}(ş)) : ş \in Ķ, ɳ\,\tilde{ş}(ş) \in [0,1]\}$ where $ɳ\,\tilde{ş}(ş)$ is a membership function (MF).

Definition 2:

A fuzzy number (FN) $\tilde{ş}$ is a fuzzy set (FS) whose membership function (MF) $ɳ\,\tilde{ş}(ş)$ meets the following condition

(1) $ɳ\,\tilde{ş}(ş)$ piecewise ongoing

(2) $ɳ\,\tilde{ş}(ş)$ is convex

(3) $ɳ\,\tilde{ş}(ş)$is normal (i.e.) $ɳ\,\tilde{ş}(ş_0) = 0$.

Definition 3:

A fuzzy number (FN) $\tilde{\text{ϙ}}$ is a trapezoidal fuzzy number (TFN) described by $\tilde{\text{ϙ}} = (\text{q}^{m}, \text{q}^{n}, ɼ_1, ɼ_2)$ where $\text{q}^{m}, \text{q}^{n}, ɼ_1, ɼ_2$ are actual points and its ϙ(ş) membership function (MF) is shown below,

$$\text{ϙ}(\text{ş}) = \begin{cases} \dfrac{\text{ş} - (\text{q}^{m} - ɼ_1)}{ɼ_1}, & \text{q}^{m} - ɼ_1 \le \text{ş} \le \text{q}^{m} \\ 1, & \text{q}^{m} \le \text{ş} \le \text{q}^{n} \\ \dfrac{(\text{q}^{n} + ɼ_2) - \text{ş}}{ɼ_2}, & \text{q}^{n} \le \text{ş} \le \text{q}^{n+ɼ_2} \\ 0, & others \end{cases}$$

3. Ranking Trapezoidal Fuzzy Numbers

The Yager ranking function proposed in [23] is used for the classification of the fuzzy trapezoidal numbers.

The ranking function (RF) ђ: F (Ḩ) ▯ Ḩ where F (Ḩ) is a set of fuzzy numbers (FN) based on the set of actual points that applies of the fuzzy number to the Actual line where the general pattern exists, that is.

(i) $\tilde{Ţ} > \tilde{Ç} \leftrightarrow ђ(\tilde{Ţ}) > ђ(\tilde{Ç})$

(ii) $\tilde{Ţ} < \tilde{Ç} \leftrightarrow ђ(\tilde{Ţ}) < ђ(\tilde{Ç})$

(iii) $\tilde{Ţ} = \tilde{Ç} \leftrightarrow ђ(\tilde{Ţ}) = ђ(\tilde{Ç})$

4. Development Ranking Function

The technique of developing the ranking function with the help of weight so that the weight value between the closed interval is zero and one.

Now let $W = \frac{9}{10}$, then applying the following Function Ranking:

$$Ḩ(Ţ) = w \int_0^1 (\inf \tilde{\text{q}}_\sigma + \sup \tilde{\text{q}}_\sigma)\, d\sigma$$

Substitute the weight value in the ordinate function as follows:

$$Ḩ(Ţ) = \frac{9}{10} \int_0^1 (\inf \tilde{\text{q}}_\sigma + \sup \tilde{\text{q}}_\sigma)\, d\sigma$$

$$Ḩ(Ţ) = \frac{9}{10} \left[\int_0^1 (\text{q}^{m} - ɼ_1(1 - \sigma^4)\, d\sigma + \int_0^1 \text{q}^{n} + ɼ_2(1 - \sigma^2)\, d\sigma \right]$$

and

$$Ḩ(Ţ) = \frac{9}{10} \left[\text{q}^{m}\sigma - ɼ_1\sigma + \frac{\sigma^5}{5}ɼ_1 + \text{q}^{n}\sigma + ɼ_2\sigma - \frac{\sigma^3}{3}ɼ_2 \Big|_0^1 \right]$$

$$Ḩ(Ţ) = \frac{9}{10} \left[\text{q}^{m} - ɼ_1 + \frac{\text{q}_1}{5} + \text{q}^{n} + ɼ_2 - \frac{ɼ_2}{3} \right]$$

and

$$\text{Ḩ}(\text{Ţ}) = \frac{9}{10}\left[\text{ɋ}^{\text{ɱ}} - \frac{4}{5}\text{ŗ}_1 + \text{ɋ}^{\text{ŋ}} + \frac{2}{3}\text{ŗ}_2\right]$$

$$\text{Ḩ}(\text{Ţ}) = \frac{9}{10}\left[\text{ɋ}^{\text{ɱ}} + \text{ɋ}^{\text{ŋ}} + \frac{2}{15}(5\text{ŗ}_2 - 6\text{ŗ}_1)\right]$$

The following formula is development ranking function (DRF):

$$\text{Ḩ}(\text{Ţ}) = \frac{9}{10}(\text{ɋ}^{\text{ɱ}} + \text{ɋ}^{\text{ŋ}}) + \frac{3}{25}(5\text{ŗ}_2 - 6\text{ŗ}_1)$$

5. **Finding the Critical Path for Solving Fuzzy Network Analysis**

A project network's Critical Path (CP) can be obtained by using the following steps:

Step1: Convert trapezoidal fuzzy numbers to the equivalent crisp value by development ranking function.

step2: Construct the project network.

Step3: Identify Project Activities.

Step4: Set precedent relationships of all activities.

Step5: Estimate the time of activity about each activity.

step6: Identify all possible production network paths, and consider these paths as alternatives. Then create matrix of decisions based on all possible paths and efficient criteria such as time, expense, quality.

Step7: Calculate network analysis by critical path method.

Step8: Comparison of the development ranking function with the other function to obtain the minimum project period in hours.

6. **Numerical Example:**

The problem arises in the search on the length of the critical path (CP) in the non-rotating project network containing 7 vertices and 9 edges with a trapezoid fuzzy number (TFN).

The following problem can be represented as a trapezoidal fuzzy number in network analysis. And the trapezoidal fuzzy number problem is transformed into a crisp value using the formula of the development ranking function in Table 1:

Table 1: The trapezoidal fuzzy number problem is converted crisp value in example

Activity (i,j)	Fuzzy duration	Crisp values using development ranking function	Crisp values using Maleki ranking function [24]
1-2	(3,4,5,6)	6.300	7.500
1-3	(5,6,7,7)	9.060	11.000
2-4	(4,5,6,6)	7.380	9.000
2-6	(8,9,10,11)	14.700	17.500
3-4	(5,6,7,7)	9.060	11.000
3-5	(10,11,12,14)	18.660	22.000
4-7	(14,15,16,18)	25.380	30.000
5-7	(14,15,15,16)	24.900	29.500
6-7	(15,16,17,18)	26.460	31.500

Build the project network (PN) with each activity's time period in Fig. 1

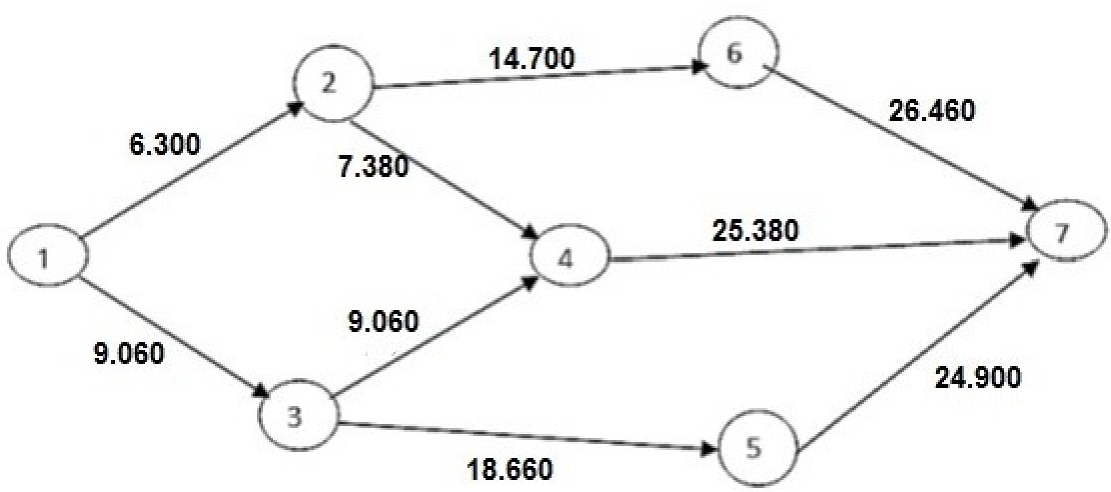

Figure1: Project network (PN) with time period for each activity based on developed ranking function calculations.

We can calculate and identify the critical path by adopted critical path method as follows in Table 2:

Table 2: The calculate and identify the critical path method

Event (i)	ESi	Lf j	Slack(Lfj -ESi)	Critical path
1	0	0	0	Yes
2	6.300	11.460	5.16	No
3	9.060	9.060	0	Yes
4	18.120	27.240	9.12	No
5	27.720	27.720	0	Yes
6	21	26.160	5.16	No
7	52.620	52.620	0	Yes

Critical path (CP) is 1 →3 →5→7

The length of the Critical Path (CP) is (52.620) hours, the minimum project period.

Build the project network (PN) with each activity's time period in Fig. 2

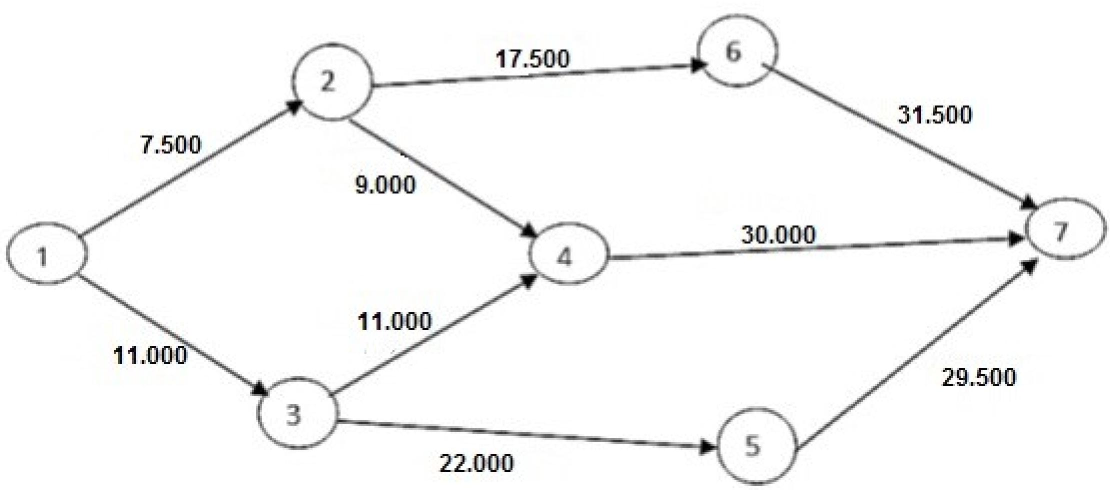

Figure2: Project network (PN) with time period for each activity based on Maleki ranking function [24] calculations.

The calculate and identify the critical path by adopted the critical path method as follows in Table 3:

Table 3: The calculate and identify the critical path method

Event (i)	ESi	Lf j	Slack(Lfj -ESi)	Critical path
1	0	0	0	Yes
2	7.500	13.500	6.000	No
3	11.000	11.000	0	Yes
4	22.000	32.000	10.000	No
5	33.000	33.000	0	Yes
6	25.000	31.000	6.000	No
7	62.000	62.000	0	Yes

Critical path (CP) is 1 →3 →5→7

The length of the Critical Path (CP) is (62.000) hours, the minimum project period

compared the two ranking functions in the following Table 4 and Fig.3

Table 4: Compared the two ranking functions

NO.	Ranking Function	The length of the project in hour
1	Development ranking function	52.620
2	Maleki ranking function	62.000

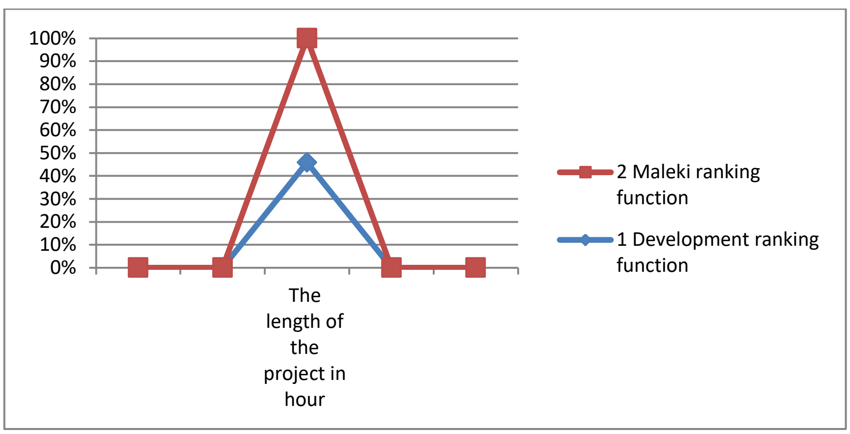

Figure3: Compared the development ranking functions (DRF) and Maleki ranking function (MRF)

It appears from the comparison Table 4, Fig.3 that the developed ranking function is the best Maleki ranking function [24].

7. **Conclusions**

An algorithm is presented and proposed to transform the fuzzy network into a clear network, and achieve the best result in the completion times of the fuzzy network with a trapezoidal fuzzy number. The proposed method is examined by numerical calculations in this paper.

References

[1] M. Shanmugasundari and K. Ganesan , Project Scheduling Problems Under Fuzzy Environment, A New Solution Approach. IJPAMS, vol. 95, (2014) , p.387-399.

[2] R. Morovatdar, A. Aghaie and S.Haji Yakhchali ,Fuzzy Network Analysis for Projects with High Level of Risks – Uncertainty in Time and Structure, IJIEPR, vol.22,(2011),p.73-82.

[3] A. Soltani and R.Haji, A Project Scheduling Method Based on Fuzzy Theory, JISE, vol.1,(2007) , p. 70-80.

[4] A.Kumar and P.Kaur, A New Method for Fuzzy Critical Path Analysis in Project Networks with a New Representation of Triangular Fuzzy Numbers, AAM, vol.05, (2010) ,p.345 – 369.

[5] D. Pandey , S K.Tyagi and V.Kumar , Reliability Analysis of a Series and Parallel Network using Triangular Intuitionistic Fuzzy Sets, AAM, vol.6, (2011),p.105 – 115.

[6] K.Selvakumari and G.Sowmiya, Fuzzy Network Problems Using Job Sequencing Technique in Hexagonal Fuzzy Numbers, IJAERD, vol.4, (2017), p.116-121.

[7] P.Jayagowri and G. Geetharamani , Using Metric Distance Ranking Method to Find Intuitionistic Fuzzy Critical Path, Hindawi , (2015), p.1-12.

[8] N R. Shankar, V. Sireesha and P Ph B. Rao, Critical Path Analysis in the Fuzzy Project Network, AFM,vol.5, (2010), p.285–294.

[9] D. Sen, D. Roy and S. Dey , Critical Path Method in the Network Analysis with Parametric Fuzzy Activity, IOSR Journal of Computer Engineering (IOSR-JCE), vol.21, (2019), p.54-60.

[10] S. Narayanamoorthy and S. Maheswari, The Intelligence of Octagonal Fuzzy Number to Determine the Fuzzy Critical Path: A New Ranking Method, Hindawi Publishing Corporation Scientific Programming,(2016), p.1-8.

[11] R.Shruti and S. Akila, Applying Metric Distance Ranking Technique in Fuzzy Critical Path Analysis. Science, Technology and Development, (2020), p.257-265.

[12] R. J. Mitlif, Ranking Function Application for Optimal Solution of Fractional Programming Problem, , Al-Qadisiyah Journal Of Pure Science , vol. 25, No.1, (2020), p.27-35.

[13] N S. GOWRI and S. AKILA, Analysis of Fuzzy Critical path Using Metric Distance Ranking Technique, Science, Technology and Development, (2020) , p. 98-107.

[14] C. Rajendran and M. Ananthanarayanan, Fuzzy Critical Path Method Using Ranking of Fuzzy Numbers, IJSR, vol.16, (2017),p.2702- 2705.

[15] R. J. Mitlif, Computation the Optimal Solution of Octagonal Fuzzy Numbers, Journal of Al-Qadisiyah for Computer Science and Mathematics, Vol.12, No.4, (2020), pp. 71–78.

[16] R. J. Mitlif, An Efficient Algorithm for Fuzzy Linear Fractional Programming Problems via Ranking Function, Baghdad Sci. J, vol.19, No.1,(2022), p.71-76.

[17] S.Narayanamoorthy and S. Maheswari, Fuzzy Critical path Method with Hexagonal and Generalised Hexagonal Fuzzy Numbers Using Ranking Method, IJAER , vol. 13, (2018), p.11877-11882.

[18] R. J. Mitlif, M. RASHEED, S. SHIHAB, T. RASHID , S. Abed Hamad, Linear Programming Method Application in a Solar Cell, Journal of Al-Qadisiyah for Computer Science and Mathematics Vol.13, No.1, (2021) , p. 10-21.

[19] R. J. Mitlif, M. Rasheed, S. Shihab , AN OPTIMAL ALGORITHM FOR A FUZZY TRANSPORTATION PROBLEM, JOURNAL OF SOUTHWEST JIAOTONG UNIVERSITY, Vol. 55, No. 3, (2020) , p.1-11.

[20] I H. Hussein and R J .Mitlif , Ranking Function to Solve a Fuzzy Multiple Objective Function, Baghdad Sci. J, vol.18, No.1 , (2021), p.144-148.

[21] R. J. Mitlif, 2016, Solving fuzzy fractional linear programming problems by ranking function methods, JOURNAL OF COLLEGE OF EDUCATION, 1, pp:93-108.

[22] I H. Hussein and Z S. Abood, Solving Fuzzy Games Problems by Using Ranking Functions, Baghdad Sci. J, vol.15, (2018), p.98-101.

[23] Karyati , D U. Wutsqa and N. Insani, Yager's Ranking Method for Solving the Trapezoidal Fuzzy Number Linear Programming, ICMSE, Journal of Physics: Conf. Series, (2018), p.1-7.

[24] H R. Maleki, Ranking Functions and Their Applications to Fuzzy Linear Programming, FJMS, vol.4 (2002), p.283-301.