Journal of Al-Qadisiyah for Computer Science and Mathematics Vol. 14(3) 2022 , pp Math. 42–58

JQCM
University Of AL-Qadisiyah

Available online at www.qu.edu.iq/journalcm
JOURNAL OF AL-QADISIYAH FOR COMPUTER SCIENCE AND MATHEMATICS
ISSN:2521-3504(online) ISSN:2074-0204(print)

On Differential and Integration of g-g-Transformation

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ARTICLE INFO

Article history:
Received: 17 /06/2022
Rrevised form: 13 /07/2022
Accepted : 22 /07/2022
Available online: 12 /08/2022

Keywords:

differential equation, integral transform, g-transform, convolution theorem, inverse integral transform.

ABSTRACT

In this work we present an integral transform, namely g-g-transform and defined as :

$$g(f(x)) = p(s)^m \int_0^\infty f(x) e^{-q(s)^n x}\, dx \quad , p(s) \neq 0 \ , q(s) > 0 \ \textit{or n even positive integrer}$$

This transformation consider generalized for g-transformation which defined as :

$$g(f(x)) = p(s) \int_0^\infty e^{-q(s)x} \ f(x) dx \quad , p(s) \neq 0 \ , q(s) > 0$$

We study differential and integration of this transformation. Also we find the inverse of this transformation by using g-g-transformati of integration . we use differential of g-g-transormation for finding g-g-transformation of some function of the form $x^k f(x)$.

MSC..44-XX

https://doi.org/10.29304/jqcm.2022.14.3.994

1. Introduction:

The method of integral transformations is widely used in the classical sections of mathematical physics, in the theory of differential equations, integral equations, in the theory automatic regulation and control, in the theory of queuing and other areas,[4],[11].

There are various types of such transformations: the Fourier transform, Laplace, Hankel, Meyer, Kontorovich-Lebedev series others. We confine ourselves to considering the integral Laplace transforms, [6],[8],[10] .

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The most important property of the integral transform is that the operations differentiation in the space of the originals corresponds to the operation of image multiplication to the operational variable s. This property allows us to replace the solution of differential equations to solve algebraic ones, which is much simpler.

In [9], H.Jafari introduce a new integral transformation as generalized for many integral transformation like Laplace transformation, Fourier transformation and others. He define it as the following:

$$\mathrm{g}\big(f(x)\big) = p(s)\int_0^{\infty} e^{-q(s)x}\ f(x)dx \quad ,p(s) \neq 0\ ,q(s) > 0$$

in this work we generalize g-transformation by introducing a g-g-transformation and we define as:

$$\mathrm{g}\big(\mathrm{f(x)}\big) = \mathrm{p(s)^m}\int_0^{\infty} \mathrm{f(x)e^{-q(s)^n x}\,dx} \quad ,\mathrm{p(s)} \neq 0\ ,\mathrm{q(s)} > 0\ or\ n\ even\ positive\ integrer$$

2. g-g-trans formation :

2.1 Definition:

g-g-transform for a function f(x) where $x \in [0.\infty]$ is defined as:

$$g\big(f(x)\big) = p(s)^m\int_0^{\infty} f(x)e^{-q(s)^n x}\,dx \quad ,p(s) \neq 0\ ,q(s) > 0\ or\ n\ even\ positive\ integrer$$

2.2 Examples:

$$g - g(1) = (p(s))^m \int_0^{\infty} e^{-(q(s))^n x}\,dx = \frac{(\mathrm{p(s)})^{\mathrm{m}}}{-(\mathrm{q(s)})^{\mathrm{n}}}\mathrm{e}^{-(\mathrm{q(s)})^{\mathrm{n}}}\mathrm{x} = \frac{(\mathrm{p(s)})^{\mathrm{m}}}{(\mathrm{q(s)})^{\mathrm{n}}}$$

2.3 Examples:

$$g - g(e^{ax}) = \big(p(s)\big)^m \int_0^{\infty} e^{-(q(s))^n x}\, e^{ax}dx$$

$$= (p(s))^m \int_0^\infty e^{-(q(s)^n - a)x}\, dx$$

$$= \frac{(p(s))^m}{-(q(s)^n - a} e^{-(q(s)^n - a)x}\Big]_0^\infty = \frac{(p(s))^m}{(q(s)^n - a)}$$

2.4 Example:

$$g - g(x) = (p(s))^m \int_0^\infty e^{-(q(s))^n x}\, x dx$$

$$=(p(s))^m \left[\frac{1}{(q(s))^n} x e^{-(q(s))^n x} - \frac{1}{(q(s))^{2n}} e^{-(q(s))^n x}\right]_0^\infty = \frac{p(s)^m}{(q(s))^{2n}}$$

2.5 The following table explain g-g-transformation for some selected functions

NO	Function f(x)	$\mathbf{g - g(f(x))}$	Condition
1	$\mathbf{x^k}$	$\frac{\mathbf{(p(s))^m k!}}{\mathbf{(q(s))^{(k+1)m}}}$	**Re(q(s)) > 0**
2	**sinax**	$\frac{\mathbf{a(p(s))^m}}{\mathbf{(q(s))^{2n} + a^2}}$	**Re(q(s)) > 0**
3	**cosax**	$\frac{\mathbf{(p(s))^m (q(s))^n}}{\mathbf{(q(s))^{2n} + a^2}}$	**Re(q(s)) > 0**
4	**sinhax**	$\frac{\mathbf{(p(s))^m a}}{\mathbf{(q(s))^{2n} - a^2}}$	$\mathbf{|Re(q(s))| > \sqrt[n]{a}}$

3. g-g-transform of Differentiation

3.1 Theorem:

Let f be a continuous function on [0 , ∞ [and $\mathrm{g - g}(f(x)) = g - g(s)$, q(s)=s , n=1 , where s is a positive constant then

$$\frac{d^k}{ds^k}\, \mathrm{g - g}(f(x)) = \frac{(-1)^k}{(p(s))^m} \sum_{i=0}^{k} C_i^k (-1)^i (p(s))^{(i)}\ \mathrm{g - g}\left(x^{k-i} f(x)\right)$$

where C_i^k is combonation of k, i

Proof:

$$g-g(f(x))=(p(s))^m\int_0^\infty e^{-(q(s))^n x}f(x)dx\ ,G(f(x))|_{q(s)=s}=(p(s))^m\int_0^\infty e^{-sx}f(x)dx$$

Then by using (Leibniz formula) we get:

$$\frac{d^k}{ds^k}\ g-g(f(x)) = \sum_{i=0}^{k} C_i^k\left((p(s))^m\right)^{(i)}\left(\int_0^\infty e^{-sx}\ f(x)dx\right)^{(k-i)}$$

Now we note:

$$\left(\int_0^\infty e^{-sx}f(x)dx\right)^{(k-i)}=(-1)^{k-i}\int_0^\infty x^{k-i}e^{-sx}f(x)\,dx$$

$$=\frac{(-1)^{k-i}}{(p(s))^m}\ g-g\left(x^{k-i}f(x)\right)$$

Thus, $$\frac{d^k}{ds^k}\ g-g(f(x))=\sum_{i=0}^{k} C_i^k\left((p(s))^m\right)^{(i)}\frac{(-1)^{k-i}}{(p(s))^m}\ g-g\left(x^{k-i}f(x)\right)$$

$$\frac{d^k}{ds^k}\ g-g(f(x))=\frac{(-1)^k}{(p(s))^m}\sum_{i=0}^{k} C_i^k(-1)^i(p(s))^{(i)}\ g-g\left(x^{k-i}f(x)\right)$$

3.2 Example:

Find $g-g(xe^x)$, *such that* $p(s)=s^2$, $m=n=1$, $q(s)=s$, $f(x)=e^x$

We note that $k=1$

$$\frac{d}{ds}\ g-g(e^x)=\frac{(-1)^1}{s^2}\sum_{i=0}^{1} C_i^k(-1)^i(s^2)^{(i)}\ g-g\left(x^{1-i}e^x\right)$$

$$=\ -2\left[\frac{1}{2}g-g(xe^x)-s^{-1}g-g(e^x)\right]=\frac{d}{ds}\left(g-g(e^x)\right).$$

$$Then, 2\,s^{-1}\,g-g(e^x)-G(xe^x)\ =\frac{d}{ds}\left(g-g(e^x)\right)$$

$$2\frac{s}{s-1}\ -g-g(xe^x)\ =\frac{d}{ds}\left(\frac{s^2}{s-1}\right)$$

$$Thus, g-g(xe^{x}) = \frac{2s}{s-1} - \frac{2s(s-1)-s^2}{(s-1)^2}$$

$$= \frac{2s(s-1)-2s^2+2s-s^2}{(s-1)^2}$$

$$g-g(xe^{x}) = \frac{2s^2-2s-3s^2+2s}{(s-1)^2} = \frac{-s^2}{(s-1)^2}$$

3.2 Example:

Find $g-g(x^2 \sin x)$, $such\ that\ p(s)=s, m=2, n=1, q(s)=s$

We note $k=2$, $f(x)= sinx$

First we find $g-g(\sin x)$

that means , $k=1$

$$\frac{d}{ds}\, g-g\big(f(x)\big) = \frac{(-1)^1}{s^2}\sum_{i=0}^{1} C_i^k(-1)^i (s^2)^{(i)}\ g-g\big(x^{1-i}\, sinx\big)$$

$$[-g-g(x\sin x) + s^{-1} g-g(sinx)] = \frac{d}{ds}\big(\ g-g(sinx)\big)$$

$$\frac{2}{s}\frac{s^2}{s^2+1} - \ g-g(xsinx) = \frac{d}{ds}\left(\frac{s^2}{s^2+1}\right) = \frac{2s(s^2+1)-2s^3}{(s^2+1)^2}$$

$$= \frac{2s^3+2s-2s^3}{(s^2+1)^2} = \frac{2s}{(s^2+1)^2}$$

$$Thus, g-g(xsinx) = \frac{2s}{s^2+1} - \frac{2s}{(s^2+1)^2} = 2s\left(\frac{s^2+1-1}{(s^2+1)^2}\right) = \frac{2s^3}{(s^2+1)^2}$$

Second , we take $k=2$

$$g-g(x^2 \sin x) - 4s^{-1}\frac{2s^3}{(s^2+1)^2} + 2s^{-2}\frac{s^2}{s^2+1} = \frac{2(1-3s^2)}{(S^2+1)^3}$$

$$Thus, g-g(x^2 \sin x) = \frac{8s^2}{(s^2+1)^2} - \frac{2}{s^2+1} + \frac{2(1-3s^2)}{(s^2+1)^3}$$

$$= \frac{2s^2(3s^2-1)}{(s^2+1)^3} = \frac{2s^2(3s^2-1)}{(s^2+1)^3}$$

4. g-g- transform of integration

4.1 Theorem

Let f(x) be a piecewise function , where $x \in [\ 0\ ,\infty\ [$ and $|f(x)| \leq Me^{kx}$ and g-g(f(x))=g-g(s) exists then

$$f(x) = \mathrm{g}-\mathrm{g}^{-1}\left(\frac{1}{\left(q(s)\right)^{n}}\ g-g(f'(x))\right)+f(0)\,,$$

where $(')$ denoted the derivative

Proof:

$$Since\quad \mathrm{g}-\mathrm{g}\big(f(x)\big) = \big(p(s)\big)^{m}\int_{0}^{\infty} e^{-\left(q(s)\right)^{n}x} f(x)dx$$

$$= \ \big(p(s)\big)^{m}\left(\frac{-f(x)}{\left(q(s)\right)^{n}}\ e^{-q(s)x}|_{0}^{\infty} + \frac{1}{\left(q(s)\right)^{n}}\int_{0}^{\infty} e^{-\left(q(s)\right)^{n}x}\ f'(x)\,dx\right)$$

$$= \frac{\left(p(s)\right)^{m}}{\left(q(s)\right)^{n}}\ \int_{0}^{\infty} e^{-\left(q(s)\right)^{n}x}\ f'(x)dx + \frac{\left(p(s)\right)^{m}}{\left(q(s)\right)^{n}} f(0),$$

$$\text{Then,}\qquad f(x) = \mathrm{g}-\mathrm{g}^{-1}\left(\frac{\left(p(s)\right)^{m}}{\left(q(s)\right)^{n}}\int_{0}^{\infty} e^{-\left(q(s)\right)^{n}x}\ f'(x)dx\right)+f(0)$$

$$= \ \mathrm{g}-\mathrm{g}^{-1}\left(\frac{1}{\left(q(s)\right)^{n}}\ g-g(f'(x))\right)+f(0)$$

$$g-g^{-1}\left(\frac{1}{q(s)}\ g-g(f'(x))\right) = f(x)-f(0)$$

4.2 Example

Let $p(s) = \mathrm{s}, \mathrm{m} = 2 and\ q(s) = \mathrm{s}, \mathrm{n} = 3$

$$Find\quad \mathrm{g}-\mathrm{g}^{-1}\left(\frac{1}{s(s^{3}+1)}\right)$$

Solution:

$\mathrm{g}-\mathrm{g}^{-1}\left(\frac{1}{s(s^{3}+1)}\right) = \mathrm{g}-\mathrm{g}^{-1}\left(\frac{1}{s^{3}}\frac{s^{2}}{(s^{3}+1)}\right)$, and

$$\text{g}-\text{g}\big(f'(x)\big)=\frac{s^2}{s^3+1}$$

Since $\text{g}-\text{g}(e^{-x})=\frac{s^2}{s^3+1}$ thus $f'(x)=e^{-x}$

$$f(x)=\int e^{-x}\,dx=-e^{-x}$$

$$f(0)=-e^0=-1\ ,G^{-1}\left(\frac{1}{s(s^3+1)}\right)=-e^{-x}+1$$

4.3 Example:

Take $p(s)=s, m=1\ ,\ q(s)=\text{s}, \text{n}=2$ and

$$Find\quad \text{g}-\text{g}^{-1}\left(\frac{1}{s(s^4+1)}\right)$$

<u>**Solution:**</u>

$$\text{g}-\text{g}^{-1}\left(\frac{1}{s(s^4+1)}\right)=g-g^{-1}\left(\frac{s}{s^2}\frac{1}{s^4+1}\right)$$

Since $\text{g}-\text{g}(\sin x)=\frac{s}{s^4+1}$ $this\ implies\ that$ $f'(x)=\sin x$

$$f(x)=\int \sin x\,dx=-\cos x\quad ,\quad f(0)=-\cos 0=-1$$

$$\text{g}-\text{g}^{-1}\left(\frac{1}{s(s^4+1)}\right)=-\cos x+1$$

5.Conclusion:

In this paper we find the general formula for finding the inverse of the g-g-transformation by using g-g-tranformation for derivative of the function and then finding it by integration. Also we get g-g-transformation for some function as form $x^n f(x)$ by using g-g-transformation of the terms $x^k f(x)\ ;\ k=0,1,\dots,n-1$.

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