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On λ-Convergent For Double Sequence Spaces of Fuzzy Numbers Described by Double Orlicz Functions

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ABSTRACT

This study introduces λ-convergent for double sequence spaces of fuzzy numbers described by double Orlicz functions and consider some properties, such as, $(\mathbb{m}\mathbb{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ is a solid , $(\mathbb{m}\mathbb{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ isn't solid , $(\mathbb{m}\mathbb{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ and $(\mathbb{m}\mathbb{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ aren't symmetric , $(\mathbb{m}\mathbb{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ and $(\mathbb{m}\mathbb{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ aren't convergence-free , $(\mathbb{m}\mathbb{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ and $(\mathbb{m}\mathbb{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M},\wp)$ are sequence algebras .

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1. Introduction :

In 1965, Zadeh [34] was the first describe the fuzzy set and fuzzy set operation concepts .
Fuzzy logic became a major research topic in a variety of mathematical disciplines later on, including metric and topological space [5,18] and approximation theory [1] .

Fuzzy set theory is utilized in computer programming [9], nonlinear dynamical systems [10], population dynamics [2], chaos control [8], quantum physics [17], and other fields of study and engineering for modeling, uncertainty, and ambiguity.

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Many other sorts of fuzzy Numbers will be introduced by workers on sequence space. Bromwich [3], the first to publish a paper on double sequence, was the first to do so..

The concept of regular convergence of a real or complex double sequence was introduced by Hardy [11]. Tripathy and Dutta [25,26] studied and constructed a variety of fuzzy real-valued double sequence spaces .

The concept of ordinary convergence is broadened to cover sequences. The concept of ideal convergence was created by Kastyrko et al [12], which is a generalization of statistical convergence based on the ideal of natural number subsets .

[20,21,28,29,30,31,33] has some work in this approach . The Orlicz function has been used to define sequence space by several writers, including [6,7,13,19,22,23,24,27,30,32] .

We discuss a few new double sequence spaces of fuzzy numbers defined by the double Orlicz function in this study . Many of the topics mentioned here are regarded rare occurrences, and the space offered here is far more broad than those currently in use .

2.Definitions and Preliminaries :

Definition 2.1[3]:

A double Orlicz function is a function that has two part $\mathbb{M}: [0,\infty) \times [0,\infty) \rightarrow [0,\infty) \times [0,\infty)$ as a result $\mathbb{M}(\mathfrak{N}, \mathfrak{M}) = (\mathbb{M}_1(\mathfrak{N}), \mathbb{M}_2(\mathfrak{M}))$, in which $\mathbb{M}_1: [0,\infty) \rightarrow [0,\infty)$ and $\mathbb{M}_2: [0,\infty) \rightarrow [0,\infty)$, these functions are non-decreasing, continuous, even, convex, and satisfy the following conditions :

i) $\mathbb{M}_1(0) = 0, \mathbb{M}_2(0) = 0 \Longrightarrow \mathbb{M}(\mathfrak{A}, \mathfrak{S}) = (\mathbb{M}_1(0), \mathbb{M}_2(0)) = (0,0)$.

ii) $\mathbb{M}_1(\mathfrak{A}) > 0, \mathbb{M}_2(\mathfrak{S}) > 0 \Longrightarrow \mathbb{M}(\mathfrak{A}, \mathfrak{S}) = (\mathbb{M}_1(\mathfrak{A}), \mathbb{M}_2(\mathfrak{S})) > (0,0), \forall\, \mathfrak{A} > 0, \mathfrak{S} > 0$, by which we mean $(\mathfrak{A}, \mathfrak{S}) > (0,0)$, that $\mathbb{M}_1(\mathfrak{A}) > 0, \mathbb{M}_2(\mathfrak{S}) > 0$.

iii) $\mathbb{M}_1(\mathfrak{A}) \to \infty, \mathbb{M}_2(\mathfrak{S}) \to \infty$ as $\mathfrak{A} \to \infty, \mathfrak{S} \to \infty$, then $\mathbb{M}(\mathfrak{A}, \mathfrak{S}) = (\mathbb{M}_1(\mathfrak{A}), \mathbb{M}_2(\mathfrak{S})) \to (\infty,\infty)$ as $(\mathfrak{A}, \mathfrak{S}) \to (\infty,\infty)$, by which we mean $\mathbb{M}(\mathfrak{A}, \mathfrak{S}) \to (\infty,\infty)$, as $\mathbb{M}_1(\mathfrak{A}) \to \infty, \mathbb{M}_2(\mathfrak{S}) \to \infty$.

Definition 2.2 [33]:

Assume $\mathbb{X}$ is a non-empty set, If 1) $\forall\, \mathbb{A}, \mathbb{B} \in \lambda$ as $\mathbb{A} \cup \mathbb{B} \in \lambda$ 2) $\forall\, \mathbb{A} \in \lambda, \mathbb{B} \subseteq \mathbb{A}$ as $\mathbb{B} \in \lambda$ then $\lambda \subseteq 2^{\mathbb{X}}$ is an ideal .

Definition 2.3 [33]:

For every $\varepsilon > 0, \exists\, n_0 = n_0(\varepsilon), m_0 = m_0(\varepsilon) \in \mathbb{N}$ in which $\bar{d}(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}, \mathfrak{Y}_0) < \varepsilon, \forall n \geq n_0, m \geq m_0$ then $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}})$ be convergent to the fuzzy real number $\mathfrak{A}_0$ in the Pringsheim sense.

Definition 2.4 [33]:

for every $\varepsilon \succ 0, \{(\mathfrak{d}, \mathfrak{h}) \in \mathbb{N}^2 : \bar{d}(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}, \mathfrak{Y}_0) \succcurlyeq \varepsilon\} \in \lambda_2$ then $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}})$ is a λ-convergent to a fuzzy number $\mathfrak{A}_0$.

Remark 2.5 [33]:

Let's $\mathbb{E}_{\mathbb{F}}^2$ be the sequence spaces of double fuzzy numbers.

Definition 2.6 [33]:

If $(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}) \in \mathbb{E}^2_{\mathbb{F}}$ whenever $\bar{d}(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}, \bar{0}) \preccurlyeq \bar{d}(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}, \bar{0})$, $\forall\, \mathfrak{d}, \mathfrak{h} \in \mathbb{N}$ and $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}) \in \mathbb{E}^2_{\mathbb{F}}$ then $\mathbb{E}^2_{\mathbb{F}}$ is a solid .

Definition 2.7 [33]:

If $\left(\mathfrak{Y}_{\curlywedge(\mathfrak{d})\curlywedge(\mathfrak{h})}\right) \in \mathbb{E}^2_{\mathbb{F}}$ whenever $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}) \in \mathbb{E}^2_{\mathbb{F}}$ in which $\curlywedge$ is a combination of $\mathbb{N} \times \mathbb{N}$ then $\mathbb{E}^2_{\mathbb{F}}$ is a symmetric .

Definition 2.8 [33]:

If $\left(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}} \otimes \mathfrak{H}_{\mathfrak{d}\mathfrak{h}}\right) \in \mathbb{E}^2_{\mathbb{F}}$, whenever $\left(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}\right), \left(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}\right) \in \mathbb{E}^2_{\mathbb{F}}$ then $\mathbb{E}^2_{\mathbb{F}}$ is a sequence algebra

Definition 2.9 [33]:

If $\left(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}\right) \in \mathbb{E}^2_{\mathbb{F}}$ whenever $\left(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}\right) \in \mathbb{E}^2_{\mathbb{F}}$ and $\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}} = \bar{0}$ implies $\mathfrak{H}_{\mathfrak{d}\mathfrak{h}} = \bar{0}$ then $\mathbb{E}^2_{\mathbb{F}}$ is a convergence-free .

Definition 2.10 [33]:

A fuzzy subset of $\mathbb{R}$ is a map $\mathbb{F}: \mathbb{R} \to \mathbb{I} = [0,1]$ linking each real number $\mathfrak{g}$ with its membership rank $\mathbb{F}(\mathfrak{g})$ that fits the following conditions :

1) $\forall\, \mathbb{F}(\mathfrak{g}) \succcurlyeq \mathbb{F}(\mathfrak{l}) \wedge \mathbb{F}(\mathfrak{f}) = \min\{\mathbb{F}(\mathfrak{l}), \mathbb{F}(\mathfrak{f})\}$, in which $\mathfrak{l} < \mathfrak{g} < \mathfrak{f}$, then $\mathbb{F}$ is a convex.
2) $\forall\, \mathfrak{g}_0 \in \mathbb{R}$ and $(\mathfrak{g}_0) = 1$, then $\mathbb{F}$ is a normal.
3) $\forall\, \epsilon \succ 0$ and $\mathbb{F}^{-1}([0, \mathfrak{k} + \epsilon))$ is open in the usual topology of $\mathbb{R}$, then $\mathbb{F}$ is a upper-semi-continuous $\forall\, \mathfrak{k} \in \mathbb{I}$.
4) $\forall\, \mathbb{F}(\mathfrak{g}) = 0$, then $\mathbb{F}$ is a non-negative number $\forall\, \mathfrak{g} < 0$. $\mathbb{R}^*(\mathbb{I})$ refers to the set of all fuzzy number that aren't negative of $\mathbb{R}(\mathbb{I})$.

Let's $\mathbb{R}(\mathbb{I})$ to be a collection of each fuzzy integer that are upper-semi-continuous, normal, and convex .

A real numbers $\mathbb{R}$ can be embedded in $\mathbb{R}(\mathbb{I})$ if we are to define $\bar{\mathfrak{q}} \in \mathbb{R}(\mathbb{I})$ by

$$\bar{\mathfrak{q}}(\mathfrak{y}) = \begin{cases} 1 & \text{, if } \mathfrak{y} = \mathfrak{q} \\ 0 & \text{, if } \mathfrak{y} \neq \mathfrak{q}. \end{cases} .$$

The following are the arithmetic operations on $\mathbb{R}(\mathbb{I})$:

$$(\mathbb{P} \oplus \mathbb{Q})(\mathfrak{y}) = \sup\{\mathbb{P}(\mathfrak{z}) \wedge \mathbb{Q}(\mathfrak{y} - \mathfrak{z})\}, \ \mathfrak{y}, \mathfrak{z} \in \mathbb{R},$$

$$(\mathbb{P} \ominus \mathbb{Q})(\mathfrak{y}) = \sup\{\mathbb{P}(\mathfrak{y}) \wedge \mathbb{Q}(\mathfrak{z} - \mathfrak{y})\}, \ \mathfrak{y}, \mathfrak{z} \in \mathbb{R},$$

$$(\mathbb{P} \otimes \mathbb{Q})(\mathfrak{y}) = \sup\left\{\mathbb{P}(\mathfrak{y}) \wedge \mathbb{Q}\left(\tfrac{\mathfrak{y}}{\mathfrak{z}}\right)\right\}, \ \mathfrak{y}, \mathfrak{z} \in \mathbb{R}, \mathfrak{z} \neq 0,$$

$$\left(\frac{\mathbb{X}}{\mathbb{Y}}\right)(\mathfrak{y}) = \sup\{\mathbb{P}(\mathfrak{z}\mathfrak{y}) \wedge \mathbb{Q}(\mathfrak{y})\}, \ \mathfrak{y}, \mathfrak{z} \in \mathbb{R}.$$

A absolute value $|\mathfrak{f}|$ of $\mathfrak{f} \in \mathbb{R}(\mathbb{I})$ with characterized by (see [33])

$$|\mathfrak{f}|(\mathfrak{y}) = \begin{cases} \max\{\mathfrak{f}(\mathfrak{y}), \mathfrak{f}(-\mathfrak{z})\}, & \forall\ \mathfrak{y} \geq 0, \\ 0, & \forall\ \mathfrak{y} < 0. \end{cases}$$

Let's $\mathbb{D}$ denote the set of all closed bounded intervals $\mathfrak{G} = [\mathfrak{G}^{\mathbb{S}}, \mathfrak{G}^{\mathbb{T}}], \mathfrak{E} = [\mathfrak{E}^{\mathbb{S}}, \mathfrak{E}^{\mathbb{T}}]$. $\mathfrak{G} \leq \mathfrak{E} \Leftrightarrow \mathfrak{G}^{\mathbb{S}} \leq \mathfrak{E}^{\mathbb{S}}$ and $\mathfrak{G}^{\mathbb{T}} \leq \mathfrak{E}^{\mathbb{T}}$.

Also $d(\mathbb{X}, \mathbb{Y}) = \max\left[|\mathfrak{G}^{\mathbb{S}} - \mathfrak{G}^{\mathbb{S}}|, |\mathfrak{E}^{\mathbb{T}} - \mathfrak{E}^{\mathbb{T}}|\right]$ .Then $(\mathbb{D}, d)$ is a $\mathbb{CMS}$. Let's $\bar{d} : \mathbb{R}(\mathbb{I}) \times \mathbb{R}(\mathbb{I}) \to \mathbb{R}^+ \cup \{0\}$ be characterized by $\bar{d}(\mathfrak{G}, \mathfrak{E}) = \sup_{0 \leq \ltimes \leq 1} d([\mathfrak{G}]^{\ltimes}, [\mathfrak{E}]^{\alpha})$. Because of this $(\mathbb{R}(\mathbb{I}), \bar{d})$ be a complete metric space is well-known .

Let's $\mathbb{M} = (\mathbb{M}_1, \mathbb{M}_2)$ be double sequence space and $\wp = (\wp_{\mathfrak{d}\mathfrak{h}})$ denoted double sequence of real numbers with positive bounds. The following are the classes of double sequence that we will discuss:

$(c^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp) = \left\{((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, (\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}) : \lambda_2 - \lim \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, \mathbb{L}_1)}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}, \mathbb{L}_2)}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} = 0, \text{for some } \rho > 0 \text{ and } \mathbb{L}_1, \mathbb{L}_2 \in \mathbb{R}(\mathbb{I})\right\}$,in which $\mathbb{M} = (\mathbb{M}_1, \mathbb{M}_2)$.

$(c_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp) = \left\{((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, (\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}) : \lambda_2 - \lim \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} = 0, \text{ for some } \rho > 0\right\}$, in which $\mathbb{M} = (\mathbb{M}_1, \mathbb{M}_2)$.

$(\ell_\infty^2)^{(\mathbb{F})}(\mathbb{M}, \wp) = \left\{((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, (\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}) : \sup_{\mathfrak{d}\mathfrak{h}} \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{Y}_2)_{\mathfrak{ce}}, \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{ce}}} < \infty, \text{ for some } \rho > 0\right\}$, in which $\mathbb{M} = (\mathbb{M}_1, \mathbb{M}_2)$.

$(\ell_\infty^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp) = \Big\{((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, (\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}})$: there is such as a real number $\mu \succ 0$ as a result the collection $\Big\{(\mathfrak{d}, \mathfrak{h}) \in \mathbb{N} \times \mathbb{N}: \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} > \mu\Big\} \in \lambda_2$, for some $\rho > 0\Big\}$, in which $\mathbb{M} = (\mathbb{M}_1, \mathbb{M}_2)$.

We also write

$(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp) = (c^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp) \cap (\ell_\infty^2)^{(\mathbb{F})}(\mathbb{M}, \wp)$

$(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp) = (c_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp) \cap (\ell_\infty^2)^{(\mathbb{F})}(\mathbb{M}, \wp)$

3. Main Results :

Theorem 3.1:

$(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ is a solid .

Proof:

Let's $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}) = ((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, (\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}) \in (\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ and $(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}) = ((\mathfrak{H}_1)_{\mathfrak{d}\mathfrak{h}}, (\mathfrak{H}_2)_{\mathfrak{d}\mathfrak{h}})$ be in which $\bar{d}(\mathfrak{S}_{\mathfrak{d}\mathfrak{h}}, \bar{0}) \preccurlyeq \bar{d}(\mathfrak{U}_{\mathfrak{d}\mathfrak{h}}, \bar{0})$, $\forall\, \mathfrak{d}, \mathfrak{h} \in \mathbb{N}$. Let's $\varepsilon \succ 0$ be given. As a result, the solidity of $(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ is determined by the following relationship:

$$\left\{(\mathfrak{d}, \mathfrak{h}) \in \mathbb{N} \times \mathbb{N}: \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{H}_1)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{H}_2)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} \geq \varepsilon\right\} \subseteq$$
$$\left\{(\mathfrak{d}, \mathfrak{h}) \in \mathbb{N} \times \mathbb{N}: \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}, \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} \geq \varepsilon\right\}.$$

Example 1:

$(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ isn't solid.

Proof:

If $\lambda_2(\rho) \subset 2^{\mathbb{N}\times\mathbb{N}}$ then the class of all subsets of $\mathbb{N} \times \mathbb{N}$ is a known as zero natural density. Let's $\lambda_2 = \lambda_2(\rho)$ and $\mathbb{A} \in \lambda_2$ and $\wp_{\mathfrak{d}\mathfrak{h}} = 1$, $\forall\, \mathfrak{d}, \mathfrak{h} \in \mathbb{N}$ and $\mathbb{M}(x_1, x_2) = (x_1^2, x_2^2)$, $\forall\, (\mathfrak{d}, \mathfrak{h}) \notin \mathbb{A} \Longrightarrow (\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}})$ is characterized with :

$$\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{v}) = \begin{cases} \big(1+(\mathfrak{d}+\mathfrak{h})(\mathfrak{v}-1), 1+(\mathfrak{d}+\mathfrak{h})(\mathfrak{v}-1)\big), & \forall\, 1-\frac{1}{\mathfrak{d}+\mathfrak{h}} \le \mathfrak{v} \le 1 \\ \big(1-(\mathfrak{d}+\mathfrak{h})(\mathfrak{v}-1), 1-(\mathfrak{d}+\mathfrak{h})(\mathfrak{v}-1)\big), & \forall\, 1 < \mathfrak{v} \le 1+\frac{1}{\mathfrak{d}+\mathfrak{h}} \\ (0,0) & \text{otherwise} \end{cases}.$$

$\forall\, (\mathfrak{d}, \mathfrak{h}) \in \mathbb{A}$ then $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}) = \bar{1}$ and $\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{x}) \in (\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$.

Let's $\mathbb{K} = \{2\mathfrak{i} : \mathfrak{i} \in \mathbb{N}\}$ and $(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}})$ has the following definition :

$$\mathfrak{H}_{\mathfrak{d}\mathfrak{h}} = \begin{cases} \mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}, & \forall\, (\mathfrak{d}+\mathfrak{h}) \in \mathbb{K} \\ 0 & \text{otherwise} \end{cases}.$$

Then $(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}) \notin (\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$. Thus $(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ isn't solid.

Example 2:

$(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ and $(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ aren't symmetric.

Proof:

$\lambda_2(\rho) \subset 2^{\mathbb{N}\times\mathbb{N}} \Longrightarrow$ the class of all subsets of $\mathbb{N} \times \mathbb{N}$ is a known as zero natural density . Let's $\lambda_2 = \lambda_2(\rho)$and $\mathbb{M}(x_1, x_2) = (x_1^2, x_2^2)$ and

$$\wp_{\mathfrak{d}\mathfrak{h}} = \begin{cases} (1,1), & \forall\, \mathfrak{d} \text{ is even and } \forall\, \mathfrak{h} \in \mathbb{N} \\ (2,2) & \text{other wise} \end{cases}.$$

$\forall\, \mathfrak{d} = \mathfrak{i}^2, \mathfrak{i} \in \mathbb{N}$ and $\forall\, \mathfrak{h} \in \mathbb{N}$ then $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}})$ is characterized with :

$$\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{v}) = \begin{cases} \left(1+\frac{\mathfrak{v}}{2\sqrt{\mathfrak{d}}-1}, 1+\frac{\mathfrak{v}}{2\sqrt{\mathfrak{d}}-1}\right), & \forall\, 1-2\sqrt{\mathfrak{d}} \le \mathfrak{v} \le 0 \\ \left(1-\frac{\mathfrak{v}}{2\sqrt{\mathfrak{d}}-1}, 1-\frac{\mathfrak{v}}{2\sqrt{\mathfrak{d}}-1}\right), & \forall\, 0 < \mathfrak{v} \le 2\sqrt{\mathfrak{d}}-1 \\ (0,0) & \text{otherwise} \end{cases}.$$

$\exists\, \mathfrak{d} \ne \mathfrak{i}^2, \mathfrak{i} \in \mathbb{N}$ and $\exists\, \mathfrak{h} \notin \mathbb{N}$ then $\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}} = \bar{0}$ and $\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{t}) \in \mathbb{Z}(\mathbb{M}, \wp)$, $\forall\, \mathbb{Z} = (\mathfrak{m}^2)^{\lambda(\mathbb{F})}, (\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}$.

$\forall\, \mathfrak{h}$ is odd and $\forall\, \mathfrak{d} \in \mathbb{N}$ then $(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}})$ reorganization of $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}})$ is characterized with :

$$\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{v}) = \begin{cases} \left(1-\frac{\mathfrak{v}}{2\mathfrak{d}-1}, 1-\frac{\mathfrak{v}}{2\mathfrak{d}-1}\right), & \forall\, 1-2\mathfrak{d} \le \mathfrak{v} \le 0 \\ \left(1-\frac{\mathfrak{v}}{2\mathfrak{d}-1}, 1-\frac{\mathfrak{v}}{2\mathfrak{d}-1}\right), & \forall\, 0 < \mathfrak{v} \le 2\mathfrak{d}-1 \\ (0,0) & \text{otherwise} \end{cases}.$$

$\exists\, \mathfrak{d}$ is even and $\exists\, \mathfrak{d} \notin \mathbb{N}$ then $\mathfrak{H}_{\mathfrak{d}\mathfrak{h}} = \bar{0}$ and $\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{x}) \notin \mathbb{Z}(\mathbb{M}, \wp)$, $\forall\, \mathbb{Z} = (\mathfrak{m}^2)^{\lambda(\mathbb{F})}, (\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}$ then $(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ and $(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ aren't symmetric .

Example 3:

$(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ and $(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ aren't convergence-free.

Proof:

If $\lambda_2(\rho) \subset 2^{\mathbb{N}\times\mathbb{N}}$ then the class of all subsets of $\mathbb{N}\times\mathbb{N}$ is a known as zero natural density. Let's $\lambda_2 = \lambda_2(\rho)$ and $\mathbb{A} \in \lambda_2$, $\wp_{\mathfrak{d}\mathfrak{h}} = \frac{1}{3}$, $\forall\, \mathfrak{d}, \mathfrak{h} \in \mathbb{N}$ and $\mathbb{M}(x_1, x_2) = (x_1, x_2)$. $\forall\, (\mathfrak{d}, \mathfrak{h}) \notin \mathbb{A}$ then $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}})$ is characterized with :

$$\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{v}) = \begin{cases} (1{+}2(\mathfrak{d}{+}\mathfrak{h})\mathfrak{v}\,, 1{+}2(\mathfrak{d}{+}\mathfrak{h})\mathfrak{v}) & , \forall -\frac{1}{2(\mathfrak{d}+\mathfrak{h})} \le \mathfrak{v} \le 0 \\ (1{-}2(\mathfrak{d}{+}\mathfrak{h})\mathfrak{v}\,, 1{-}2(\mathfrak{d}{+}\mathfrak{h})\mathfrak{v}) & , \forall\ 0 < \mathfrak{v} \le \frac{1}{2(\mathfrak{d}+\mathfrak{h})} \\ (0,0) & \text{otherwise} \end{cases}.$$

$\forall(\mathfrak{d}\,, \mathfrak{h}) \in \mathbb{A}$ then $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}) = \bar{0}$ and $\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{v}) \in \mathbb{Z}(\mathbb{M}, \wp)$, $\forall\, \mathbb{Z} = (\mathfrak{m}^2)^{\lambda(\mathbb{F})}, (\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}$

$\forall(\mathfrak{d}\,, \mathfrak{h}) \notin \mathbb{A}$ then $(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}})$ is characterized with:

$$\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{v}) = \begin{cases} \left(1{+}\frac{2\mathfrak{v}}{(\mathfrak{d}+\mathfrak{h})}, 1{+}\frac{2\mathfrak{v}}{(\mathfrak{d}+\mathfrak{h})}\right), \forall -\frac{\mathfrak{d}+\mathfrak{h}}{2} \le \mathfrak{v} \le 0 \\ \left(1{-}\frac{2\mathfrak{v}}{(\mathfrak{d}+\mathfrak{h})}, 1{-}\frac{2\mathfrak{v}}{(\mathfrak{d}+\mathfrak{h})}\right), \forall\ 0 < \mathfrak{v} \le \frac{\mathfrak{d}+\mathfrak{h}}{2} \\ (0,0) \quad \text{otherwise} \end{cases}.$$

$\forall\, (\mathfrak{d}\,, \mathfrak{h}) \in \mathbb{A}$ then $(\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}) = \bar{0}$ and $\mathfrak{H}_{\mathfrak{d}\mathfrak{h}}(\mathfrak{v}) \notin \mathbb{Z}(\mathbb{M}, \wp)$, $\forall\, \mathbb{Z} = (\mathfrak{m}^2)^{\lambda(\mathbb{F})}, (\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}$.Thus $(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ and $(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ aren't convergence-free.

Theorem 3.2 :

$(\mathfrak{m}^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ and $(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ are sequence algebras.

Proof:

Takes $(\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$. Let's $(\mathfrak{Y}_{\mathfrak{d}\mathfrak{h}})$, $(\mathfrak{d}\mathfrak{h}) \in (\mathfrak{m}_0^2)^{\lambda(\mathbb{F})}(\mathbb{M}, \wp)$ and $0 < \varepsilon < 1$. The result is obtained by applying the following inclusion relation :

$$\left\{(\mathfrak{d}\,, \mathfrak{h}) \in \mathbb{N}\times\mathbb{N}: \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}}\,,\ \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}}\,,\ \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} < \varepsilon\right\} \cap$$

$$\left\{(\mathfrak{d}\,, \mathfrak{h}) \in \mathbb{N}\times\mathbb{N}: \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{H}_1)_{\mathfrak{d}\mathfrak{h}}\,,\ \bar{0})}{\rho}\right) \curlyvee \mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{H}_2)_{\mathfrak{d}\mathfrak{h}}\,,\ \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} < \varepsilon\right\} \subseteq \left\{(\mathfrak{d}\,, \mathfrak{h}) \in \mathbb{N}\times\mathbb{N}: \left[\mathbb{M}_1\left(\frac{\bar{d}((\mathfrak{Y}_1)_{\mathfrak{d}\mathfrak{h}} \otimes (\mathfrak{H}_1)_{\mathfrak{d}\mathfrak{h}}\,,\ \bar{0})}{\rho}\right) \curlyvee\right.\right.$$

$$\left.\left.\mathbb{M}_2\left(\frac{\bar{d}((\mathfrak{Y}_2)_{\mathfrak{d}\mathfrak{h}} \otimes (\mathfrak{H}_2)_{\mathfrak{h}}\,,\ \bar{0})}{\rho}\right)\right]^{\wp_{\mathfrak{d}\mathfrak{h}}} < \varepsilon\right\}$$

. We may also demonstrate the outcome in other cases .

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