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On Nb-Separation Axioms

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ABSTRACT

The main purpose of this paper is to express a new class of separation axioms based on open sets, known as Nb open, as well as to analyze and verify several essential ideas linked with this class.

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1- Introduction

In a topological space, Andrijevi'c [2] created a new class of generalized open sets known as b-open sets, and T. A. Al-Hawary, A. Al-Omari also[9] worked on same field. All semi-open sets and pre-open sets are included in the class of b-open sets. The b-open set class yields the same topology as the preopen set class, A subset S of a space X is called b-open if S$\subseteq \overset{\circ}{\overline{S}} \cup \overline{S^{\circ}}$[2], A subset S of a space X is b-closed if X-S is b-open. Thus S is b-closed if and only if $\overset{\circ}{\overline{S}} \cap \overline{S^{\circ}} \subset S$[2], The union of any family of b-open sets is a b-open set and The intersection of an open and a b-open set is a b-open set[2], and A.AL-Omari and M.S.Md. Noorani [1] establish the idea of N – open sets, which are defined as follows: "A subset A of a space X is said to be an N – open if for every x∈A, there exist an open set $U_x \subseteq X$ containing x such that $U_x - X$ is a finite set, The complement of an N-open set is said to be N-closed, For every open set is an N-open set[1], Let X be a topological space, then X with the set of all N-open subsets of X is a topological space[1], Let X be a topological space, then the intersection of an open set with an N-open set is an N-open set and the union of N-open sets is also N-open[1]. They prove that the family of all N – open subset of a space X, denoted by $\mathcal{T}_N$ Forms a topology on X finer than $\mathcal{T}$. Moreover, we find a mutual work about ωb-open sets merging ω-open and b-open by [8] and they concluded this new concept, so we merged b-open and N-open sets to propose a new concept called Nb-open that combines all previous attributes in a new definition depending on these concepts and satisfies the basic properties of topological space like interior and closure and so on".

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2-preliminaries

Now we represent a new definition by merging the definitions of b-open and N-open:

Definition(2.1):"A subset A of a space X is said to be an Nb-open set if for each x∈A there exists a b-open set U in X with x∈ U and U-A= finite."

Proposition(2.2):

1- Every N-open set is also an Nb-open set, although the converse is not always true.

2- Every b-open set is also an Nb-open set, although the converse may not always be accurate in practice.

Example(2.3): Consider the indiscrete space (R,τ_{ind}), the set {1} is Nb-open since {1} is b-open containing 1 and {1}-{1}=∅ which is finite but {1} is not N-open since R the only open set which contain 1 but R-{1}=infinite.

3- Main Results

Definition(3.1):A space X is said to be Nb-$T_\circ$ space if for each distinct points x and y in X we $have\ Nb-open\ set\ contains\ one\ but\ not\ the\ other$.

Example(3.2):"Let X={1,2,3} and $\tau = \{\phi, X, \{1\}, \{2\}, \{1,2\}\}, then\ X\ is\ Nb - T_\circ\ space$"

Remark(3.3): Every $T_\circ\ space\ is\ Nb - T_\circ\ space\ but\ the\ converse\ may\ be\ not\ true$.

Example(3.4):"LetX={a,b,c},$\tau = \{\emptyset, X, \{a\}\}$". clearly X is not $T_\circ$ space but {b}and {c}are Nb-open sets, therefore X is Nb-$T_\circ$.

Theorem(3.5):$(X,\tau)\ is\ NbT_\circ - space\ iff\ \overline{\{x\}}^{Nb} \neq \overline{\{y\}}^{Nb}\ for\ each\ x, y \in X\ , x \neq y$.

Proof:$(\Rightarrow) Suppose\ that\ X\ is\ NbT_\circ - space, to\ prove\ \overline{\{x\}}^{Nb} \neq \overline{\{y\}}^{Nb} for\ each$

$x, y \in X\ , x \neq y\ , since\ X\ is\ NbT_\circ - space\ and\ x \neq y\ then\ there\ exists\ U \in \tau$

$such\ that\ (x \in U\ and\ y \notin U) or (x \notin U and\ y \in U)$

$suppose\ that\ (x \in U\ and\ y \notin U)\ which\ implies\ (x \in U\ and\ y \in X - U)$

$X - U\ closed\ set\ since\ U\ is\ open\ which\ implies\ \{y\} \subseteq X - U$

$and\ \overline{\{y\}}^{Nb} \subseteq \overline{X - U}^{Nb} = X - U\ (since\ X - U\ closed\ and\ \overline{X - U}^{Nb} = X - U)$

$then\ \overline{\{y\}}^{Nb} \subseteq X - U \wedge x \in U \Rightarrow \{x\} \nsubseteq X - U \Rightarrow \overline{\{x\}}^{Nb} \nsubseteq X - U, \therefore\ \overline{\{x\}}^{Nb} \neq \overline{\{y\}}^{Nb}$

Similarly, if we take $(x \notin U \wedge y \in U)$

(conversely) $suppos\ that\ \overline{\{x\}}^{Nb} \neq \overline{\{y\}}^{Nb}\ \forall x \neq y \in X, to\ prove\ X\ is\ NbT_\circ - space$

Suppose that X is not Nb$T_\circ$-space $then$ (there exist x,y∈X such that for each U∈τ such that x∈U ⇒y∈U)

$let\ z \in X\ such\ that\ z \in \overline{\{x\}}^{Nb}\ \ldots\ldots\ldots (*)$

$then\ for\ each\ U \in \tau\ such\ that\ z \in U \wedge U \cap \{x\} \neq \emptyset$ (by true: $z \in \overline{A} \Leftrightarrow for\ each\ U \in \tau\ such\ that\ z \in U \wedge U \cap A \neq \emptyset\ , but\ U \cap \{x\} \neq \emptyset \Rightarrow x \in U$

(since the only element in $\{x\} is\ x$)

hence every set contains z must contains x. So, we have the following two

statements:

Every Nb-open set contains $z\ must\ contains\ x\ and\ every\ Nb-open$ set contains x must contains $y.\ so\ every$ Nb-open set contains z must contains $y.\ \Rightarrow$

$for\ each\ U \in \tau\ such\ that\ z \in U \wedge U \cap \{y\} \neq \emptyset, z \in \overline{\{y\}}^{Nb}$ … … … . . (∗∗)"

$then\ for\ each\ z \in \overline{\{x\}}^{Nb} \Rightarrow z \in \overline{\{y\}}^{Nb} \Rightarrow \overline{\{x\}}^{Nb} \subseteq \overline{\{y\}}^{Nb}$ "

Similarly, we prove $\overline{\{y\}}^{Nb} \subseteq \overline{\{x\}}^{Nb}$

Hence $\overline{\{x\}}^{Nb} = \overline{\{y\}}^{Nb}$ $C!\left(since\ \overline{\{x\}}^{Nb} \neq \overline{\{y\}}^{Nb}\right), hence\ X\ is\ NbT_\circ - space.$

"Definition"(3.6):[3]"A space X is called bT_1-space if for each $x \neq y$ in X, there exist two b-open sets $U\ and\ V$ such that $x \in U, y \notin U$ and $y \in V, x \notin V$".

Definition(3.7):[1]A space X is called NT_1-space if and only if for each $x \neq y$ in X there exist N-open sets $U\ and\ V$ such that $x \in U, y \notin U, and\ y \in V, x \notin V$.

By the same context, we can define the definition of Nb$T_1 - space$

Definition(3.8):"A space X is called NbT_1-space if for each $x \neq y\ in\ X$, there exist Nb-open sets $U\ and\ V$ such that $x \in U, y \notin U\ and\ y \in V, x \notin V$".

Proposition(3.9):"Every T_1-space is NbT_1-space".

Proof: "let (X, τ) be T_1 space and $x, y \in X \ni x \neq y. Then$ there exist two open sets $U\ and\ V$ such that $x \in U, y \notin U\ and\ y \in V, x \notin V$, since every open set is b-open thus, $U\ and\ V$ are two b-open sets such that $x \in U, y \notin U\ and\ y \in V, x \notin V$ therefore (X, τ) be NbT_1-space".

The case of N$T_1\ is\ similar$.

Remark(3.10): In general, the opposite of the preceding claim is not valid.

Example(3.11):"Let X={1,2,3,4} , $\tau = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$"

"Proposition"(3.12):[3]"Let X be a topological space and Y⊆ X. If G is a b-open set in X and Y is an open set in X, then $G \cap Y$ is b-open set in Y."

Proof: Let x∈ $A \cap Y\ which\ implies\ x \in A\ and\ x \in Y, since\ A\ is\ Nb-open\ in\ X, then\ there$

Exists b-open set U in X containing x and U-A is finite, $U \cap Y\ is\ b-open\ in\ Y$ then

U$\cap Y - (A \cap Y) \subset U - A = finite. then\ we\ have\ A \cap Y\ is$ Nb-open in Y.

Theorem(3.14): suppose M is an open subset of X, Then M is NbT_1-subspace if X is NbT_1-space.

"Proof": "Let x,y ∈ M such that $x \neq y$ since X is NbT_1-space, then there exist two Nb-open sets U, V in X such that x ∈ U but .$y \notin U$ and $y \in V$ but $x \notin V$ Let $A = U \cap M, B = V \cap M$. Thus A, B are Nb-open set in M and $x \in$ A .but $y \notin A$ and $y \in B$ but $x \notin B$, therefore, M is NbT_1-space".

Definition(3.15):A function f: "$X \to Y$ is said to be Nb-continuous if $f^{-1}(U)$ is Nb-open in X whenever U is an open set in Y".

Remark(3.16): Every continuous function is also Nb-continuous, although the reverse may not be accurate in some cases.

Example(3.17): Let $X = \{1,2,3\}, \tau_x = indiscrete\ topology$

$Y = X \quad , \tau_y = discrete\ topology$

Then $I_X: (X, \tau_{ind}) \to (X, \tau_D) is\ an\ Nb - continuous\ but\ not\ continuous.$

Definition(3.18): $a\ function\ f: X \to Y\ is\ Nb - open\ if$ f(U) is Nb-open in Y, whenever U is open set in X.

Remark(3.19): Every open function is an Nb-open function.

Theorem(3.20): Let $f: X \to Y$ be a one-to-one Nb-continuous function. If Y is T_1-space then X is NbT_1-space.

"**Proof**:let $x_1, x_2 \in X\ such\ that\ x_1 \neq x_2$", $since\ f: X \to Y\ is$ one-to-onefunction $and\ x_1 \neq x_2\ then\ f(x_1) \neq f(x_2) and f(x_1), f(x_2) \in Y since\ Y\ is\ T_1$ space then ∃U,V open sets in Y , $f(x_1) \in U\ but\ f(x_2) \notin U, and\ f(x_2) \in V but\ f(x_1) \notin V, since\ f\ is$ Nb-continuous function then $f^{-1}(U), f^{-1}(v) are\ Nb - open\ sets\ in\ X$, since $f(x_1) \in U, thus\ x_1 \in f^{-1}(U)$, and also since $f(x_2) \notin U\ thus\ x_2 \notin f^{-1}(U), and\ also\ since\ f(x_2) \in V, then\ x \in f^{-1}(V), and\ since\ f(x_1) \notin V\ then\ x_1 \notin f^{-1}(V)$, therefore $X\ is\ NbT_1 - space.$

"Definition"(3.21):[3]"A space X is called bT_2-space (b-Hausdorff space)" if for each $x \neq y\ in\ X, there\ exist\ disjoint\ b - open\ sets\ U, V\ such\ that\ x \in U, y \in V.$

Definition(3.22):[5]A space X is called N-Hausdorff if any two distinct X points have disjoint N-open neighborhoods.

By the same context, we can define the definition of Nb$T_2 - space$

Definition(3.23): Aspace X is called NbT2-space (Nb-Hausdorff) if for each x≠y in X $there\ exist\ disjoint\ Nb - open\ sets\ U, V\ such\ that$ x∈U, y∈V containing $x \& y\ respectively$.

Remark(3.24): Any T_2 space is also an NbT_2-space, but the reverse may not be accurate.

Example(3.25): Let $X = \{1,2,3\}, \tau = \{X, \tau_{ind}\}$ is $NbT_2 - space\ but\ not\ T_2 - space.$

Proposition(3.26):$let\ f: X \to Y\ be\ a\ bijective\ function$

1- if f is Nb-open and X is T_2-space, then Y is NbT_2-space.

2- if f is Nbcontinuous and Y is T_2space, then X is NbT_2-space.

"Proof":$let\ f: X \to Y\ be\ a\ bijective\ function\ then,$

1- $suppose\ f\ is\ Nb - open\ and\ X\ is\ T_2 - space, let$ $y_1 \neq y_2 \in Y$, since f is bijective, then there exist x_1, x_2 in X, $such\ that\ f(x_1) = y_1\ and\ f(x_2) = y_2\ and\ x_1 \neq x_2$, since X is T_2-space then there exists two disjoint open sets U and V in X such that $x_1 \in U\ and\ x_2 \in V$, since f is Nb-open then f(U)and f(V)are Nb-open sets in Y $hence\ f(x_1) = y_1 \in f(U) and\ f(x_2) = y_2 \in f(V), since\ f\ is\ bijective\ so\ f(U) and\ f(V) are\ disjoint\ in\ Y, f(U) \cap f(V)$

$= f(U \cap V) = f(\emptyset) = \emptyset,\ thus\ Y\ is\ NbT_2 - space.$

2- similar to prove of (1).

Proposition(3.27): Every NbT_2-space is NbT_1-space.

Proof: *let* (X,τ) *be an* $NbT_2-space$, let x and y be two distinct in X, since X is NbT_2-space

then there exist disjoint Nb-open sets U and V such that $x \in U$ *and* $y \in V$, *since U and V are disjoint then* $x \in U$ *but* $y \notin U$, *and* $y \in V$ *and* $x \notin V$, so X is

NbT_1-space".

Theorem(3.28): Let M be an open subspace of X, then M is NbT_2subspace if X is NbT_2-space".

Proof: *let* $x, y \in M$, $x \neq y$ *then* $x, y \in X$ *so* there exist B_1, B_2 such that $B_1 \cap B_2=\emptyset$ such that $x \in B_1$, $y \in B_2$ where B_1,B_2 are Nb-open sets in X

let $E_1=B_1 \cap M$, $E_2=B_2 \cap M$ are Nb-open subsets in M, and $x \in E_1$, $y \in E_2$, then $E_1 \cap E_2=(B_1 \cap M) \cap (B_2 \cap M)=(B_1 \cap B_2) \cap M=\emptyset \cap M=\emptyset$, hence M is NbT_2-space.

Definition(3.29): *let* $f: X \to Y$ be a function of a topological space (X,τ) *into a topological*

space (Y,τ^*) then f is called an Nb-irresolute function if $f^{-1}(A)$ *is an* $Nb-open$ *set in* X, *for every* $Nb-open$ *set* A *in* Y.

Proposition(3.30): *let* $f: X \to Y$ *be one* $-$ *to* $-$ *one* Nb-irresolute function and Y is NbT_2-space *then X is* $NbT_2-space$.

Proof: *suppose* $f: X \to Y$ *is* $1-1$ *and f is* $Nb-irresolute$ *and Y is* $NbT_2-space$,

let $x_1, x_2 \in X$ *with* $x_1 \neq x_2$ *since f is* $1-1$ *then*

$y_1 = f(x_1) \neq f(x_2) = y_2$ *for some* $y_1, y_2 \in Y$,

since Y is $NbT_2-space$ then there exist disjoint Nb-open sets U and V *such that* $y_1 = f(x_1) \in U$ *and* $y_2 = f(x_2) \in V$ *then* $x_1 = f^{-1}(y_1) \in f^{-1}(U), x_2 = f^{-1}(y_2) \in f^{-1}(V)$, *and* since f is Nb-irresolute , $f^{-1}(U)$ *and* $f^{-1}(V)$ *are* $Nb-open$ *sets in X, hence*

$f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\emptyset) = \emptyset$, *then X is* $NbT_2-space$.

"Definition"(3.31):[3] A space $\mathcal{X}$ is said to be b-regular space if for each $x \in \mathcal{X}$ *and A closed subset of* $\mathcal{X}$ *such that* $x \notin A$ then there exist disjoint b-open sets U and V such that $x \in U$ *and* $A \subseteq V$.

Definition(3.32):[6] A space X is said to be N-regular space iff for each $p \in X$ and C closed subset in X such that $p \notin C$, there exist disjoint N-open sets U,V in X such that $p \in U$and $C \subseteq V$.

By the same context, we can define the definition of Nb$Regular-space$

Definition(3.33): *A space X is said to be* $Nb-regular$ *space if for each x in X and*

A closed set such that $x \notin A$ there exist disjoint Nb-open sets U,V such that $x \in U, A \subseteq V$.

Remark(3.34): There is no such thing as an Nb-regular space in general, and there is no such thing as a regular Nb-space.

Example(3.35): X=$\{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$, $1 \in X$ *and* $\{3\}$ *is* $Nb-closed$ *in X* ,$1 \notin \{3\}$

but {1}*and* {3}are Nb-open sets which contain it selfs, since X is not regular , $\{3\} \subseteq_{closed} X$ *and* $1 \notin \{3\}$but there no exist two disjoint open sets Contain 1 and {3}.

Definition(3.36):A function $f: X \to Y$ *is said to be* $Nb-closed$, *if* f(A)is an Nb-closed set

in Y for every closed subset A of X.

Theorem(3.37):*let X and Y be homeomorphism topology, if X is regular space then*

Y is $Nb-$ *regular space*.

Proof:*let X and Y be homeomorphic* topological space and let X be regular space , to prove Y is Nb-regular space, let y∈Y and A closed set in Y such that y∉A , since f is onto

then there exists x∈X such that f(x)=y, since f is continuous function and $f^{-1}(A)$ closed in Y, and $x \notin f^{-1}(A)$*and X is regular space then there exist open sets U and V* , $U \cap V = \emptyset$

such that $x \in U, f^{-1}(A) \subseteq V$, *then* $f(U)$ *and* $f(V)$ *are open sets in Y* ," *hence* $y = f(x) \in f(U), A = f(f^{-1}(A)) \subseteq f(V)$*but every open is*$Nb-$ *open therefore Y is* Nb-regular space.

Proposition(3.38): A topological space X is Nb-regular space iff for every x∈X and each

open U in X such that x∈U there exists *an* $Nb-$ *open set L such that* $x \in L \subseteq \bar{L}^{Nb} \subseteq U$.

Proof:*let X be* $Nb-$ *regular space and* $x \in X, U$ *be open set in X such that*

$x \in U$ *then* U^c *is closed set in X and* $x \notin U^c$*thus there exist disjoint* $Nb-$ *open sets*

L, V *hence* $x \in L, V^c \subseteq U$*therefore* $x \in L \subseteq \bar{L}^{Nb} \subseteq V^c \subseteq U$,

conversely; let $x \in X$ *and M be a closed set in X such that* $x \notin M$ *then* M^c *is an open*

set in X and $x \in M^c (L \cap V = \emptyset \to L \subseteq V^c \to \overline{L}^{Nb} \subseteq \overline{V^c}^{Nb} = V^c (V^c$ *is* $Nb-closed))$ thus there exists an Nb-open set L such that $x \in L \subseteq \bar{L}^{Nb} \subseteq M^c$ *hence*$x \in L, M \subseteq (\bar{L}^{Nb})^c$ *but L and* $(\bar{L}^{Nb})^c$ *are disjoint* $Nb-$ *open sets therefore X is* $Nb-$ *regular*.

Definition(3.39):[3]A topological space X is called b-normal space, if for every disjoint closed set c1,c2 there exist disjoint b-open sets V1, V2 such that $c1 \subseteq V1, c2 \subseteq V2$.

Definition(3.40):[6]A space $\mathcal{X}$ is said to be N-normal space if and only if for every disjoint

closed sets C_1, C_2 there exist disjoint N-open sets V_1, V_2 such that $C_1 \subseteq V_1$

and $C_2 \subseteq V_2$".

By the same context, we can define the definition of Nb$Normal-space$

Definition(3.41):"A space $\mathcal{X}$ is said to be Nb-normal space if for every disjoint closed sets c_1,c_2 there exist disjoint Nb-open sets V_1, V_2 such that $c_1 \subseteq V_1$, $c_2 \subseteq V_2$.

Remark(3.42): Nb-normal spaces can be found in every common space. However, this is not always the case.

Example(3.43): *let* $X = \{1,2,3\}$*and* $\tau = \{\emptyset, X, \{2,3\}, \{1,2\}, \{2\}\}$is Nb-normal but not normal.

"Proposition"(3.44): A topological space X is Nb-normal space, iff for every closed set D$\subseteq X$ *and each open set* U *in* X, such that D$\subseteq U$ there exists an Nb-open set V such that $D \subseteq V \subseteq \overline{V}^{Nb} \subseteq U$.

Proof: Let X be Nb-normal space and let D be closed set and U open set in X such that D$\subseteq U$ then D and U^c are disjoint closed sets in X since X is Nb-normal space thus, there exist disjoint Nb-open sets V, L hence D$\subseteq V, U^c \subseteq L$ *therefore* $D \subseteq V \subseteq \overline{V}^{Nb} \subseteq \overline{L^C}^{Nb} = L^C \subseteq U$, Conversely":

Let D_1, D_2 be disjoint closed sets in X, then D_2^c *is open set in* X *and* $D_1 \subseteq D_2^c$ there exists an Nb-open set V such that $D_1 \subseteq V \subseteq \overline{V}^{Nb} \subseteq D_2^c$ *hence*

$D_1 \subseteq V, D_2 \subseteq \left(\overline{V}^{Nb}\right)^c$ *and* $V, \left(\overline{V}^{Nb}\right)^c$ *are disjoint* $Nb-$*open sets therefore* X

Is Nb-normal space.

Definition(3.45): A space X is said to be Nb-compact if for every Nb-open cover has a finite subcover. So every Nb-compact space is compact although the converse may not always be accurate in practice.

Example(3.46): The indiscrete space is compact space but not Nb-compact, since if C={{x}:x$\in R$} is Nb-open cover to R which has no finite subcover where {x} is Nb-open set.

Proposition(3.47): Every Nb-compact subset of T_2-space is closed.

Proof: Let A be Nb-compact subset of a T_2-space X. To prove that A is closed that is; X-A is open, let p$\in X-A$, *so* for every point q in A is distinct with p. But X is T_2-space, then there exist two disjoint open sets M(p) and N(q) containing p and q respectively. The collection {N(q): q$\in A$} is an open cover to A, so the cover is Nb-open to A(every open set is Nb-open) which is Nb-compact, so there exists a finite subcover to A, $A \subset \bigcup_{i=1}^{n} N(q_i) = N$ *and suppose* $M = \bigcap_{i=1}^{n} M(p_i)$, but the finite intersection of open sets is open. So M is open set containing p and contained in X-A. therefore it is open so A is closed.

Proposition(3.48): Every Nb-closed subset of Nb-compact space is Nb-compact.

Proof: Suppose A be an Nb-closed subset of Nb-compact space X and let $c = \{U_\propto : \propto \in \Lambda\}$

be an Nb-open cover to A that is ; A$\subset \cup \{U_\propto : \propto \in \Lambda\}$ *but* $X - A$ *is* $Nb-open$. So

X$\subset \bigcup_{\propto \in \Lambda} U_\propto \cup (X - A)$ but X is Nb-compact, which lead us to X$\subset (\bigcup_{i=1}^{n} U_{\propto i}) \cup (X - A)$

which means that A$\subset \bigcup_{i=1}^{n} U_{\propto i}$. *then we have* A *is* $Nb-compact$.

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