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Some Results in Fuzzy 2- Metric Space

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ABSTRACT

The notion of fuzzy 2-metric space and its essential features are initially examined in this study. Following that, the ideas of compact fuzzy2- metric space and fuzzy totally bounded space are discussed. Furthermore, the essential theorems relating to these concepts are established. In particular, we prove that a fuzzy 2- metric space $(\mathbb{W}, \mathfrak{J}, ⋇)$ which is compact must be complete.

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1. Introduction and Preliminaries

The fuzzy set notion was created in 1965. Since then, researchers have applied this key set to a variety of fields. Fuzzy sets have also become a recognized study topic in both pure and practical mathematics and statistics, demonstrating how this theory is very adaptable and fruitful in a wide range of applications, see[5,7,11,18,30]. In functional analysis, the main tools are metrics, norms, and inner product structures. As a result, fuzzy metrics, fuzzy norms, and fuzzy inner products are essential in the development of fuzzy functional analysis. Numerous authors have proposed various papers for fuzzy normed spaces, like [9,13,17,19,21,22,24]. The notion of a fuzzy metric space, which can be thought of as a generalization of the statistical metric space, was initially described by Kramosil

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and Michálek [16] in 1975. From various perspectives, many authors have introduced and debated various concepts of fuzzy metric space. (see[6,8,14]). On the other hand, a large number of research works on fuzzy metric spaces have been published, such as [3,4, 10,12, ,20, 23, ,25,28,29]. In 2002 [27]Sharma proposed the notion of fuzzy 2-metric space and proved a common fixed point theory in this space. Recently Kocinac et al.[15] introduced and investigated various selective covering properties in fuzzy 2-metric spaces.

The major goal of this work is to continue looking at some of the properties of Sharma's fuzzy2- metric space definition. we will define the compact 2-fuzzy metric space and fuzzy totally bounded space, as well as examine some essential characteristics associated with these notions.

Gahler established the concept of 2-metric space in 1960 [1, 2].

Assume that $\mathbb{W}$ be a nonempty set. If a function $\mathfrak{D}: \mathbb{W} \times \mathbb{W} \times \mathbb{W} \to R$ fulfill the following rules, it is termed a 2-metric function on $\mathbb{W}$:

($\mathfrak{D}$1)For all $x, y \in \mathbb{W}$ there exists $z \in \mathbb{W}$ such that $\mathfrak{D}(x, y, z) \neq 0$.

($\mathfrak{D}$2)$\mathfrak{D}(x, y, z) = 0$ if at least two of the three points are the same.

($\mathfrak{D}$3)$\mathfrak{D}(x, y, z) = \mathfrak{D}(x, z, y) = \mathfrak{D}(y, x, z) = \mathfrak{D}(y, z, x) = \mathfrak{D}(z, x, y) = \mathfrak{D}(z, y, x)$, for all $x, y, z \in \mathbb{W}$

($\mathfrak{D}$4)$\mathfrak{D}(x, y, z) \leq \mathfrak{D}(w, z, y) + \mathfrak{D}(x, w, z) + \mathfrak{D}(x, y, w)$, for all $x, y, z, w \in \mathbb{W}$.

The pair $(\mathbb{W}, \mathfrak{D})$ is then referred to as a 2-metric space.

To define fuzzy 2-metric spaces, we need to use the well-known concept of triangular or t-norm.

Definition 1.1[26]: Assume that $\circledast: [0,1] \times [0,1] \to [0,1]$ is a binary operation. If $\circledast$ achieve the following:

1- $\mathcal{I} \circledast \eta = \eta \circledast \mathcal{I}$, for each $\mathcal{I}, \eta \in [0,1]$.

2- $(\mathcal{I} \circledast \eta) \circledast \varsigma = \mathcal{I} \circledast (\eta \circledast \varsigma), \varsigma \in [0,1]$.

3- $\mathcal{I} \circledast 1 = \mathcal{I}$.

4- $\mathcal{I} \circledast \eta \leq \varsigma \circledast \tau$ whenever $\mathcal{I} \leq \varsigma$ and $\eta \leq \tau$, for each $\mathcal{I}, \eta, \varsigma, \tau \in [0,1]$.

Then $\circledast$ is termed continuous t-norm.

Definition 1.2[27]: A binary operation $\circledast: [0,1] \times [0,1] \times [0,1] \to [0,1]$ is termed a continuous t-norm if $([o,1], \circledast)$ is an abelian topological monoid with unit 1 such that $\mathcal{I}_1 \circledast \eta_1 \circledast \varsigma_1 \leq \mathcal{I}_2 \circledast \eta_2 \circledast \varsigma_2$ whenever $\mathcal{I}_1 \leq \mathcal{I}_2$, $\eta_1 \leq \eta_2$, $\varsigma_1 \leq \varsigma_2$ for all $\mathcal{I}_1, \mathcal{I}_2, \eta_1, \eta_2$ and ς_1, ς_2 are in [0,1].

Definition 1.3[27]: If $\mathbb{W}$ is a nonempty set, $\circledast$ is a continuous t-norm, and $\mathfrak{J}$ is a fuzzy set in $\mathbb{W} \times \mathbb{W} \times \mathbb{W} \times (0, \infty)$ meeting the following criteria for all $x, y, z \in \mathbb{W}$ and $\vartheta, \vartheta_1, \vartheta_2, \vartheta_3 > 0$, then the triplet $(\mathbb{W}, \mathfrak{J}, \circledast)$ is known as fuzzy 2-metric space briefly $(\hat{\mathfrak{F}}(\mathbb{W})2 - \text{space})$

$(\mathfrak{J}_1)$ $\mathfrak{J}(x, y, z, 0) = 0$.

$(\mathfrak{J}_2)$ $\mathfrak{J}(x, y, z, \vartheta) = 1$, if at least two of x, y, z are equal;

$(\mathfrak{J}_3)$ $\mathfrak{J}(x, y, z, \vartheta) = \mathfrak{J}(x, z, y, \vartheta) = \mathfrak{J}(y, z, x, \vartheta)$

$(\mathfrak{J}_4)$ $\mathfrak{J}(x, y, z, \vartheta_1 + \vartheta_2 + \vartheta_3) \geq \mathfrak{J}(x, y, u, \vartheta_1) \circledast \mathfrak{J}(x, u, z, \vartheta_2) \circledast \mathfrak{J}(u, y, z, \vartheta_3)$

$(\mathfrak{I}_5)$ $\mathfrak{I}(x, y, z, .) : [0,1) \to [0,1]$is continuous.

A sample example of a fuzzy 2-metric is shown below.

Example 1.4[15]: Let $(\mathbb{W}, \mathfrak{D})$ be a 2-metric space. Then the mapping $\mathfrak{I}: \mathbb{W} \times \mathbb{W} \times \mathbb{W} \times (0, \infty) \to [0,1]$

defined by

$$\mathfrak{I}(x, y, z, \vartheta) = \frac{\frac{\mathfrak{I}(x,z,y,\vartheta)}{\vartheta}}{\vartheta + \mathfrak{D}(x, y, z)}$$

is a fuzzy 2-metric on $\mathbb{W}$ induced by the 2-metric $\mathfrak{D}$.

Definition1.5[15]: Let $(\mathbb{W}, \mathfrak{I}, ⊛)$ be an $\hat{\mathfrak{F}}(\mathbb{W})$2-space. For $x \in \mathbb{W}, e \in (0,1), \vartheta > 0$, the open ball is given as follow:

$B(x, e, \vartheta) = \{y \in \mathbb{W} : \mathfrak{I}(x, y, z, \vartheta) > 1 - e$ for each $z \in \mathbb{W}\}$.

Definition 1.6[27]: Suppose that $(\mathbb{W}, \mathfrak{I}, ⊛)$ be an $\hat{\mathfrak{F}}(\mathbb{W})$2-space and $\{x_n\}$ be a sequence in $\mathbb{W}$. If there exists $x \in \mathbb{W}$ such that $\lim_{n\to\infty} \mathfrak{I}(x_n, x, u, \vartheta) = 1$, for all $u \in \mathbb{W}$ and $\vartheta > 0$, then $\{x_n\}$ is called a convergent sequence.

Definition 1.7[27]: Suppose that $(\mathbb{W}, \mathfrak{I}, ⊛)$ be an $\hat{\mathfrak{F}}(\mathbb{W})$2-space and $\{x_n\}$ be a sequence in $\mathbb{W}$. Then $\{x_n\}$ is called Cauchy sequence if $\lim_{n\to\infty} \mathfrak{I}(x_{n+p}, x_n, u, \vartheta) = 1$, for all $u \in \mathbb{W}$ and $\vartheta > 0, p > 0$.

This definition of a Cauchy sequence is equivalent to $\lim_{n,m\to\infty} \mathfrak{I}(x_m, x_n, u, \vartheta) = 1$, for all $\vartheta > 0$.

We use this as the definition of the Cauchy sequence throughout the paper.

Definition 1.8[27]: A complete fuzzy 2-metric space$(\mathbb{W}, \mathfrak{I}, ⊛)$ is defined as an $\hat{\mathfrak{F}}(\mathbb{W})$2-space in which every Cauchy sequence is a convergent sequence.

2. Compact Fuzzy2- metric space

This section introduces the concept of compact fuzzy 2- metric space. The fundamental properties will then be investigated. We'll start by explaining the concept of an open and closed set.

Definition2.1: Let $(\mathbb{W}, \mathfrak{I}, ⊛)$ be an $\hat{\mathfrak{F}}(\mathbb{W})$2-space and $\mathcal{H}$ indicate a subset in $\mathbb{W}$. If for any $x \in \mathcal{H}$ there exists $0 < e < 1$, and $\vartheta > 0$ such that $B(x, e, \vartheta) \subseteq \mathcal{H}$, then $\mathcal{H}$ is called an open set in $(\mathbb{W}, \mathfrak{I}, ⊛)$.

Definition2.2: Let $(\mathbb{W}, \mathfrak{I}, ⊛)$ be an $\hat{\mathfrak{F}}(\mathbb{W})$2-space and $\mathcal{H}$ be a subset of $\mathbb{W}$. If any sequence $\{x_n\}$ in $\mathcal{H}$ is convergence to $x \in \mathcal{H}$, then the subset $\mathcal{H}$ is called closed.

Now we introduced the notion of an open cover.

Definition 2.3: Let$(\mathbb{W},\mathfrak{I},\circledast)$ be an $\widehat{\mathfrak{F}}(\mathbb{W})2$-space and $\mathcal{H}$ indicate a subset in$\mathbb{W}$. Let $\widehat{\mathcal{L}}$ be a collection of open sets in $\mathbb{W}$ that have the characteristic that $\mathcal{H} \subseteq \bigcup_{L\in\mathcal{L}} L$. Then a family $\mathcal{L}$ is termed an open cover of $\mathcal{H}$.

Definition 2.4: Let $(\mathbb{W},\mathfrak{I},\circledast)$ be an $\widehat{\mathfrak{F}}(\mathbb{W})2$-space. If each open cover has a finite sub-collection $\{L_1, L_2, \dots, L_k\} \subseteq \widehat{\mathcal{L}}$ with $\mathbb{W} = \bigcup_{i=1}^{k} L_i$, then $(\mathbb{W},\mathfrak{I},\circledast)$ is called compact $\widehat{\mathfrak{F}}(\mathbb{W})2$-space.

Definition 2.5: Assume that $(\mathbb{W},\mathfrak{I},\circledast)$ be an $\widehat{\mathfrak{F}}(\mathbb{W})2$ -space and $\mathcal{H} \subseteq \mathbb{W}$. If each sequence in $\mathbb{W}$ has a convergent subsequence whose limit is a point of $\mathbb{W}$, then $\mathbb{W}$ is compact.

Proposition 2.6: Assume that $(\mathbb{W},\mathfrak{I},\circledast)$ be a compact $\widehat{\mathfrak{F}}(\mathbb{W})2$-space and $\mathcal{H} \subseteq \mathbb{W}$. If $\mathcal{H}$ is closed set in $\mathbb{W}$ then $\mathcal{H}$ is compact.

Proof:

Consider $(\mathbb{W},\mathfrak{I},\circledast)$ be a compact $\widehat{\mathfrak{F}}(\mathbb{W})2$-space. Let $\mathcal{H}$ be a closed subset of $\mathbb{W}$. Suppose that $\{x_n\} \in \mathcal{H}$ then $\{x_n\} \in \mathbb{W}$. By Definition (2.5), Since $\mathbb{W}$ is compact then $\{x_n\}$ has a subsequence $\{x_{nk}\}$ converges to x_{n_i}. Then $x_{n_i} \in \mathcal{H}$ since $\mathcal{H}$ is a closed subset, hence $\mathcal{H}$ is compact.

Definition 2.7: Let $(\mathbb{W},\mathfrak{I},\circledast)$ be an $\widehat{\mathfrak{F}}(\mathbb{W})2$-space and $\mathcal{H}$ is a subset of $\mathbb{W}$ then $\mathcal{H}$ is said to be fuzzy totally bounded(briefly ($\widehat{\mathfrak{F}}T -$ bounded))if for any $m \in (0,1)$ and $\vartheta > 0$ there exists a finite set of points $\{\ell_1, \ell_2, \dots, \ell_n\} \subset \mathcal{H}$ that is whenever x in $\mathbb{W}$, $\mathfrak{I}(x, \ell_i, u, \vartheta) > 1 - m$ for some $\ell_i \in \{\ell_1, \ell_2, \dots, \ell_n\}$ where $u \in \mathbb{W}$. This set of points $\{\ell_1, \ell_2, \dots, \ell_n\}$ is called m- fuzzy net of $(\mathbb{W},\mathfrak{I},\circledast)$.

Proposition 2.8: Let $(\mathbb{W},\mathfrak{I},\circledast)$ be $\widehat{\mathfrak{F}}(\mathbb{W})2$-space. Then $\mathbb{W}$ is $\widehat{\mathfrak{F}}T -$ bounded if it is compact.

Proof:

Suppose that $\mathbb{W}$ be compact. For any $x \in \mathbb{W}$ and radius $e \in (0,1)$, the collection of all balls $\{B(x, e, \vartheta)\}$ is an open cover of $\mathbb{W}$ where $\vartheta > 0$.

Now since $\mathbb{W}$ is compact then by Definition(2.4) it follows that this cover includes finite subcover say $\{B(x_1, e, \vartheta), B(x_2, e, \vartheta), \dots, B(x_k, e, \vartheta)\}$ so $\{x_1, x_2, \dots, x_k\}$ represent an m- fuzzy net of $\mathbb{W}$ and by Definition(2.7) it follows that $\mathbb{W}$ is $\widehat{\mathfrak{F}}T -$ bounded.

Proposition 2.9: Let $(\mathbb{W},\mathfrak{I},\circledast)$ be compact $\widehat{\mathfrak{F}}(\mathbb{W})2$-space. Then $(\mathbb{W},\mathfrak{I},\circledast)$ is complete.

Proof:

Assume that $(\mathbb{W},\mathfrak{I},\circledast)$is a compact $\widehat{\mathfrak{F}}(\mathbb{W})2$-space that is not complete. Then there's a Cauchy sequence $\{x_n\}$ in $(\mathbb{W},\mathfrak{I},\circledast)$ that doesn't have a limit in $\mathbb{W}$. Let $y \in \mathbb{W}$ and $\vartheta > 0$ because $\{x_n\}$ does not converge to y there is $0 < \delta < 1$ such that $\mathfrak{I}(x_n, y, u, \vartheta) > 1 - \delta$ for infinitely many values of $\mathfrak{n}$ where $u \in \mathbb{W}$. Since $\{x_n\}$ is Cauchy, there exists an integer N such that $\mathfrak{n}, \mathfrak{m} \geq N$. As a result of this $\mathfrak{I}(x_m, x_n, u, \vartheta) > 1 - \delta$. Choose $\mathfrak{m} \geq N$ such that $\mathfrak{I}(x_m, y, u, \vartheta) > 1 - \delta$. So, the open ball $B(y, \delta, \vartheta)$ includes x_n for only finitely many values of $\mathfrak{n}$.

In this way, we may correlate a ball $B(y, \delta(y), \vartheta)$ with each $y \in \mathbb{W}$ where $0 < \delta(y) < 1$ relies on y, and the ball $B(y, \delta(y), \vartheta)$ has x_n for only a finite number of $\mathfrak{n}$ values. Notice that $\mathbb{W} = \bigcup B(y, \delta(y), \vartheta)$ implying that

$\{B(\mathcal{y}, \delta(\mathcal{y}), \vartheta): \mathcal{y} \in \mathbb{W}\}$ is a covering for $\mathbb{W}$. Now, because $\mathbb{W}$ is compact, a finite sub covering $B(\mathcal{y}_i, \delta_i(\mathcal{y}), \vartheta)$, $i = 1, 2, \ldots, n$ exists. Since every ball possesses x_n for only a finite number of values of n, the balls are in the finite sub covering, consequently $\mathbb{W}$ must also have x_n for only a finite number of values of n. This, on the other hand, is not possible. As a result, $(\mathbb{W}, \mathfrak{J}, \circledast)$ must be complete.

Proposition 2.10: Let $(\mathbb{W}, \mathfrak{J}, \circledast)$ be an $\hat{\mathfrak{F}}(\mathbb{W})2$-space and $\mathcal{H} \subseteq \mathbb{W}$. If $\mathbb{W}$ is $\hat{\mathfrak{F}}T -$ bounded then $\mathcal{H}$ is $\hat{\mathfrak{F}}T -$ bounded.

Proof:

Suppose that $(\mathbb{W}, \mathfrak{J}, \circledast)$ be an $\hat{\mathfrak{F}}(\mathbb{W})2 -$ space and $\mathbb{W}$ is $\hat{\mathfrak{F}}T -$ bounded. Consider $\{x_1, x_2, \ldots, x_k\}$ represent an m-fuzzy net of $\mathbb{W}$ where $m \in (0,1)$. Then for any $x_1 \in \mathbb{W}$ and some $\mathcal{y}_i \in \mathcal{H}$ we have:

$$\mathfrak{J}(x_1, \mathcal{y}_i, u, \vartheta/3) > 1 - m \quad \text{where } u \in \mathbb{W}.$$

Putting $\mathcal{D} = \{z_1, z_2, \ldots, z_m\}$ which is a finite subset of $\mathcal{H}$.

Then for every $1 < i < m$, we have $\mathfrak{J}(x_1, z_i, u, \vartheta/3) > 1 - m$ for some $z_i \in \mathcal{H}$.

Now ,

$$\begin{aligned}\mathfrak{J}(x_1, \mathcal{y}_i, u, \vartheta) &\geq \mathfrak{J}(x_1, \mathcal{y}_i, z, \vartheta/3) \circledast \mathfrak{J}(x_1, z_i, u, \vartheta/3) \circledast \mathfrak{J}(z_i, \mathcal{y}_i, u, \vartheta/3) \\ &\geq (1 - m) \circledast (1 - m) \circledast (1 - m) \\ &> 1 - \delta\end{aligned}$$

for some $\delta \in (0,1)$. This indicates that $\mathcal{H}$ is $\hat{\mathfrak{F}}T -$ bounded.

Proposition 2.11: Let $(\mathbb{W}, \mathfrak{J}, \circledast)$ be an $\hat{\mathfrak{F}}(\mathbb{W})2$-space and $\mathcal{H} \subseteq \mathbb{W}$. If $\mathcal{H}$ is compact then every infinite subset $\mathcal{D} \subseteq \mathcal{H}$ has a limit point.

Proof:

Consider $(\mathbb{W}, \mathfrak{J}, \circledast)$ be an $\hat{\mathfrak{F}}(\mathbb{W})2$-space and $\mathcal{H}$ be a compact subset in $(\mathbb{W}, \mathfrak{J}, \circledast)$ and let $\mathcal{D} \subseteq \mathcal{H}$ be an infinite set. We state that $\mathcal{D}$ does not include any limit point, and then we demonstrate that this is false. Because $\mathcal{D}$ does not have a limit point, then for all $x \in \mathcal{D}$ there exists $e \in (0,1)$ such that the open ball $B(x, e, \vartheta)$ does not include any points of $\mathcal{H}$ different from x , that is for all $x \in \mathcal{D}$ we have $B(x, e, \vartheta) \cap \mathcal{D} \setminus \{x\} = \emptyset$.

Now for each $x \in \mathcal{D}$, let $B(x, e, \vartheta): e \in (0,1), \vartheta > 0\}$ represents the family of the open ball. Then we have $\mathcal{D} \subseteq \bigcup_{x \in \mathcal{D}} B(x, e, \vartheta)$ which is an infinite open covering of $\mathcal{D}$ and each element in this covering of $\mathcal{D}$ includes only one point x , consequently, there is no finite open sub covering implying that $\mathcal{D}$ is not compact and this contradiction, hence statement that the infinite subset $\mathcal{D}$ of $\mathcal{H}$ dose does not include any limit point is wrong. Hence $\mathcal{D}$ has a limit point.

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