Available online at www.qu.edu.iq/journalcm
JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS
ISSN:2521-3504(online) ISSN:2074-0204(print)

Double α-g-Transformation and Its Properties

Ahmed Mahdi Abbood[a], Assist.Prof. Dr. Methaq Hamza Geem[b]

[a,b] *Iraq ,Al-Qadisiyah university , Education College , Mathematics Department,* ***Methaq.Geem@qu.edu.Iq***

ARTICLE INFO

Article history:
Received: 15 /06/2022
Rrevised form: 13 /07/2022
Accepted : 22 /07/2022
Available online: 12 /08/2022

Keywords:

integral transform, double transformation, mittage-Leffler, expontial function, g-transform.

ABSTRACT

In this paper, we introduce a new transformation , namely , **α-g-transformation** and denote it as Dg_α. This transformation consider as generalized to double g-transformation. We defined it as the following form :

$$Dg_\alpha(f(x,t) = p_1(s)p_2(s)\int_0^\infty\int_0^\infty E_\alpha\big(-(q_1(s)x)^\alpha . E_\alpha(-(q_2(s)t)^\alpha)\big)f(x,t)(dx)^\alpha(dt)^\alpha$$

MSC..44-XX

https://doi.org/ 10.29304/jqcm.2022.14.3.993

1. Introduction:

The integral transformations are one of import methods for solving many of problems. Such that we can convert ordinary differential equation to algebraic equation and then we back by inverse of this integral transformation, but the case partial differential equations we use the integral transformation to convert partial differential equations to ordinary differential equations. By using double integral transformation we can convert partial differential equations to algebraic equations , thus the double integral transformation is has import applications. This transformation is distinguished by the generalities of the most known integral transformations and the possibility of finding new

∗Corresponding author

Email addresses:

Communicated by 'sub etitor'

integral transformations from it, which helps in solving differential equations with variable coefficients and in different orders.

The kerenal of any transformation is considered import part because by the kerenel we can distinguish type of this transformation. In [4] , H.Jaferi present general integral transformation and he called it g-transformation also he studied properties of g-transformation and its applications in differential equations. mitttag-leffler function is import function . In this work we introduce a generalized double gtransformation by using mitttag-leffler function as a substitute for exponential fuction in g-transformation . Also we present many theorem and examples related with topic of paper.

2. Double α-g-transformation

2.1 Definition [4]:

g-transformation $g\big(f(x)\big)$ for a piecewise function $f(x)$ where $x \in [\ 0 , \infty\ [$ and $|f(x)| \leq Me^{kx}$ is defined by the following integral

$$g\big(f(x)\big) = p(s)\int_0^\infty e^{-q(s)x}\ f(x)dx \quad , p(s) \neq 0 \qquad (1)$$

such that the integral is convergent for some q(s), s is positive constant, and

$$\big\|g\big(f(x)\big)\big\| \leq \frac{p(s)M}{k - q(s)}$$

2.2 Definition [7]:

We say that r-gamma function and denote it as Γ_r and it is defined as:

$$\Gamma_\alpha(n) = \frac{1}{\Gamma(\alpha+1)}\int_0^\infty x^{(n-1)\alpha} E_\alpha(x)^\alpha \,(dx)^\alpha \quad , Re(n) > 0, r > 1 \qquad (2)$$

We note that if α =1 we get the classical gamma function $\Gamma(n)$

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} \, dx \quad , Re(n) > 0$$

2.3 Definition:

Let f(x,y) be a function such that x ,y >0 we define a fractional Double α-g-transformation which it is denoted as Dg_α and it is defined as following:

$$Dg_\alpha(f(x,t) = p_1(s)p_2(s)\int_0^\infty\int_0^\infty E_\alpha\big(-(q_1(s)x + q_2(s)t)^\alpha\big) f(x,t)(dx)^\alpha(dt)^\alpha \quad (3)$$

Where $E_\alpha(z)$ is mittag-liffler function $E_\alpha(Z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}$ where $p_1(s), p_2(s) > 0$ $q_1, q_2 > 0$

2.4 Remark :

By properties of Mittag-leffler function then we can Define the fractional

double g-transformation of order α

$$Dg_\alpha(f(x,t) = p_1(s)p_2(s)\int_0^\infty\int_0^\infty E_\alpha\big(-(q_1(s)x)^\alpha . E_\alpha(-(q_2(s)t)^\alpha)\big) f(x,t)(dx)^\alpha(dt)^\alpha$$

2.5 Proposition:

If $f_1(x,t)$ and $f_2(x,t)$ are a functions of the vairables x , t then

$$Dg_\alpha\big(a_1 f_1(x,t) + a_2 f_2(x,t)\big) = a_1 \alpha - Dg(f_1(x,t) + a_2 \alpha - Dg(f_2(x,t))$$

Proof:

the proof is performed from Definition (2.3)

2.6 Example:

1) $Dg_\alpha(1) = \left[p_1 \int_0^\infty E_\alpha(-q_1 x)^\alpha (1)(dx)^\alpha\right]\left[p_2 \int_0^\infty E_\alpha(-q_2 x)^\alpha\,(dt)^\alpha\right]$

$$= \frac{p_1}{q_1^\alpha}\Gamma_\alpha(1)\Gamma(\alpha+1).\frac{p_2}{q_2^\alpha}\Gamma_\alpha(1)\Gamma(\alpha+1)$$

$$\frac{p_1 p_2}{q_1^\alpha q_2^\alpha}\Gamma_\alpha^2(1)\Gamma^2(\alpha+1)$$

2) $$Dg_\alpha(x^n) = p_1 p_2 \int_0^\infty \int_0^\infty E_\alpha(-q_1 x)^\alpha E_\alpha(-q_1 x)^\alpha (x^n)(dx)^\alpha (dt)^\alpha$$

$$Dg_\alpha(x^n) = \left[p_1 \int_0^\infty E_\alpha(-q_1 x)^\alpha (x^n)(dx)^\alpha\right]\left[p_2 \int_0^\infty E_\alpha(-q_2 x)^\alpha\,(dt)^\alpha\right]$$

$$\frac{p_1}{q_1^{\alpha(n+1)}}\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma(\alpha+1)\ .\frac{p_2}{q_2^\alpha}\Gamma(1)\Gamma(\alpha+1)\ .$$

$$\frac{p_1 p_2}{q_1^{\alpha(n+1)} q_2^\alpha}\Gamma_\alpha(\frac{n}{\alpha}+1)\Gamma_\alpha(1)\Gamma^2(\alpha+1)$$

3) $$Dg_\alpha(x^n t^m) = p_1 p_2 \int_0^\infty \int_0^\infty E_\alpha(-q_1 x)^\alpha E_\alpha(-q_1 x)^\alpha (x^n t^m)(dx)^\alpha (dt)^\alpha$$

$$Dg_\alpha(x^n t^m) = \left[p_1 \int_0^\infty E_\alpha(-q_1 x)^\alpha (x^n)(dx)^\alpha\right]\left[p_2 \int_0^\infty E_\alpha(-q_2 x)^\alpha\,(t^m)\,(dt)^\alpha\right]$$

$$\frac{p_1}{q_1^{\alpha(n+1)}}\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma(\alpha+1)\ .\frac{p_2}{q_2^{\alpha(m+1)}}\Gamma\left(\frac{m}{\alpha}+1\right)\Gamma(\alpha+1)\ .$$

$$\frac{p_1p_2}{q_1^{\alpha(n+1)}q_2^{\alpha(m+1)}}\Gamma_\alpha(\frac{n}{\alpha}+1)\Gamma_\alpha(\frac{m}{\alpha}+1)\Gamma^2(\alpha+1)$$

2.7 Proposition:

$Dg_\alpha\big(f(ax,bt)\big)=\frac{p_1p_2}{a^\alpha b^\alpha}F_\alpha\left(\frac{q_1}{a},\frac{q_2}{b}\right)$ where a, b are constants

Proof:

$Dg_\alpha\big(f(ax,bt)\big)=p_1p_2\int_0^\infty\int_0^\infty E_\alpha\big(-(q_1(s)x+q_2(s)t^\alpha)\big)f(ax,bt)(dx)^\alpha(dt)^\alpha$

let u=ax , v=bt

$$Dg_\alpha f(ax+bt)=\frac{p_1p_2}{a^\alpha b^\alpha}\int_0^\infty\int_0^\infty E_\alpha\left(-\left(\frac{q_1u}{a}+\frac{q_2v}{b}\right)^\alpha\right)f(u,v)(du)^\alpha(dv)^\alpha$$

$$=\frac{p_1p_2}{a^\alpha b^\alpha}F_\alpha\left(\frac{q_1}{a},\frac{q_2}{b}\right)$$

2.8 Proposition:

$Dg_\alpha(E_\alpha(-(ax+bt)^\alpha)f(x,t))=p_1p_2F_\alpha(q_1+a,q_2+b)$

Proof:

$Dg_\alpha(E_\alpha(-(ax+bt)^\alpha)f(x,t))=p_1p_2\int_0^\infty\int_0^\infty E_\alpha(-(ax+bt)^\alpha E_\alpha(-(q_1x+$
$q_2t)^\alpha f(x,t)(dx)^\alpha(dt)^\alpha$

By using the equality $E_\alpha(k(x+t)^\alpha)=E_\alpha(kx^\alpha)E_\alpha(kt^\alpha)$

Then we have

$$\mathrm{Dg}_\alpha(\mathrm{E}_\alpha(-(\mathrm{ax}+\mathrm{bt})^\alpha)\mathrm{f}(\mathrm{x},\mathrm{t}) = \mathrm{p}_1\mathrm{p}_2\int_0^\infty\int_0^\infty \mathrm{E}_\alpha(-(\mathrm{a}+\mathrm{q}_1)\mathrm{x}+(\mathrm{b}+\mathrm{q}_2)\mathrm{t})^\alpha \mathrm{f}(\mathrm{x},\mathrm{t})(\mathrm{dx})^\alpha(\mathrm{dt})^\alpha$$

Therefor $\mathrm{Dg}_\alpha(\mathrm{E}_\alpha(-(\mathrm{ax}+\mathrm{bt})^\alpha)\mathrm{f}(\mathrm{x},\mathrm{t}) = \mathrm{p}_1\mathrm{p}_2\mathrm{F}_\alpha(\mathrm{q}_1+\mathrm{a},\mathrm{q}_2+\mathrm{b})$

2.9 Proposition:

$$\mathrm{Dg}_\alpha\left(\mathrm{x}^\alpha\mathrm{t}^\alpha\mathrm{f}(\mathrm{x},\mathrm{t})\right) = \frac{\mathrm{p}_1\mathrm{p}_2\,\partial^{2\alpha}}{\partial q_2^\alpha\partial q_1^\alpha}\mathrm{Dg}_\alpha(\mathrm{f}(\mathrm{x},\mathrm{t})$$

Proof :

$$\mathrm{Dg}_\alpha(\mathrm{x}^\alpha\mathrm{t}^\alpha\mathrm{f}(\mathrm{x},\mathrm{t}) = \mathrm{p}_1(\mathrm{s})\mathrm{p}_2(\mathrm{s})\int_0^\infty\int_0^\infty \mathrm{x}^\alpha\mathrm{E}_\alpha\left(-(q_1\mathrm{x})^\alpha\mathrm{t}^\alpha\mathrm{E}_\alpha(-(q_2\mathrm{t})^\alpha)\right)\mathrm{f}(\mathrm{x},\mathrm{t})(\mathrm{dx})^\alpha(\mathrm{dt})^\alpha$$

By using the equality $\mathrm{D}_s^\alpha\left(\mathrm{E}_\alpha(-\mathrm{sx}^\alpha)\right) = -\mathrm{x}^\alpha\mathrm{E}_\alpha(-\mathrm{sx}^\alpha)$

Thus $\mathrm{Dg}_\alpha\left(\mathrm{x}^\alpha\mathrm{t}^\alpha\mathrm{f}(\mathrm{x},\mathrm{t})\right) =$

$$\mathrm{p}_1\mathrm{p}_2\int_0^\infty\int_0^\infty \frac{\partial^\alpha}{\partial \mathrm{q}_1^\alpha}\mathrm{E}_\alpha(-\mathrm{q}_1\mathrm{x})^\alpha\frac{\partial^\alpha}{\partial \mathrm{q}_2^\alpha}\mathrm{E}_\alpha(-(\mathrm{q}_2\mathrm{t})^\alpha\mathrm{f}(\mathrm{x},\mathrm{t})(\mathrm{dx})^\alpha(\mathrm{dt})^\alpha$$

$$= \mathrm{p}_1\mathrm{p}_2\int_0^\infty\int_0^\infty \frac{\partial^{2\alpha}}{\partial \mathrm{q}_1^\alpha\mathrm{q}_2^\alpha}\mathrm{E}_\alpha(-\mathrm{q}_1\mathrm{x})^\alpha\mathrm{E}_\alpha(-(\mathrm{q}_2\mathrm{t})^\alpha\mathrm{f}(\mathrm{x},\mathrm{t})(\mathrm{dx})^\alpha(\mathrm{dt})^\alpha$$

$$= \frac{\mathrm{p}_1\mathrm{p}_2\,\partial^{2\alpha}}{\partial q_1^\alpha q_2^\alpha}\mathrm{Dg}_\alpha\left(\mathrm{f}(\mathrm{x},\mathrm{t})\right)$$

2.10 Definition [3]:

Let f , h be a function such that x , t >0 we can define the fractional double convolution is defined as following:

$$\left(\mathrm{f}(\mathrm{x},\mathrm{t}) **_\alpha \mathrm{h}(\mathrm{x},\mathrm{t})\right) = \int_0^\alpha\int_0^\alpha \mathrm{f}(\mathrm{x}-\theta,\mathrm{t}-\tau)\mathrm{h}(\tau,\theta)(\mathrm{d}\tau)^\alpha(\mathrm{d}\theta)^\alpha$$

2.11 Theorem:

let f , h be a function with Dg_α transformation f and h are known

Then : $Dg_\alpha\ (f(x,t) **_\alpha h(x,t) = Dg_\alpha(f(x,t)\ Dg_\alpha(h(x,t))$

Proof

$Dg_\alpha(f **_\alpha g)(x,t) = p_1p_2 \int_0^\infty \int_0^\infty E_\alpha((q_1t)^\alpha E_\alpha(-((q_2x)^\alpha(f **_\alpha h)(x,t)(dx)^\alpha\ (dt)^\alpha =$
$p_1p_2 \int_0^\infty \int_0^\infty E_\alpha(-(q_1t)^\alpha E_\alpha\big(-((q_2x)^\alpha)\big[\int_0^\infty \int_0^\infty f(x-\theta, t-$
$\tau\big)h(\tau,\theta)(d\tau)^\alpha(d\theta)^\alpha\big](dx)^\alpha\ (dt)^\alpha$

let $u = t - \tau$, $v = x - \theta$ *a nd taking the limit from* 0 *to* $= \infty$,

$$= p_1p_2 \int_0^\infty \int_0^\infty E_\alpha(-q_1^\alpha(u+\tau)^\alpha)E_\alpha(-q_2^\alpha(v + \theta)^\alpha) \int_0^\infty \int_0^\infty f(u,v)h(\tau,\theta)(d\tau)^\alpha(d\theta)^\alpha(du)^\alpha(dv)^\alpha$$

$$= p_1p_2 \int_0^\infty \int_0^\infty E_\alpha(-q_1^\alpha u^\alpha)E_\alpha(-q_2^\alpha v^\alpha)f(u,v)(du)^\alpha(dv)^\alpha \int_0^\infty \int_0^\infty E_\alpha(-q_1^\alpha\tau^\alpha)E_\alpha(-q_2^\alpha\theta^\alpha)h(\tau,\theta)$$

$$=\frac{1}{p_1p_2} Dg_\alpha\big(f(x,t)\big)\ Dg_\alpha\big(h(x,t)\big)$$

2.12 Definition:

Two variables delta function $\delta_\alpha(x-a, t-b)$ of fractional order α $0 < a \le 1$ we can defined

$$\int_R \int_R f(x,t)\delta_\alpha(x-a,t-b)(dx)^\alpha(dt)^\alpha = \alpha^2 f(a,b)$$

2.13 Example :

We take the delta function $\delta_\alpha(x-a, y-b)$

$$Dg_\alpha\{(x-a, y-b)\} = \int_0^\infty \int_0^\infty E_\alpha(-(q_1x+q_2y)^\alpha\, \delta_\alpha(x-a, y-b)(dx)^\alpha(dy)^\alpha$$

$$= \alpha^2 E_\alpha(-(q_1x+q_2y)^\alpha)$$

REFERENCES

1. Apelblat , "Differentiation of the Mittag-Leffler Functions with Respect to Parameters in the Laplace Transform Approach", J.Mathematics 8(657), 1-22 (2020).
2. G. Jumarie , "Laplace's transform of fractional order via the Mittag-Leffler function and modified Riemann-Liouville derivative", J.Applied Mathematics Letters 22 (2009) , 1659-1664 (2009).
3. G. D. Medina at el , "Fractional Laplace Transform and Fractional Calculus", J.International Mathematical Forum, 991-999 (2017).
4. H.Jafari , "Yang Xiao-Jun, 2021, Towards new general double integral transform and its applications to differential equations", J.Mathematical Methods in the Applied Sciences 2021(1), 1-8(2021).
5. H. J. Haubold , "Mittag-Leffler Functions and Their Applications", J.Hindawi Publishing Corporation, Journal of Applied Mathematics 1(2011), 1-51(2011).
6. J. West et al. , "Physics of Fractal Operators" ,© Springer-Verlag (New York), Inc., (2003).
7. M. A. Khan and S. Ahmed , "On some properties of the generalized Mittag-Leffler function", Springer Plus (2013).