Journal of Al-Qadisiyah for Computer Science and Mathematics Vol. 14(2) 2022 , pp Math. 53–70

JQCM
University Of AL-Qadisiyah

Available online at www.qu.edu.iq/journalcm

JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS

ISSN:2521-3504(online) ISSN:2074-0204(print)

Some Properties Of Soft Modular Space

Rand Raheem huniwi[1] and Noori F. Al-Mayahi[2] *

Department of Mathematics,""College of Science" "University of Al-Qadisiyah, Diwaniyah-Iraq .Email: ma20.post6@qu.edu.iq[1]

Department of Mathematics,""College of Science""University of Al-Qadisiyah, Diwaniyah-Iraq.Email: nafm60@yahoo.comi[2]

ARTICLE INFO

Article history:
Received: 13 /05/2022
Rrevised form: 10/06/2022
Accepted : 21/06/2022
Available online: 06/07/2022

Keywords: soft set, modular space, soft metric space, soft modular space.

ABSTRACT

In this paper, we will present a definition of soft modular space and some properties of soft modular (convergent, continuous, bounded) are explained instead of the prevailing definition.

MSC..

https://doi.org/10.29304/jqcm.2022.14.2.970

1.Introduction:

Russian mathematician Molodtsov [1] came up with the idea of soft set theory in 1999, which is a Mathematical tool for coping with uncertainty and making decisions. For example, the subjects of social sciences, physics, engineering, economics and computers all make use of soft set theory in some way. Many operations on soft sets were defended by Maji at al [8] when applying soft set theory to decision problems. For a soft ideal topological space, Yildirim et al. [13] introduced the concept of a soft ideal and defined soft I-Baire spaces. Many researchers have focused their attention on soft topology and soft metric spaces in the previous decade. [10], [7],[6],[4].

2.1 Basic concepts about soft sets

Definition (2.1) [1]:

A pair (F, E) is said to be a soft set over X, where F is a function given by : $E \to P(X)$.

Example (2.2) [8]: Suppose the following

X is The set of houses

E is the set of parameters. Each parameter is a word or a sentence.

*Corresponding author

Email addresses:

Communicated by 'sub etitor'

E = {expensive, beautiful, wooden, cheap, in the green surroundings}.

Then X = { h_1 , h_2, h_3, h_4, h_5, h_6} and E = { e_1, e_2, e_3, e_4, e_5}.

Where

e_1 stands for the parameter ' expensive '

e_2 stands for the parameter ' beautiful '

e_3 stands for the parameter ' wooden '

e_4 stands for the parameter ' cheap '

e_5 stands for the parameter ' in the green surroundings '

Suppose that F(e_1) ={h_2, h_4} , F(e_2) ={h_1, h_3}, F(e_3) ={h_3, h_4 , h_5}, F(e_4) ={h_1, h_3 , h_5}

F(e_5) ={h_1}.

Then the soft set (F, E) is a parametrized family of X and gives us a collection of approximate of descriptions an object.

Thus , we can view the soft set as (F, E)={ {h_2, h_4},{h_1, h_3},{h_3, h_4 , h_5},{h_1, h_3 , h_5},{h_1}}

Definition (2.3) [5]:

A soft set (F, E) over X is said to be a null soft set denoted by $\breve{\emptyset}$, if for all $e \tilde{\in} E$, F(e)= $\breve{\emptyset}$.

Definition (2.4) [5]:

A soft set (F, E)over X is said to be an absolute soft set denoted by $\breve{X}$, if for all $e \tilde{\in} E$, F(e)= X.

Definition (2.5) [5]:

Let (F,A) and (G,B) be two soft sets over $universe\ X$, we say (F,A) is a soft subset of (G,B) and denoted by $(\mathrm{F}, \mathrm{A}) \tilde{\subseteq} (\mathrm{G}, \mathrm{B})$ if:

(i) $A \tilde{\subseteq} B$.

(ii) $F(e) \tilde{\subseteq} G(e)$, $\forall e \tilde{\in} A$.

also, one says that (F, A) and (G, B) are soft equal denoted by $(\mathrm{F}, \mathrm{A}) = (\mathrm{G}, \mathrm{B})$, if $(\mathrm{F}, \mathrm{A}) \tilde{\subseteq} (\mathrm{G}, \mathrm{B})$ and $(\mathrm{G}, \mathrm{B}) \tilde{\subseteq} (\mathrm{F}, \mathrm{A})$.

Example (2.6) [8]: Suppose the following

Let A= { e_1, e_3, e_5}$\subset E$ and B = { e_1, e_2, e_3, e_5}$\subset E$ .Clearly A$\subset B$.

Let (F,A) and (G,B) be two soft sets over the same $universe$ X $= \{ h_1 , h_2, h_3, h_4, h_5, h_6\}$

and E = { e_1, e_2, e_3, e_4, e_5} such that .

G(e_1) ={h_2, h_4} , G(e_2) ={h_1, h_3}, G(e_3) ={h_3, h_4 , h_5}, G(e_5) ={h_1}, and

F(e_1) ={h_2, h_4} , F(e_3) ={h_3, h_4 , h_5}, F(e_5) ={h_1}.

Therefore, $(\mathrm{F}, \mathrm{A}) \tilde{\subseteq} (\mathrm{G}, \mathrm{B})$.

Definition (2.7) [9,2]:

The soft complement of a soft set F_E over a universe X is denoted by $(F_E)^c$ and it is defined by $(F_E)^c = (F^c, E)$, where F^c is function given by $F^c: E \to P(X)$, $F^c(e) = X \backslash F(e)$, for all $e \tilde{\in} E$.

i.e. $(F_E)^c = \{(e, X \backslash F(e)) : \forall\ e \in E\}$. It is clear that:

(i) $\breve{\emptyset}.^{c} = \tilde{X}_E$; $\tilde{X}_E{}^{c} = \breve{\emptyset}$.

(ii) If $\breve{x}_{e1} \tilde{\in} F_E$, then $\breve{x}_{e1} \notin (F_E)^c$.

Remark (2.8) [5]:

1. $\breve{\emptyset}$ is a soft subset of any soft set F_E .
2. any soft set F_E is a soft subset of $\tilde{X}_E$.

Definition (2.9) [9]:

1. The intersection of the two soft sets (F,A) and (G,B) over the common universe X is the soft s et (H,C) , where $C = A \cap B$ and for all $e \tilde{\in} C$, H(e) =F$(e) \cap G(e)$. We write

(F,A) $\tilde{\cap}$ (G, B) = (H, C).

2. The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set

(H,C), where $C = A \cup B$ and for all $e \tilde{\in} C$,

$C = A \cup B$ and one writes $H_C = F_A \tilde{\cup} G_B$ such that

$$H(e) = \begin{cases} F(e) & , if\ e \tilde{\in} A \backslash B \\ G(e) & , if\ e \tilde{\in} B \backslash A \\ F(e) \cup G(e) & , if\ e \tilde{\in} A \cap B \end{cases} .$$

We express it as (F,A) $\tilde{\cup}$ (G, B) = (H, C).

3. The difference (H,E) of two soft sets (F,E) and (G,B) over X , denoted by (F,A) \(G,B),

Is defined by $H(e) = F(e) \backslash G(e)$, for all $e \tilde{\in} E$.

Theorem (2.10) [9]:

Let (F, E) be soft set then the following hold:

1. $(F, E)^c \tilde{\cup} (F, E) = (\breve{X}, E)$.
2. $(F, E) \tilde{\cap} (F, E)^c = \tilde{\emptyset}_E$.
3. $(F, E) \tilde{\cap} (\breve{X}, E) = (F, E)$.
4. $(F, E) \tilde{\cap} \tilde{\emptyset}_E = \tilde{\emptyset}_E$.
5. $(F, E) \tilde{\cup} \tilde{\emptyset}_E = (F, E)$.

Remark (2.11)[4]:

1.A soft set F_E for which $F(e)$ is a singleton set, for all $\boldsymbol{e} \tilde{\in} \boldsymbol{E}$, is called singleton soft set.

2. In sense, when $|E| = 1$ a soft set F_E behaves similarly to a set. In this case the soft set is the same as the set $F(e)$, where $E = \{e\}$.

Example (2.12)[7]:

Let$X = \{1,2,3\}, E = \{e_1, e_2\}$then $F_E = \{(e_1, \{1,2\}), (e_2, \{2,3\})\}$ is not a singleton soft set .But $G_E = \{(e_1, \{1\}), (e_2, \{2\})\}$ is a singleton soft set.

Definition (2.13) [9]:

Let $\mathbb{R}$ be the set of real numbers, $B(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and E taken as a set of parameters. Then a function $F: E \to B(\mathbb{R})$ is called a soft real set .If a soft real set is a singleton soft set, it will be said a soft real number and denoted by $\tilde{r}$, $\tilde{s}$, $\tilde{t}$ etc. $\breve{0}$and $\breve{1}$are the soft real numbers where $\breve{0}(e) = 0$, $\breve{1}(e) = 1$ for all $e \tilde{\in} E$.

Remark (2.14)[6]:

The set of all soft real numbers is denoted by $\mathcal{R}(E)$ and the set of all non-negative soft real numbers by $\mathcal{R}^*(E)$. **Definition (2.15) [9]:**

Let $\tilde{r}$, $\tilde{s}$ two soft real numbers, then ,the following statements :

1. $\tilde{r} \tilde{\leq} \tilde{s}$ if $\tilde{r}(e) \leq \tilde{s}(e)$, $\forall\, e \tilde{\in} E$.
2. $\tilde{r} \tilde{\geq} \tilde{s}$ if $\tilde{r}(e) \geq \tilde{s}(e)$, $\forall\, e \tilde{\in} E$.
3. $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(e) < \tilde{s}(e)$, $\forall\, e \tilde{\in} E$.
4. $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(e) > \tilde{s}(e)$, $\forall\, e \tilde{\in} E$.

Definition (2.16)[11]:

The soft set F_E is said to be (finite soft set) if the set of parameters E is finite and $F(e_i)$ is finite set, $\forall i \in \xi$.

Remark (2.17)[12]:

Let $\tilde{r}, \tilde{s}, \tilde{t} \tilde{\in} \mathcal{R}(E)$. Then the soft addition $\tilde{r} \boxplus \tilde{s}$ of $\tilde{r}, \tilde{s}$ and soft scaler multiplication $\tilde{t} \boxdot \tilde{r}$ of $\tilde{t}$ and $\tilde{r}$ are defined by:

1. $(\tilde{r} \boxplus \tilde{s})(e) = \tilde{r}(e) \boxplus \tilde{s}(e)$, for all $e \tilde{\in} E$.
2. $(\tilde{r} \boxminus \tilde{s})(e) = \tilde{r}(e) \boxminus \tilde{s}(e)$, for all $e \tilde{\in} E$.
3. $(\tilde{r} \boxdot \tilde{s})(e) = \tilde{r}(e) \boxdot \tilde{s}(e)$, for all $e \tilde{\in} E$.
4. $(\tilde{r} \tilde{/} \tilde{s})(e) = \tilde{r}(e) / \tilde{s}(e)$, and $\tilde{s}(e) \neq 0$ for all $e \tilde{\in} E$.

Remark (2.18)[6]:

For two soft real numbers $\tilde{r}, \tilde{s}$, we have:

1. If $\tilde{r} \tilde{\leq} \tilde{s}$, then $\tilde{r} \boxplus \tilde{t} \tilde{\leq} \tilde{s} \boxplus \tilde{t}$; for all $\tilde{t} \tilde{\in} \mathcal{R}(A)$.
2. If $\tilde{r} \tilde{\leq} \tilde{s}$, then $\tilde{r} \boxdot \tilde{t} \tilde{\leq} \tilde{s} \boxdot \tilde{t}$; for all $\tilde{t} \tilde{\in} \mathcal{R}(A)^*$.

Theorem (2.19)[9]:

Let F_E, G_E, H_E are soft sets in (F, E), $\breve{x}_e \neq \breve{\emptyset}$. Then the following hold:

(i) $\forall e \in E, (e, \emptyset) \tilde{\in} F_E$.

(ii) $\breve{x}_e \tilde{\in} [F_E \tilde{\cup} Gn_E]$ iff $\breve{x}_e \tilde{\in} F_E \tilde{\vee} \breve{x}_e \tilde{\in} G_E$.

(iii) $\breve{x}_e \tilde{\in} [F_E \tilde{\cap} G_E]$ iff $\breve{x}_e \tilde{\in} F_E \tilde{\wedge} \breve{x}_e \tilde{\in} G_E$.

(iv) $\breve{x}_e \tilde{\in} [F_E \tilde{\setminus} G_E]$ iff $\breve{x}_e \tilde{\in} F_E \tilde{\wedge} \breve{x}_e \tilde{\notin} G_E$.

Definition (2.20)[10]:

A soft point (F,E) over X is said to be a soft point if there is exactly one $e \tilde{\in} E$, such that

F(e)={x} for some x∈ X and F(e)=∅ , ∀$e \in E$ / $\{e\}$. It will be denoted by $\breve{x}_e$.

Theorem (2.21)[10]:

Every soft set can be expressed as union of all soft points belonging to it . Conversely, any

set of soft points can be considers as a soft set.

Definition (2.22)[12]:

Let (F, E) be a soft set over $\breve{X}$. The set soft (F, E) is said to be a soft vector and denoted by $\breve{x}_e$ if there is exactly one e $\widetilde{\in}$ E. Such that F(e) = $\{x\}$ for some x ∈ X and F(e)=$\breve{\emptyset}$,

∀ $e \,\widetilde{\in}\, E \setminus \{e\}$ set. The set of all soft vector over $\breve{X}$ will be denoted by SV($\breve{X}$) .

Definition (2.23) [12]:

The set SV($\breve{X}$) is called soft vector space.

3.1 Basic properties of Soft Modular

In this section We will define a soft modular space and some properties

Definition (3.1):

Let SV($\breve{X}$) be a soft vector space , A function $\breve{\mathcal{M}}$: SV($\breve{X}$) → $\mathcal{R}$(E)*is said soft modular on SV($\breve{X}$) if satisfies the following condition:

1. $\breve{\mathcal{M}}(\breve{x}_{e1}) = \breve{0} \Leftrightarrow \breve{x}_{e1} = \breve{0}$.
2. $\breve{\mathcal{M}}(\alpha\breve{x}_{e1}) = \breve{\mathcal{M}}(\breve{x}_{e1})$ for $\alpha \in F$ with $|\alpha| = 1$
3. $\breve{\mathcal{M}}(\alpha\breve{x}_{e1} \boxplus \beta\breve{y}_{e2}) \,\widetilde{\leq}\, \breve{\mathcal{M}}(\breve{x}_{e1}) \boxplus \breve{\mathcal{M}}(\breve{y}_{e2})$ iff , $\beta \geq 0$, for all $\breve{x}_{e1}, \breve{y}_{e2} \,\widetilde{\in}\,$ SV($\breve{X}$).

The soft vector space SV($\breve{X}$) with the soft modular $\breve{\mathcal{M}}$ on $\breve{X}$ is said to be a soft modular liner space and denoted by $(\breve{X}, \breve{\mathcal{M}})$.

Definition (3.2)[2]:

A function $\breve{\mathfrak{H}}$: SP($\breve{X}$) × SP($\breve{X}$) → $\mathcal{R}^*$(E) is said to be soft metric on SP($\breve{X}$) if $\breve{\mathfrak{H}}$ satisfies the following conditions

1. $\breve{\mathfrak{H}}(\breve{x}_{e1}, \breve{y}_{e2}) \,\widetilde{\geq}\, \breve{0}$for all $\breve{x}_{e1}, \breve{y}_{e2} \,\widetilde{\in}\,$ SP($\breve{X}$).
2. $\breve{\mathfrak{H}}(\breve{x}_{e1}, \breve{y}_{e2}) = \breve{0}$if and only if $\breve{x}_{e1} = \breve{y}_{e2} \,\widetilde{\in}\,$ SP($\breve{X}$).
3. $\breve{\mathfrak{H}}(\breve{x}_{e1}, \breve{y}_{e2}) = \breve{\mathfrak{H}}(\breve{y}_{e2}, \breve{x}_{e1})$ for all $\breve{x}_{e1}, \breve{y}_{e2} \,\widetilde{\in}\,$ SP($\breve{X}$) .
4. $\breve{\mathfrak{H}}(\breve{x}_{e1}, \breve{z}_{e3}) \,\widetilde{\leq}\, \breve{\mathfrak{H}}(\breve{x}_{e1}, \breve{y}_{e2}) \boxplus \breve{\mathfrak{H}}(\breve{y}_{e2}, \breve{z}_{e3})$ for all $\breve{x}_{e1}, \breve{y}_{e2}, \breve{z}_{e3} \,\widetilde{\in}\,$ SP($\breve{X}$).

The soft set $\breve{X}$ with a soft metric $\breve{\mathfrak{H}}$ on $\breve{X}$is said to be a soft metric space and denoted by $(\breve{X}, \breve{\mathfrak{H}}, E)$ or $(\breve{X}, \breve{\mathfrak{H}})$.

Example (3.3):

Let SV($\breve{X}$) = $\mathbb{R}^2$ with $\breve{\mathcal{M}}(\breve{x}_{e1}, \breve{y}_{e2}) = \breve{x}_{e1} \boxplus \breve{y}_{e2}$, for any pair $(\breve{x}_{e1}, \breve{y}_{e2})$ in SV($\breve{X}$), then $(\breve{X}, \breve{\mathcal{M}})$ is soft modular space .

Solution:

Let $(\breve{x}_{e1}, \breve{y}_{e2})$, ∈ $\mathbb{R}^2$ and $\gamma, \beta, \lambda \in \mathbb{K}$ with $\gamma + \beta = 1$

1. Since $\mathcal{M}(\breve{x}_{e1}, \breve{y}_{e2}) = \breve{0}$if and only if $\breve{x}_{e1} \boxplus \breve{y}_{e2} = \breve{0}$ and since

$\breve{x}_{e1} \boxplus \breve{y}_{e2} = \breve{0}$ if and only if $\breve{x}_{e1} = \breve{y}_{e2} = \breve{0}$, then $\breve{\mathcal{M}}(\breve{x}_{e1}, \breve{y}_{e2}) = \breve{0}$ if and only if the pair $(\breve{x}_{e1}, \breve{y}_{e2})$ the zero in $\mathbb{R}^2$

2. $\breve{\mathcal{M}}(\gamma(\breve{x}_{e1}, \breve{y}_{e2})) = \mathcal{M}((\gamma\breve{x}_{e1}, \gamma\breve{y}_{e2})) = \gamma\breve{x}_{e1} \boxplus \gamma\breve{y}_{e2}$.

Since $|\gamma| = 1$, then $\breve{\mathcal{M}}((\breve{x}_{e1}, \breve{y}_{e2})) = \breve{x}_{e1} \boxplus \breve{y}_{e2} = \breve{\mathcal{M}}((\breve{x}_{e1}, \breve{y}_{e2}))$.

3. $\breve{\mathcal{M}}\left(\alpha(\breve{x}_{e1}, \breve{y}_{e2}) \boxplus \beta(\breve{z}_{e3}, \breve{d}_{e_4})\right) = \alpha(\breve{x}_{e1}, \breve{y}_{e2}) \boxplus \beta(\breve{z}_{e3}, \breve{d}_{e_4})$

$$= (\alpha\breve{x}_{e1} \boxplus \alpha\breve{y}_{e2}) \boxplus (\beta\breve{z}_{e3} \boxplus \beta\breve{d}_{e_4})$$

$$\leq (\breve{x}_{e1} \boxplus \breve{y}_{e2}) \boxplus (\breve{z}_{e3} \boxplus \breve{d}_{e_4})$$

$$\breve{\mathcal{M}}\left(\alpha(\breve{x}_{e1}, \breve{y}_{e2}) \boxplus \beta(\breve{z}_{e3}, \breve{d}_{e_4})\right)$$

$$= \breve{\mathcal{M}}(\alpha(\breve{x}_{e1}, \breve{y}_{e2})) \boxplus \breve{\mathcal{M}}(\beta(\breve{z}_{e3}, \breve{d}_{e_4}))$$

Thus $\mathbb{R}^2$ is soft modular space.

Definition (3.4):

A sequence of soft vectors $\{\breve{x}_{e_n}\}$ in $(\breve{X}, \breve{\mathcal{M}})$ is said to be convergent to $\breve{x}_0$ if $\forall\, \breve{\varepsilon} \,\tilde{>}\, \breve{0}$,

$\exists\, k \in Z$ *such that* $\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \,\tilde{<}\, \breve{\varepsilon}, \forall n \,\tilde{\geq}\, k$ and is denoted by $\breve{x}_{e_n} \to \breve{x}_{e_0}$ *as* $n \to \infty$ or $\lim_{n\to\infty} \breve{x}_{e_n} = \breve{x}_{e_0}$, $\breve{x}_{e_0}$ *is said to be* the limit of sequence $\breve{x}_{e_n}$ as $n \to \infty$.

Definition (3.5):

A sequence $\{\breve{x}_{e_n}\}$ in $(\breve{X}, \breve{\mathcal{M}})$ is said to be a Cauchy sequence if corresponding to every $\breve{\varepsilon} \,\tilde{>}\, \breve{0}, \exists\, m \in N$ *such that* $\breve{\mathcal{M}}\left(\breve{x}_{e_n} \boxminus \breve{x}_{e_j}\right) \,\tilde{<}\, \breve{\varepsilon}, \forall\, n, j \geq m$, *i.e*

$\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_j}) \to \breve{0}$, *as* $n, j \to \infty$.

Definition (3.6):

Let $(\breve{X}, \breve{M})$ be a soft modular space .Then $(\breve{X}, \breve{M})$ is said to be complete if every Cauchy sequence in $\breve{X}$ convergent to a soft vector of $\breve{X}$

Theorem (3.7)

Every soft modular space $(\breve{X}, \breve{M})$ is soft metric space $(\breve{X}, \breve{\mathfrak{H}})$.

Proof

Let $(\breve{X}, \breve{M})$ be a soft modular space, defined $\breve{\mathfrak{H}} : \mathrm{SV}(\breve{X}) \times \mathrm{SV}(\breve{X}) \to \mathcal{R}(\mathrm{E})^*$ by

$\breve{\mathfrak{H}}(\breve{x}_{e_1}, \breve{y}_{e_2}) = \breve{\mathcal{M}}(\breve{x}_{e_1} \boxminus \breve{y}_{e_2})$ for all $\breve{x}_{e_1}, \breve{y}_{e_2} \,\tilde{\in}\, \mathrm{SV}(\breve{X})$

1. Let all $\breve{x}_{e_1}, \breve{y}_{e_2} \,\tilde{\in}\, \mathrm{SV}(\breve{X})$

$\breve{x}_{e_1} \boxminus \breve{y}_{e_2} \to \breve{\mathcal{M}}(\breve{x}_{e_1} \boxminus \breve{y}_{e_2}) \,\tilde{\geq}\, \breve{0} \to \breve{\mathfrak{H}}(\breve{x}_{e_1}, \breve{y}_{e_2}) \,\tilde{\geq}\, \breve{0}$.

2. Let all $\breve{x}_{e_1}, \breve{y}_{e_2} \,\tilde{\in}\, \mathrm{SV}(\breve{X})$

$\check{\mathfrak{H}}(\check{x}_{e_1},\check{y}_{e_2}) = \check{0} \Leftrightarrow \check{\mathcal{M}}(\check{x}_{e_1} \boxminus \check{y}_{e_2}) = \check{0} \Leftrightarrow \check{x}_{e_1} \boxminus \check{y}_{e_2} = \check{0} \Leftrightarrow \check{x}_{e_1} = \check{y}_{e_2}$.

3. Let $\check{x}_{e_1},\check{y}_{e_2} \,\tilde{\in}\, SV(\check{X})$

$\check{\mathfrak{H}}(\check{x}_{e_1},\check{y}_{e_2}) = \check{\mathcal{M}}(\check{x}_{e_1} \boxminus \check{y}_{e_2}) = \check{\mathcal{M}}(\boxminus(\check{y}_{e_2} \boxminus \check{x}_{e_1})) = \check{\mathcal{M}}(\check{y}_{e_2} \boxminus \check{x}_{e_1})$

$= \check{\mathfrak{H}}(\check{y}_{e_2},\check{x}_{e_1})$.

4. .Let $\check{x}_{e_1},\check{y}_{e_2},\check{z}_{e3} \,\tilde{\in}\, SV(\check{X})$

$$\check{\mathcal{M}}(\check{x}_{e_1} \boxminus \check{y}_{e_2}) = \check{\mathcal{M}}\left((\check{x}_{e_1} \boxminus \check{z}_{e3}) \boxplus (\check{z}_{e3} \boxminus \check{y}_{e_2})\right)$$

$$\tilde{\leq} \check{\mathcal{M}}(\check{x}_{e_1} \boxminus \check{z}_{e3}) \boxplus \check{\mathcal{M}}(\check{z}_{e3} \boxminus \check{y}_{e_2})$$

$$\Rightarrow \check{\mathfrak{H}}(\check{x}_{e_1},\check{y}_{e_2}) \tilde{\leq} \check{\mathfrak{H}}(\check{x}_{e_1} \boxminus \check{z}_{e3}) \boxplus \check{\mathfrak{H}}(\check{z}_{e3} \boxminus \check{y}_{e_2}) .$$

It follow that$(\check{X},\check{\mathfrak{H}})$ is soft metric on $\check{X}$ and this fuzzy metric is called the soft metric induced by soft modular.

Definition (3.8):

Let $(\check{X},\check{M})$ be a soft modular space . *The soft open ball with center* $\check{x}_{e_1} \,\tilde{\in}\, SV(\check{X})$ *and radius* $\check{r} \,\tilde{>}\, \check{0}$ *is denoted and defined by*

$B(\check{x}_{e_1},\check{r}) = \{\check{y}_{e_2} \,\tilde{\in}\, \check{X} : \check{\mathcal{M}}(\check{x}_{e_1} \boxminus \check{y}_{e_2}) \,\tilde{<}\, \check{r}\}$.

Similarly, the soft closed ball with center $\check{x}_{e_1} \,\tilde{\in}\, SV(\check{X})$ *and radius* $\check{r} \,\tilde{>}\, \check{0}$ *is denoted and defined by*

$\overline{B(\check{x}_{e_1},\check{r})} = \{\check{y}_{e_2} \,\tilde{\in}\, \check{X}_M : \check{\mathcal{M}}(\check{x}_{e_1} \boxminus \check{y}_{e_2}) \,\tilde{\leq}\, \check{r}\}$.

Definition (3.9):

Let $(\check{X},\check{M})$ be a soft modular space and $\check{A} \,\tilde{\subseteq}\, \check{X}$ we say that $\check{A}$ is soft open set if for every $\check{x}_{e_1} \,\tilde{\in}\, \check{A}$ there exist $\check{r} \,\tilde{>}\, \check{0} \ni B(\check{x}_{e_1},\check{r}) \,\tilde{\subset}\, \check{A}$. A subset $\check{A}$ of $\check{X}$ is said to be soft closed if its complement is soft open , that is , $\check{A}^c = \check{X} \boxminus \check{A}$ is soft closed .

Theorem (3.10):

The intersections finite number of soft open sets in soft modular space is soft open sets.

Proof:

Let $SV(\check{X})$ be a soft modular space and let $\{\widetilde{G_m}: m=1, 2,\ldots, n\}$ be a finite collection of soft open set in $SV(\check{X})$

Let $\check{H} = \tilde{\cap}\, \{\check{G}_m , m = 1,2, \ldots, n\}$

to prove $\check{H}$ is an soft open set

let $\check{x}_{e_1} \in \check{H} \Rightarrow \check{x}_{e_1} \in \check{G}_m \forall\, m=1,2,\ldots n$

Since $\check{G}_m$ open soft set $\forall m \Rightarrow \exists\, \check{r} \,\tilde{>}\, \check{0} \ni B(\check{x}_{e_1},\check{r}) \,\tilde{\subset}\, \check{G}_m$

$B(\check{x}_{e_1},\check{r}) \,\tilde{\subset}\, \tilde{\cap}\, \check{G}_m \Rightarrow B(\check{x}_{e_1},\check{r}) \,\tilde{\subset}\, \check{H}$

Then $\check{H}$ is soft open set.

Theorem (3.11):

The union of an arbitrary collections of open soft set in soft modular space is soft open sets .

Proof:

Let $SV(\check{X})$ be a soft modular space and let $\{\check{\gamma}_\lambda : \lambda \,\tilde{\in}\, \wedge\}$ be an arbitrary collection of soft open sets in $SV(\check{X})$.

Let $\check{G} = \tilde{\cup}\, \{\check{\gamma}_\lambda : \lambda \,\tilde{\in}\, \wedge\}$

We must to prove $\breve{G}$ is soft open set

Let $\breve{x}_{e_1} \breve{\in} \breve{G} \Longrightarrow \breve{x}_{e_1} \breve{\in} \breve{\gamma}_{\lambda}$ for some $\breve{\lambda} \breve{\in} \breve{\Lambda}$

Since $\breve{\gamma}_{\lambda}$ is soft open set

Then there exist $\tilde{r} \breve{>} \breve{0} \ni \mathrm{B}(\breve{x}_{e_1}, \breve{r}) \breve{\subseteq} \breve{\gamma}_{\lambda}$

Since $\breve{\gamma}_{\lambda} \breve{\subseteq} \breve{G}$ then $\mathrm{B}(\breve{x}_{e_1}, \breve{r}) \breve{\subseteq} \breve{G}$

Then $\breve{G}$ is soft open set .

Theorem (3.12):

Every single set in soft modular space is soft closed set.

Proof:

Let $SV(\breve{X})$ be a soft modular space

Let $\breve{B} = \{ \breve{x}_{e_1} \}$, to prove $\breve{B}$ is soft closed set

Let $\breve{z}_{e2} \in \breve{B}^c \Longrightarrow \breve{z}_{e2} \neq \breve{x}_{e_1}$

$\breve{M}(\breve{z}_{e2} \boxminus \breve{x}_{e_1}) = \tilde{r} \breve{>} \breve{0} \Longrightarrow \tilde{r} \breve{>} \breve{0}$ (since $SV(\breve{X})$ is soft modular space)

$\breve{x}_{e_1} \breve{\notin} \mathrm{B}(\breve{z}_{e2}, \breve{r}) = \{a \in SV(\breve{X}): \breve{M}(a \boxminus \breve{z}_{e2}) \breve{<} \tilde{r} \}$

$\breve{B} \breve{\cap} \mathrm{B}(\breve{z}_{e2}, \breve{r}) = \breve{\emptyset} \Longrightarrow \mathrm{B}(\breve{z}_{e2}, \breve{r}) \breve{\subseteq} \breve{B}^c$

$\breve{z}_{e2} \breve{\in} \mathrm{B}(\breve{z}_{e2}, \breve{r}) \breve{\subseteq} \breve{B}^c$

Then $\breve{B}^c$ is soft open set $\Longrightarrow \breve{B}$ is soft closed set .

Corollary (3.13):

Every finite set in soft modular space is soft closed set.

Proof:

Let $SV(\breve{X})$ be a soft modular space

If $\breve{B} = \{\breve{x}_{e_1}, \breve{x}_{e_2}, \dots, \breve{x}_{e_n}\} \Longrightarrow \breve{B} = \breve{\cup} \{\breve{x}_{e_i}\}$

Since $\{\breve{x}_{e_i}\}$ is soft closed set (by Theorem (3.11))

then $\breve{\cup} \{\breve{x}_{e_i}\}$ is soft closed set $\Longrightarrow \breve{B}$ is soft closed set .

Definition (3.14):

A subset $\breve{C}$ of a vector space $SV(\breve{X})$ over F is called soft convex set if

$\alpha \breve{x}_{e_1} \breve{+} (1 \boxminus \alpha) \breve{y}_{e_2} \breve{\in} \breve{C}$ for all $\breve{x}_{e_1}, \breve{y}_{e_2} \breve{\in} \breve{C}, \breve{0} \breve{\leq} \alpha \breve{\leq} \breve{1}$.

Theorem (3.15):

Every soft open and closed balls in soft convex modular space are soft convex sets.

Proof:

Let $\breve{y}_{e_2}, \breve{y}_{e_3} \in \mathrm{B}(\breve{x}_{e_1}, \breve{r})$ such that $\breve{r} \breve{>} \breve{0}$, $\breve{0} \leq \alpha \leq \breve{1}$.

$\breve{M}(\breve{y}_{e_2} \boxminus \breve{x}_{e_1}) \breve{<} \breve{r}$ and $\breve{M}(\breve{y}_{e_3} \boxminus \breve{x}_{e_1}) \breve{<} \breve{r}$.

To prove $\alpha \breve{y}_{e_2} \boxplus (1 \boxminus \alpha) \breve{y}_{e_3} \breve{\in} \mathrm{B}(\breve{x}_{e_1}, \breve{r})$

$\breve{M}(\alpha \breve{y}_{e_2} \boxplus (1 \boxminus \alpha) \breve{y}_{e_3} \boxminus \breve{x}_{e_1})$

$= M(\alpha \breve{y}_{e_2} \boxplus (1 \boxminus \alpha) \breve{y}_{e_3} \boxminus \alpha \breve{x}_{e_1} \boxplus \alpha \breve{x}_{e_1} \boxminus \breve{x}_{e_1})$

$= \breve{M}(\alpha(\breve{y}_{e_2} \boxminus \breve{x}_{e_1}) \boxplus (1 \boxminus \alpha)(\breve{y}_{e_3} \boxminus \breve{x}_{e_1})$,

$\breve{\leq} \alpha \breve{M}(\breve{y}_{e_2} \boxminus \breve{x}_{e_1}) \boxplus (1 \boxminus \alpha) \breve{M}(\breve{y}_{e_3} \boxminus \breve{x}_{e_1})$,

$\breve{<} \alpha \breve{r} \boxplus (1 \boxminus \alpha) \breve{r} = \breve{r}$

$\Longrightarrow \alpha \breve{y}_{e_2} \boxplus (1 \boxminus \alpha) \breve{y}_{e_3} \breve{\in} \mathrm{B}(\breve{x}_{e_1}, \breve{r})$

Then is $\mathrm{B}(\breve{x}_{e_1}, \breve{r})$ soft convex .

similary, we can prove $\overline{B(\breve{x}_{e_1},\breve{r})}$ is soft convex set .

Theorem (3.16):

If $\breve{A}$ is soft convex set in soft convex modular space then the closed convex set $\breve{A}$ is soft convex set.

Proof:

Let $\breve{x}_{e_1}, \breve{y}_{e_2} \,\tilde{\in}\, soft\ closed\ convex, \breve{0} \lesseqqgtr \alpha \lesseqqgtr \breve{1} . \Rightarrow \exists\ \breve{a}, \breve{b} \,\tilde{\in}\, \breve{A}$ such that

$\breve{M}(\breve{x}_{e_1} \boxminus \breve{a}) \lessapprox \breve{r}, \breve{M}(\breve{x}_{e_1} \boxminus \breve{b}) \lessapprox \breve{r}$

Since $\breve{A}$is convex$\Rightarrow \breve{\alpha}\breve{a} \boxplus (1 \boxminus \breve{\alpha})\breve{b} \,\tilde{\in}\, \breve{A}$

$(\breve{\alpha}\breve{x}_{e_1} \boxplus (1 \boxminus \breve{\alpha})\breve{y}_{e_2} \boxminus (\breve{\alpha}\breve{a} \boxplus (1 \boxminus \breve{\alpha})\breve{b}) = \breve{\alpha}(\breve{x}_{e_1} \boxminus \breve{a}) \boxplus (1 \boxminus \breve{\alpha})(\breve{y}_{e_2} \boxminus \breve{b})$

$\breve{M}\left(\breve{\alpha}\breve{x}_{e_1} \boxplus (1 \boxminus \breve{\alpha})\breve{y}_{e_2} \boxminus (\breve{\alpha}\breve{a} \boxplus (1 \boxminus \breve{\alpha})\, \breve{b})\right)$

$= \breve{M}(\breve{\alpha}(\breve{x}_{e_1} \boxminus \breve{a}) \boxplus (1 \boxminus \breve{\alpha})(\breve{y}_{e_2} \boxminus \breve{b}))$

$\lesseqqgtr \breve{\alpha}\, \breve{M}(\breve{x}_{e_1} \boxminus \breve{a}) \boxplus (1 \boxminus \breve{\alpha})\breve{M}(\breve{y}_{e_2} \boxminus \breve{b}) \lessapprox \breve{\alpha}\, \breve{r} \boxplus (1 \boxminus \breve{\alpha})\breve{r} = \breve{r}$

$\Rightarrow \breve{\alpha}\breve{x}_{e_1} \boxplus (1 \boxminus \breve{\alpha})\breve{y}_{e_2} \,\tilde{\in}\, \overline{(\breve{A})} \Rightarrow\ \breve{A}$is soft closed convex set.

Definition (3.17):

Let $(\breve{X},\breve{\mathcal{M}}),(\breve{Y},\breve{\mathcal{M}})$ be two soft modular spaces .The linear function $f:\breve{X} \to \breve{Y}$ is said bounded if $f(\breve{A})$ is bounded set in $\breve{Y}$ for all $\breve{A}$ bounded set in $\breve{X}$.

i-e: $\forall\{\breve{A}$ *bounded set in* $\breve{X} \Rightarrow f(\breve{A})$ *bounded set in* $\breve{Y}$ *}.*

Theorem (3.18):

Let $(\breve{X},\breve{\mathcal{M}})$ be a soft modular space ,then

1. Every M-convergent sequence is M-Cauchy sequence in $(\breve{X},\breve{\mathcal{M}})$.
2. Every sequence in $\breve{X}$ has a unique limit .
3. If $\breve{x}_{e_n} \to \breve{x}_{e_0}, \breve{y}_{e_n} \to \breve{y}_{e_0}$,then $\breve{x}_{e_n} \boxplus \breve{y}_{e_n} \to \breve{x}_{e_0} \boxplus \breve{y}_{e_0}$
4. If $\breve{x}_{e_n} \to \breve{x}_{e_0}$ then $\breve{x}_{e_n} \to c\breve{x}_{e_0}, c \,\tilde{\in}\, F/\{0\}$.

Proof:

Let $\{\breve{x}_{e_n}\}$ be a sequence in $\breve{X}$ such that $\breve{x}_{e_n} \to \breve{x}_{e_0}$,

$\forall \breve{\varepsilon} \gtrapprox \breve{0}\ \exists k \in Z^+ \ni \breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \lessapprox \frac{\breve{\varepsilon}}{2} \quad \forall n \gtreqqless k$.

$\forall n, j \geq k\ , \breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \lessapprox \frac{\breve{\varepsilon}}{2}\ , \breve{M}(\breve{x}_{e_j} \boxminus \breve{x}_{e_0}) \lessapprox \frac{\breve{\varepsilon}}{2}$.

$\breve{M}\left(\breve{x}_{e_n} \boxminus \breve{x}_{e_j}\right) = \breve{M}\left((\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \boxplus (\breve{x}_{e_0} \boxminus \breve{x}_{e_j})\right) \lesseqqgtr \breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \boxplus \breve{M}(\breve{x}_{e_0} \boxminus \breve{x}_{e_j})$

$< \frac{\breve{\varepsilon}}{2} \boxplus \frac{\breve{\varepsilon}}{2} = \breve{\varepsilon}$

Then $\breve{M}(x_n \boxminus x_j) \lessapprox \breve{\varepsilon}$ as $n, j \to \infty$.

therefore$\{\breve{x}_{e_n}\}$ is Cauchy sequence in $\breve{X}$.

2. Let $\{\breve{x}_{e_n}\}$ be a sequence in$\breve{X}$ such that $\breve{x}_{e_n} \to \breve{x}_{e_0}$and $\breve{x}_{e_n} \to \breve{y}_{e_0}$ and $\breve{x}_{e_0} \neq \breve{y}_{e_0}$, then $\breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \to \breve{0}$ and $\breve{M}(\breve{x}_{e_n} \boxminus \breve{y}_{e_0}) \to \breve{0}$as $n \to \infty$

$\breve{M}(\breve{x}_{e_0} \boxminus \breve{y}_{e_0}) \lesseqqgtr \breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0})\breve{M}(\breve{x}_{e_n} \boxminus \breve{y}_{e_0})$

Since $\breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \to \breve{0}$ and $\breve{M}(\breve{x}_{e_n} \boxminus \breve{y}_{e_0}) \to \breve{0}$ as $n \to \infty$

Then $\breve{M}(\breve{x}_{e_0} \boxminus \breve{y}_{e_0}) \to \breve{0}$

Then $\breve{x}_{e_0} \boxminus \breve{y}_{e_0} = \breve{0} \Longrightarrow \breve{x}_{e_0} = \breve{y}_{e_0}$.

3 . Let $\breve{x}_{e_n} \to \breve{x}_{e_0}$ and $\breve{y}_{e_n} \to \breve{y}_{e_0}$

$\breve{M}((\breve{x}_{e_n} \boxplus \breve{y}_{e_n}) \boxminus (\breve{x}_{e_0} \boxplus \breve{y}_{e_0})) \lesssim \breve{M}((\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \boxplus (\breve{y}_{e_n} \boxminus \breve{y}_{e_0}))$

Since $\breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \to \breve{0}$ and $\breve{M}(\breve{y}_{e_n} \boxminus \breve{y}_{e_0}) \to \breve{0}$

Then $\breve{M}((\breve{x}_{e_n} \boxplus \breve{y}_{e_n}) \boxminus (\breve{x}_{e_0} \boxplus \breve{y}_{e_0})) \to \breve{0}$ as $n \to \infty$

Then $\breve{x}_{e_n} \boxplus \breve{y}_{e_n} \to \breve{x}_{e_0} \boxplus \breve{y}_{e_0}$.

4. Let $\breve{x}_{e_n} \to \breve{x}_{e_0}$

$\breve{M}(c\breve{x}_{e_n} \boxminus cx) = \breve{M}(c(\breve{x}_{e_n} \boxminus \breve{x}_{e_0})) = \breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0})$

Since $\breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \to \breve{0}$ as n $\to \infty$, then $\breve{M}(c\breve{x}_{e_n} \boxminus c\breve{x}_{e_0}) \to \breve{0}$ as $n \to \infty$

Then $c\breve{x}_{e_n} \to c\breve{x}_{e_0}$.

Theorem (3.19):

Let $(\breve{X}, \breve{M})$ *and* $(\breve{Y}, \breve{M})$ *be a two soft modular spaces and let*

$\breve{x}_{e_n} \to \breve{x}_{e_1}$, $\breve{y}_{e_n} \to \breve{y}_{e_1}$, *such that* $\{\breve{x}_{e_n}\}$ *and* $\{\breve{y}_{e_n}\}$ *are two sequences in* $SV(\breve{X})$ *and*

$\alpha, \beta \in F/\{0\}$ *then* $\alpha f(\breve{x}_{e_n}) \boxplus \beta g(\breve{y}_{e_n}) \to \alpha f(\breve{x}_{e_1}) \boxplus \beta g(\breve{y}_{e_1})$ *whenever* f *and* g *are two identity functions.*

Proof:

Let $\breve{x}_{e_n} \to \breve{x}_{e_1}$ and $\breve{y}_{e_n} \to \breve{y}_{e_1}$

$\breve{\mathcal{M}}\left(\left(\alpha f(\breve{x}_{e_n}) \boxplus \beta g(\breve{y}_{e_n})\right) \boxminus \left(\alpha f(\breve{x}_{e_1}) \boxplus \beta g(\breve{y}_{e_1})\right)\right)$.

$= \breve{\mathcal{M}}\left(\alpha\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1})\right) \boxplus \beta\left(g(\breve{y}_{e_n}) \boxminus g(\breve{y}_{e_1})\right)\right)$.

$\leq \breve{M}\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1})\right) \boxplus \breve{M}\left(g(\breve{y}_{e_n}) \boxminus g(\breve{y}_{e_1})\right) = \breve{\mathcal{M}}((\breve{x}_{e_n} \boxminus \breve{x}_{e_1}) \boxplus \breve{M}(\breve{y}_{e_n} \boxminus \breve{y}_{e_1})$.

Since $\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e1}) \to \breve{0}$ *and* $\breve{\mathcal{M}}(\breve{y}_{e_n} \boxminus \breve{y}_{e_1}) \to \breve{0}$

Then $\breve{\mathcal{M}}((\alpha(f(\breve{x}_{e_n}) \boxplus \beta g(\breve{y}_{e_n}) \boxminus \left(\alpha f(\breve{x}_{e_1}) \boxplus \beta g(\breve{y}_{e_1})\right) \longrightarrow \breve{0}$ *as* n $\longrightarrow \infty$.

Then $\alpha f(\breve{x}_{e_n}) \boxplus \beta g(\breve{y}_{e_n}) \to \alpha f(\breve{x}_{e_1}) \boxplus \beta g(\breve{y}_{e_1})$.

Definition (3.20):

Let $(\breve{X}, \breve{\mathcal{M}}), (\breve{Y}, \breve{\mathcal{M}})$ be two soft Modular spaces. The function

$f : \breve{X} \longrightarrow \breve{X}$ is said to be continuous at $\breve{x}_{e_0} \breve{\in}$ SV($\breve{X}$) if for all $\breve{\varepsilon} \gtrsim 0$ and there exists $\breve{\delta} \gtrsim 0$ such that for all $\breve{x}_{e_1} \breve{\in}$ SV($\breve{X}$)

$\breve{M}(\breve{x}_{e_1} \boxminus \breve{x}_{e_0}) \lesssim \breve{\delta} \Longrightarrow \breve{M}\left(f(\breve{x}_{e_1}) \boxminus f(\breve{x}_{e_0})\right) \lesssim \breve{\varepsilon}$.

The function f is called continuous function, if it continuous at every point of $SV(\breve{X})$.

Remark (3.21):

Every identity function in soft modular space $(\breve{X}, \breve{M})$ is continuous functions in soft modular space.

Proof:

For all $\breve{\varepsilon} > \breve{0}$ take $\breve{\varepsilon} = \breve{\delta}$. $\breve{\delta} \gtrsim \breve{0}$ $\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_1}) \lesssim \breve{\delta} \Rightarrow \breve{\mathcal{M}}\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1})\right) = \breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_1}) \lesssim \breve{\delta} \lesssim \breve{\varepsilon}$ then f is soft continuous at $\breve{x}_{e_1}$ Since $\breve{x}_{e_1}$ is an arbitrary point then f is continuous function

Theorem (3.22):

Let $\breve{X}$ be soft modular space over F. Then the function $f: \breve{X} \to \breve{X}$,

$f(\breve{x}_{e_1}, \breve{y}_{e_2}) = \breve{x}_{e_1} \boxplus \breve{y}_{e_2}$ is continuous functions.

Proof:

Let $\breve{x}_{e_0}, \breve{y}_{e_0} \,\breve{\in}\, SV(\breve{X})$ and $\{\breve{x}_{e_n}\}, \{\breve{y}_{e_n}\} \,\breve{\in}\, SV(\breve{X})$ such that $\breve{x}_{e_n} \to \breve{x}_{e_0}$ and $\breve{y}_{e_n} \to \breve{y}_{e_0}$ as $n \to \infty$

$$\breve{\mathcal{M}}\left(f(\breve{x}_{en}, \breve{y}_{e_n}) \boxminus f(\breve{x}_{e_0}, \breve{y}_{e_0})\right) = \breve{\mathcal{M}}\left((\breve{x}_{e_n} \boxplus \breve{y}_{e_n}) \boxminus (\breve{x}_{e_0} \boxplus \breve{y}_{e_0})\right)$$
$$= \breve{\mathcal{M}}\left((\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \boxplus (\breve{y}_{e_n} \boxminus \breve{y}_{e_0})\right) \lesseqqgtr \breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \boxplus \breve{\mathcal{M}}(\breve{y}_{e_n} \boxminus \breve{y}_{e_0})$$

Since $\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \to \breve{0}$ and $\breve{\mathcal{M}}(\breve{y}_{e_n} \boxminus \breve{y}_{e_0}) \to \breve{0}$ as $n \to \infty$, we have

$\breve{\mathcal{M}}(f(\breve{x}_{e_n}, \breve{y}_{e_n}) \boxminus f(\breve{x}_{e_0}, \breve{y}_{e_0})) \to \breve{0}$ *as* $n \to \infty$

Then $f(\breve{x}_{e_n}, \breve{y}_{e_n}) \to f(\breve{x}_{e_1}, \breve{y}_{e_1})$ as $n \to \infty$,f is continuous function at $(\breve{x}_{e_1}, \breve{y}_{e_2})$ and $(\breve{x}_{e_1}, \breve{y}_{e_2})$ is any point in $\breve{X} \times \breve{X}$,therefore f is continuous function.

Theorem (3.23)

Let $(X, \mathcal{M}), (Y, \mathcal{M})$ be a soft modular spaces ,then the function $f: \breve{X} \to \breve{Y}$ is continuous at $\breve{x}_{e_0} \,\breve{\in}\, SV(\breve{X})$ if and only if for all sequence $\{\breve{x}_{e_n}\}$ convergent to $\breve{x}_{e_0} \,\breve{\in}\, SV(\breve{X})$ then the sequence $\{f(\breve{x}_{e_n})\}$ is convergent to $f(\breve{x}_{e_0})$ in $SV(\breve{Y})$

Proof:

Suppose the function f is continuous in x_0 and let $\{\breve{x}_{e_n}\}$ is a sequence in $SV(\breve{X})$ such that

$\breve{x}_{e_n} \to \breve{x}_{e_0}$.

Let $\breve{\varepsilon} \,\breve{\in}\, (0,1)$, since f is continuous in $\breve{x}_{e_0} \Rightarrow$ there exist $\bar{\delta} \gtrsim \breve{0}$, such that for all $\breve{x}_{e_1} \in SV(\breve{X}) : \breve{\mathcal{M}}(\breve{x}_{e_1} \boxminus \breve{x}_{e_0}) \lesssim \breve{\delta} \Rightarrow \breve{\mathcal{M}}\left(f(\breve{x}_{e_1}) \boxminus f(\breve{x}_{e_0})\right) \lesssim \breve{\varepsilon}$ Since $\breve{x}_{e_n} \to x_0$, $\breve{\delta} \gtrsim \breve{0}$, there exist $k \in Z^+$ such that

$\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \lesssim \breve{\delta}$ *for all* $n \geqq k$ *hence* $\breve{\mathcal{M}}(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_0})) \lesssim \breve{\varepsilon}$ *for all* $n \geqq k$

Then $f(\breve{x}_{e_n}) \to f(\breve{x}_{e_0})$.

Conversely suppose the condition in the theorem is true.

Suppose f is not continuous at $\breve{x}_{e_0}$.

There exist $\breve{\varepsilon} \succeq \breve{0}$ *such that for all* $\breve{\delta} \succeq \breve{0}$, *there exist* $\breve{x}_{e_1} \in SV(\breve{X})$ *and*

$\breve{\mathcal{M}}(\breve{x}_{e_1} \boxminus \breve{x}_{e_0}) \precsim \breve{\delta} \Rightarrow \breve{\mathcal{M}}\left(f(\breve{x}_{e_1}) \boxminus f(\breve{x}_{e_0})\right) \succsim \breve{\varepsilon}$

That is mean $\breve{x}_{e_n} \to \breve{x}_{e_0}$ *in* $SV(\breve{X})$ *but* $f(\breve{x}_{e_n}) \nrightarrow f(\breve{x}_{e_0})$ *in* $\breve{Y}$ *this contradiction , f is continuous at* $\breve{x}_{e_0}$.

Theorem (3.24):

Let $(\breve{X}, \breve{\mathcal{M}}), (\breve{Y}, \breve{\mathcal{M}})$ *be soft modular spaces and let* $f: \breve{X} \to \breve{Y}$ *be a linear function. Then f is continuous either at every point of* $SV(\breve{X})$ *or at no point of* $SV(\breve{X})$.

proof:

Let $\breve{x}_{e_1}$ *and* $\breve{x}_{e_2}$ *be any two point of* $SV(\breve{X})$ *and suppose f is continuous at* $\breve{x}_{e_1} \in SV(\breve{X})$

Then for each $\breve{\varepsilon} \succeq \breve{0}$ *there exist* $\breve{\delta} \succeq \breve{0}$ *such that* $\breve{x}_{e_0} \in SV(\breve{X})$,

$\breve{\mathcal{M}}(\breve{x}_{e_0} \boxminus \breve{x}_{e_1}) \precsim \breve{\delta} \Rightarrow \breve{\mathcal{M}}(f(\breve{x}_{e_0}) \boxminus f(\breve{x}_{e_1})) \precsim \breve{\varepsilon}$,

Now $\breve{\mathcal{M}}(\breve{x}_{e_0} \boxminus \breve{x}_{e_2}) \precsim \breve{\delta}$. $\breve{\mathcal{M}}(\breve{x}_{e_0} \boxplus \breve{x}_{e_1} \boxminus \breve{x}_{e_2}) \boxminus \breve{x}_{e_1}) \precsim \breve{\delta}$,

$$\Rightarrow \breve{\mathcal{M}}\left(f(\breve{x}_{e_0}) \boxplus f(\breve{x}_{e_1}) \boxminus f(\breve{x}_{e_2}) \boxminus f(\breve{x}_{e_1})\right) \precsim \breve{\varepsilon} \Rightarrow \breve{\mathcal{M}}\left(f(\breve{x}_{e_0}) \boxminus f(\breve{x}_{e_2})\right) \precsim \breve{\varepsilon}$$

then f is continuous at $\breve{x}_{e_2} \in SV(\breve{X})$ *, since* $\breve{x}_{e_2}$ *is an arbitrary point, then f is continuous.*

Corollary (3.25):

Let $(\breve{X}, \breve{\mathcal{M}}), (\breve{Y}, \breve{\mathcal{M}})$ *be two soft modular spaces and let* $f: \breve{X} \to \breve{Y}$ *be a linear function . if f is continuous at* 0 *then it is continuous at every point.*

Proof:

Let $\{\breve{x}_{e_n}\}$ *be a sequence in* $SV(\breve{X})$ *such that* $\breve{x}_{e_n} \to \breve{x}_{e_0}$,

Since f is continuous at 0, *then :*

For all $\breve{\varepsilon} \succeq \breve{0}$, *there exist* $\breve{\delta} \succeq \breve{0}, : (\breve{x}_{e_n} \boxminus \breve{x}_{e_1}) \in SV(\breve{X})$

$\breve{\mathcal{M}}((\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \boxminus 0) \precsim \breve{\delta} \Rightarrow \breve{\mathcal{M}}(f(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \boxminus f(0)) \precsim \breve{\varepsilon}$,

$\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \precsim \breve{\delta} \Rightarrow \breve{\mathcal{M}}(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_0}) \boxminus f(0)) \precsim \breve{\varepsilon}$,

$\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \precsim \breve{\delta} \Rightarrow \breve{\mathcal{M}}(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_0}) \boxminus (0)) \precsim \breve{\varepsilon}$,

$\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_0}) \precsim \breve{\delta} \Rightarrow \breve{\mathcal{M}}\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_0})\right) \precsim \breve{\varepsilon}$,

$\breve{x}_{e_n} \to \breve{x}_{e_n} \Rightarrow f(\breve{x}_{e_n}) \to f(\breve{x}_{e_0})$ *Then f is continuous at* $\breve{x}_{e_0}$

Since $\breve{x}_{e_0}$ *is arbitrary point, therefore f is continuous function .*

Theorem (3.26):

Let $(\breve{X}, \breve{\mathcal{M}})$, $(\breve{Y}, \breve{\mathcal{M}})$ *be two soft modular spaces . If the function*

$f: \breve{X} \to \breve{Y}, g: \breve{X} \to \breve{Y}$ *are two continuous functions then:*

1. g *is continuous function.*
2. kf *where* $k \in F/\{0\}$ *is continuous function .*

Proof:

Let $\{\breve{x}_{e_n}\}$ *be a sequence in* $\mathrm{SV}(\breve{X})$ *such that* $\breve{x}_{e_n} \to \breve{x}_{e_1}$ *Since* f *and* g *are two continuous functions at* $\breve{x}_{e_1}$ *,Then for all* $\breve{\varepsilon} \gtrsim \breve{0}$ *there exist* $\breve{\delta} \gtrsim \breve{0}$ *such that for all* $\breve{x}_{e_1} \in \mathrm{SV}(\breve{X})$:

$\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_1}) \lesssim \breve{\delta} \Rightarrow \breve{\mathcal{M}}\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1})\right) \lesssim \breve{\varepsilon}$,

And $\breve{\mathcal{M}}(\breve{x}_{e_n} \boxminus \breve{x}_{e_1}) \lesssim \breve{\delta} \Rightarrow \breve{\mathcal{M}}\left(g(\breve{x}_{e_n}) \boxminus g(\breve{x}_{e_1})\right) \lesssim \breve{\varepsilon}$,

Now $\breve{\mathcal{M}}\left((f \boxplus g)(\breve{x}_{e_n}) \boxminus (f \boxplus g)(\breve{x}_{e_1})\right) = \breve{\mathcal{M}}\left(f(\breve{x}_{e_n}) \boxplus g(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1}) \boxminus g(\breve{x}_{e_1})\right)$

$\lesseqgtr \breve{\mathcal{M}}\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1})\right) \boxplus \breve{\mathcal{M}}\left(g(\breve{x}_{e_n}) \boxminus g(\breve{x}_{e_1})\right)$

$\breve{\varepsilon} \boxplus \breve{\varepsilon} = \breve{\varepsilon}$

Therefore $f \boxplus g$ *is continuous function .*

2. Let $\{\breve{x}_{e_n}\}$ *be a sequence in* X *such that* $\breve{x}_{e_n} \to \breve{x}_{e_1}$ *,then for all* $\breve{\varepsilon} \gtrsim \bar{0}$

there exist $\breve{\delta} \gtrsim \breve{0}$ *such that* $\breve{M}(\breve{x}_{e_n} \boxminus \breve{x}_{e_1}) \lesssim \breve{\delta}$ *implies* $\breve{M}\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_0})\right) \lesssim \breve{\varepsilon}$

Then for all $\breve{\varepsilon} \gtrsim \breve{0}$ *there exist* $\breve{\delta} \gtrsim \breve{0}$ *such that* $\breve{M}(x_n \boxminus x) \lesssim \breve{\delta}$ *implies*

$$\breve{M}\left((kf)(\breve{x}_{e_n}) \boxminus (kf)(\breve{x}_{e_1})\right) = \breve{M}\left(k\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1})\right)\right) = \breve{M}\left(f(\breve{x}_{e_n}) \boxminus f(\breve{x}_{e_1})\right) \lesssim \breve{\varepsilon}$$

Therefore kf *is continuous function .*

References

[1] D. Molodtsov, Soft set-theory results, Comput. Math. App., 37(1999) 19-31.

[2] D., Sujoy and S., Smanta, soft metric, Ann. fuzzy math. Inf., (2013), 1-18.
[3] D. , Sujoy and S. ,Smanta, soft real sets, soft real numbers and their properties, J. fuzzy mathematics, 20(2012), 551-576.
[4] E., Peyghan, B. Samadi and A. Tayebi, some results Related to soft topological Spaces, Facta Univercity,29(4) (2014),325-336.
[5] H., Hazra, P., Mujumdar and S. Smanta, soft topology, Fuzzy Inform. Eng., (2012), 105-1154.
[6] K. H. Mohammed, on soft topological linear spaces, MSc. Thesis of Al-Qadisiyah University , 2018 .
[7] K. H. Mohammed, Some Properties Related with Soft Banach Algebra, MSC. Thesis of Al-Qadisiyah University, (2018).
[8] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl.,45 (2003) 555-562.
[9] S., Das and S., Samanta, Soft real sets, soft real numbers and their properties, J. Fuzzy Math.,20 (3) (2012), 551-576.

[10] S. ,Das and S., Samanta, soft metric, Annals of Fuzzy Mathematics and Informatics,6(1) (2013),77-94.
[11] Shbeeb, M., H., some results on soft topological spaces, Ph.D. Dissertation, Al-Mustansiriya University, 2015.
[12] Tantawy, O. and Hassan, R., soft real analysis J. of Progressive Research in Mathematics,8 (2016) ,77-94.
[13] Yazar, M. and Bayramov, S. and Gunduz,G., A New View on Soft Normed Spaces, Comput. Math. App.,24(2014) , 1149-1159.
[14]E.,Yildirim,A.,Guler and O.,Ozbakir,on soft I Baire spaces,Annals of Fuzzy Mathematics And Informatics,10(1)(2015),109-121.