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Geometric Properties for a family of p −valent Holomorphic Functions with Negative Coefficients for Operator on Hilbert Space

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Abstract

The purpose of the present investigation is to introduce and study a certain subclass $\mathcal{A}k_p(\alpha,\beta,\delta,T)$ of p-valent holomorphic functions with negative coefficients of the operators on Hilbert space in U. Moreover, we get a number of geometric properties.

Mathematics Subject Classification: 30C45, 30C50.

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1 Introduction

Let $\mathcal{A}_p$ be the class of functions f of the form:

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{n+p} z^{n+p} \quad (p \in \mathbb{N} = \{1,2,\dots\}), \qquad (1.1)$$

which are holomorphic and p-valent in the open unit disk $U = \{z \in \mathbb{C}: |z| < 1\}$.

Let k_p denote the subclass of $\mathcal{A}_p$ consisting of functions of the form:

$$f(z) = z^p - \sum_{n=1}^{\infty} a_{n+p} z^{n+p} \quad (a_{n+p} \geq 0,\ p \in \mathbb{N} = \{1,2,\dots\}). \qquad (1.2)$$

Definition 1.1: *A function $f \in k_p$ is said to be in the class $\mathcal{A}k_p(\alpha,\beta,\delta)$ if it satisfies*

$$\left| \frac{f'(z) - p z^{p-1}}{\alpha(f'(z) - \beta) + p - \beta} \right| < \delta,$$

where $0 \leq \alpha < 1,\ 0 \leq \beta < p, 0 < \delta \leq 1$ *and* $z \in U$.

Let H be a Hilbert space on the complex field. Let T be a linear operator on H. For a complex holomorphic function f on the unit disk U, we denoted $f(T)$, the operator on H defined by the usual Riesz-Dunford integral [2]

$$f(T) = \frac{1}{2\pi i} \int_c f(z)(zI - T)^{-1}\, dz,$$

where I is the identity operator on H, c is a positively oriented simple closed rectifiable contour lying in U and containing the spectrum $\sigma(T)$ of T in its interior domain [3]. Also $f(T)$ can be defined by the series

$$f(T) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} T^n,$$

which converges in the norm topology [4].

Definition 1.2: *Let H be a Hilbert space and T be an operator on H such that such that $T \neq \emptyset$ and $\|T\| < 1$. Let α, β be real numbers such that $0 \leq \alpha < 1,\ 0 \leq \beta < p, 0 < \delta \leq 1$. An holomorphic function f on the unit disk is said to belong to the class $\mathcal{A}k_p(\alpha,\beta,\delta,T)$ if it satisfy the inequality*

$$\|f'(T) - pT^{p-1}\| < \delta \|\alpha(f'(T) - \beta) + p - \beta\|,$$

where $\emptyset$ denote the zero operator on H.

The operator on Hilbert space were consider recently be Xiaopei [8], Joshi [6], Chrakim et al. [1], Ghanim and Darus [5] and Selvaraj et al. [7].

2 Main Results

Theorem 2.1: *Let $f \in k_p$ be defined by (1.2). Then $f \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$ for all $T \neq \emptyset$ if and only if*

$$\sum_{n=1}^{\infty} (n+p)(1+\delta\alpha) a_{n+p} \leq \delta(p-\beta)(1+\alpha). \qquad (2.1)$$

where $0 \leq \alpha < 1,\ 0 \leq \beta < p, 0 < \delta \leq 1$.
The result is sharp for the function f given by

$$f(z) = z^p - \frac{\delta(p-\beta)(1+\alpha)}{(n+p)(1+\delta\alpha)} z^{n+p},\quad n \geq 1. \qquad (2.2)$$

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Proof: Suppose that the inequality (2.1) holds. Then, we have

$$\|f'(T)-pT^{p-1}\| - \delta\|\alpha(f'(T)-\beta)+p-\beta\| = \left\|-\sum_{n=1}^{\infty}(n+p)\,a_{n+p}T^{n+p-1}\right\| - \delta\left\|\alpha pT^{p-1} - \sum_{n=1}^{\infty}\alpha(n+p)\,a_{n+p}T^{n+p-1} + p-\beta(1+\alpha)\right\|$$
$$\leq \sum_{n=1}^{\infty}(n+p)\,(1+\delta\alpha)a_{n+p} - \delta(p-\beta)(1+\alpha) \leq 0.$$

Hence, $f \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$.
To show the converse, let $f \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$. Then

$$\|f'(T)-pT^{p-1}\| < \delta\|\alpha(f'(T)-\beta)+p-\beta\|,$$

gives

$$\left\|-\sum_{n=1}^{\infty}(n+p)\,a_{n+p}T^{n+p-1}\right\| < \delta\left\|\alpha pT^{p-1} - \sum_{n=1}^{\infty}\alpha(n+p)\,a_{n+p}T^{n+p-1} + p-\beta(1+\alpha)\right\|$$

Setting $T=rI\ (0<r<1)$ in the above inequality, we get

$$\frac{\sum_{n=1}^{\infty}(n+p)\,a_{n+p}r^{n+p-1}}{\alpha pr^{p-1}-\sum_{n=1}^{\infty}\alpha(n+p)\,a_{n+p}r^{n+p-1}+p-\beta(1+\alpha)} < \delta. \qquad (2.3)$$

Upon clearing denominator in (2.3) and letting $r \longrightarrow 1$, we obtain

$$\sum_{n=1}^{\infty}(n+p)\,a_{n+p} < \delta(p-\beta)(1+\alpha) - \sum_{n=1}^{\infty}\delta\alpha(n+p)\,a_{n+p}.$$

Thus

$$\sum_{n=1}^{\infty}(n+p)\,(1+\delta\alpha)a_{n+p} \leq \delta(p-\beta)(1+\alpha),$$

which completes the proof.

Corollary 2.1: *If $f \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$, then*

$$a_{n+p} \leq \frac{\delta(p-\beta)(1+\alpha)}{(n+p)(1+\delta\alpha)}\,,\quad n \geq 1.$$

Theorem 2.2: *If $f \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$ and $\|T\|<1, T \neq \emptyset$, then*

$$\|T\|^p - \frac{\delta(p-\beta)(1+\alpha)}{(p+1)(1+\delta\alpha)}\|T\|^{p+1} \leq \|f(T)\| \leq \|T\|^p + \frac{\delta(p-\beta)(1+\alpha)}{(p+1)(1+\delta\alpha)}\|T\|^{p+1}$$

and

$$p\|T\|^{p-1} - \frac{\delta(p-\beta)(1+\alpha)}{1+\delta\alpha}\|T\|^p \leq \|f'(T)\| \leq p\|T\|^{p-1} + \frac{\delta(p-\beta)(1+\alpha)}{1+\delta\alpha}\|T\|^p.$$

The result is sharp for the function f given by

$$f(z) = z^p - \frac{\delta(p-\beta)(1+\alpha)}{(p+1)(1+\delta\alpha)}z^{p+1}\,.$$

Proof: According to the Theorem 2.1, we get

$$\sum_{n=1}^{\infty}a_{n+p} \leq \frac{\delta(p-\beta)(1+\alpha)}{(p+1)(1+\delta\alpha)}.$$

Hence

$$\|f(T)\| \geq \|T\|^p - \sum_{n=1}^{\infty}a_{n+p}\,\|T\|^{n+p} \geq \|T\|^p - \|T\|^{p+1}\sum_{n=1}^{\infty}a_{n+p} \geq \|T\|^p - \frac{\delta(p-\beta)(1+\alpha)}{(p+1)(1+\delta\alpha)}\|T\|^{p+1}.$$

Also,

$$\|f(T)\| \leq \|T\|^p + \sum_{n=1}^{\infty} a_{n+p}\,\|T\|^{n+p} \leq \|T\|^p + \frac{\delta(p-\beta)(1+\alpha)}{(p+1)(1+\delta\alpha)}\|T\|^{p+1}.$$

In view of Theorem 2.1, we have

$$\sum_{n=1}^{\infty}(n+p)a_{n+p} \leq \frac{\delta(p-\beta)(1+\alpha)}{1+\delta\alpha}.$$

Thus

$$\|f'(T)\| \geq p\|T\|^{p-1} - \sum_{n=1}^{\infty}(n+p)a_{n+p}\,\|T\|^{n+p-1} \geq p\|T\|^{p-1} - \|T\|^p\sum_{n=1}^{\infty}(n+p)a_{n+p} \geq p\|T\|^{p-1} - \frac{\delta(p-\beta)(1+\alpha)}{1+\delta\alpha}\|T\|^p$$

and

$$\|f'(T)\| \leq p\|T\|^{p-1} + \|T\|^p\sum_{n=1}^{\infty}(n+p)a_{n+p} \leq p\|T\|^{p-1} + \frac{\delta(p-\beta)(1+\alpha)}{1+\delta\alpha}\|T\|^p.$$

Therefore the proof is complete.

Theorem 2.3: *Let $f_0(z) = z^p$ and*

$$f_n(z) = z^p - \frac{\delta(p-\beta)(1+\alpha)}{(n+p)(1+\delta\alpha)}z^{n+p}, \quad n \geq 1.$$

Then $f \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$ if and only if it can be expressed in the form

$$f(z) = \sum_{n=0}^{\infty}\lambda_n f_n(z), \qquad (2.4)$$

where $\lambda_n \geq 0$ and $\sum_{n=0}^{\infty}\lambda_n = 1$.

Proof: Assume that f can be expressed by (2.4). Then, we have

$$f(z) = \sum_{n=0}^{\infty}\lambda_n f_n(z) = z^p - \sum_{n=0}^{\infty}\frac{\delta(p-\beta)(1+\alpha)}{(n+p)(1+\delta\alpha)}\lambda_n z^{n+p}.$$

Thus

$$\sum_{n=0}^{\infty}\frac{(n+p)(1+\delta\alpha)}{\delta(p-\beta)(1+\alpha)}\frac{\delta(p-\beta)(1+\alpha)}{(n+p)(1+\delta\alpha)}\lambda_n = \sum_{n=0}^{\infty}\lambda_n = 1-\lambda_0 \leq 1,$$

and so $f \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$.

Conversely, suppose that f given by (1.2) is in the class $\mathcal{A}k_p(\alpha,\beta,\delta,T)$. Then by Corollary 2.1, we have

$$a_{n+p} \leq \frac{\delta(p-\beta)(1+\alpha)}{(n+p)(1+\delta\alpha)}.$$

Setting

$$\lambda_n = \frac{(n+p)(1+\delta\alpha)}{\delta(p-\beta)(1+\alpha)}a_n, \quad n \geq 1,$$

and $\lambda_0 = 1 - \sum_{n=1}^{\infty}\lambda_n$. Then

$$f(z) = \sum_{n=0}^{\infty}\lambda_n f_n(z).$$

This completes the proof of the theorem.

Theorem 2.4: *The class $\mathcal{A}k_p(\alpha,\beta,\delta,T)$ is a convex set.*

Proof: Let f_1 and f_2 be the arbitrary elements of $\mathcal{A}k_p(\alpha,\beta,\delta,T)$. Then for every t $(0 \leq t \leq 1)$, we show that $(1-t)f_1 + tf_2 \in \mathcal{A}k_p(\alpha,\beta,\delta,T)$. Thus, we have

$$(1-t)f_1 + tf_2 = z^p - \sum_{n=1}^{\infty}\Big((1-t)a_{n+p} + tb_{n+p}\Big)z^{n+p}.$$

Hence

$$\sum_{n=1}^{\infty}(n+p)(1+\delta\alpha)\Big((1-t)a_{n+p} + tb_{n+p}\Big) = (1-t)\sum_{n=1}^{\infty}(n+p)(1+\delta\alpha)a_{n+p} + t\sum_{n=1}^{\infty}(n+p)(1+\delta\alpha)b_{n+p} \leq (1-t)\delta(p-\beta)(1+\alpha) + t\delta(p-\beta)(1+\alpha).$$

This completes the proof.

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خواص هندسية لعائلة من الدوال متعددة التكافؤ ذات معاملات سالبة لمؤثر على فضاء هلبرت

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المستخلص :

الغرض من العمل الحالي هو تقديم ودراسة صنف جزئي مؤكد $\mathcal{A}k_p(\alpha,\beta,\delta,T)$ من الدوال متعددة التكافؤ ذات معاملات سالبة لمؤثرات على فضاء هلبرت في U . علاوة على ذلك حصلنا على عدد من الخواص الهندسية.

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A New Electronic Voting Protocol Using Secret Sharing Based on Set of Path Domination

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Abstract

For secret electronic voting protocol we always need a secure system. Secret sharing scheme are proved to be safe guarding through distributing the input key to number of participants then reconstructing the shares through a secure process. In this paper, the set of dominating paths in special graph is used to share the votes among a set of candidates, such that, each candidate represents an edge in the graph. Based on this new technique, we have shown that it is secure and confidential. The efficiency of the new protocol is demonstrated in terms of time and cost.

Keywords: Electronic voting systems; secret sharing scheme; set of dominating path.

Mathematics subject classification: 94-xx, 05-xx.

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1. Introduction

The theory of domination is considered as one of the major research area in graph theory. Historically, the first domination type problems came from chess. In 1958, Berge[1] was the first how introduced the concept of domination in graph. After that this concept takes high popularity by many mathematicians and scientists, and it is utilized in many applications. Various types of domination of a graph have been defined and studied by many researchers. In the appendix of Haynes [2] more than 75 models are listed.

Technical systems like computer communication network, radio station and traffic management system have a structure like a graph. These systems needs many requirements to achieve the security, speed, accuracy and privacy, for example, we need a minimum number of computers to control the communication network, these can be performed through the dominating set in grapg. Electronic voting system is one of the technical systems that required many efforts to minimize the cost and time while maintaining security and confidentiality, all these objective are achieved by secret sharing schemes (SSS).

Secret sharing was introduced in 1979 by Shamir [3] and Blakley [4], it has important applications in cryptography as a protocol. A secret sharing scheme is a method to keep safe a secret value (key) by partitioned it into shares and distributes it among several participants in such a way that only confirmed qualified subsets of participants can regain the secret by pooling their shares together. This distribution increases the safety, reliability, security and convenience.

In 1987, Benelux[5] was introduce the first E-voting system that based on secret sharing scheme. E-voting systems are fundamentally different but any system should guarantee the privacy and security to protect the confidential data. In other words, it must be guaranteed that no one can discover the identity of the voter .

During twenty years ago, various techniques are used in E-voting systems that based on multiple key cipher[6], secret sharing technique and zero knowledge protocol[7], publicly verifiable secret sharing scheme[8], chinese remainder theorem[9] and some other techniques. Recently, many researchers pay attention to E-voting systems that based on secret sharing scheme and discrete logarithm problem[10]. In 2014, Pan et al.[11] introduced an improved scheme in the same field with high privacy and confidentiality.

The classical approaches that based on graph theory used to consider the set of vertices in the graph as a set of participants. In 2016, Al saidi et al.[12] proposed a new system based on secret sharing scheme that depends on the edge dominating sets as an access structure by representing the participants as a set of edges in the graph.

In this work, secret sharing scheme that based on path dominating set in a given graph is used to design a new secure E-voting system, such that, the voter's identity is protected, where each casted vote is divided into shares to be distributed to multiple parties and each vote is represented as bitwise pattern according to number of voters. The random share given to each participant according to one element in the minimum path dominating set, without giving any information about the personality of the voter, that resulted in a perfectly secure system. The proposed protocol of electric voting is based on secret sharing scheme depending on the set of minimum dominating paths, such that, this system is represented as a special graph (C_n) which is a cycle of order n, where each candidate (participant) represents an edge in the graph.

This paper includes four additional sections ordered as follows: section 2, contains basic concepts in graph theory and secret sharing scheme; in section 3, the proposed system is introduced; implementation and analysis is given in section 4; finally, our work is concluded in section 4.

2. Basic concepts

A major concepts regarding to topics presented in this work are abstracted as an overview in this section, for more details, we refer the reader to see [1, 13].

a) Graph theory

Let $G(V,E)$ be a finite, undirected, simple, connected graph. The order and the size of G is the number of vertices V (i.e. $|G|=n$), and the number of edges E respectively. The degree of a vertex v in G is the number of edges incident on v denoted by $deg(v)$. When all vertices of the graph has the same degree, it is called regular, otherwise it is irregular. An open neighbor set, $N(u)=\{v|(u,v)\in E\}$, is the set of vertices that are neighbor to u. A closed neighbor set, $N[u]=N(u)\cup\{u\}$, is the set of neighbors of u in addition to u itself. A path is an alternative sequence of vertices and edges, beginning at a vertex and ending at another one, and it doesn't visit any vertex more than one time. A cycle is just like a path except that is starts and ends at the same vertex. The length of a path (or cycle) is defined as the number of edges in it. A cycle graph with n vertices is a graph consists of a single cycle and denoted by C_n. A subset H of a graph G is denoted by $H\leq G$ with $V(H)\subseteq V(G)$ and $E(H)\subseteq E(G)$.

A subset D of vertices in a graph G is a dominating set if every vertex not in D has a neighbor in D. if the subgraph induced by D is connected, then D is called a connected dominating set. A path P is called dominating path if every vertex outside P has a neighbor in P. There is an efficient algorithm for finding the set of dominating paths in a graph [14].

b) Secret sharing scheme

In secret sharing scheme the secret information is distributed and shared among the participants in such a way that only appointed sets of participants can reconstructed the secret, besides that, no one of them has any information about the secret S. For more details see [15].

In the domain of secret sharing scheme that based on graph access structure we'll introduce two types which are:

i. ***Sun et al. Scheme [16]: It is summarized as follows:***

1) Let P= $\{p_1,p_2,\ldots,p_n\}$ be a set of participants and Γ is a uniform access structure of rank m on those participants, where Γ_0 is the basis of Γ.

2) The decomposition of Γ_0 is Γi's, for $1\leq i\leq n$, where $\boldsymbol{\Gamma_i} = \{\boldsymbol{X}: \boldsymbol{X} \in \boldsymbol{\Gamma_0}\ \boldsymbol{and}\ \boldsymbol{p_i} \in \boldsymbol{X}\}$. Thus, $\boldsymbol{\Gamma} = \boldsymbol{cl}(\boldsymbol{\Gamma_0}) = \boldsymbol{cl}(\boldsymbol{\Gamma_1}) \cup \ldots \cup \boldsymbol{cl}(\boldsymbol{\Gamma_n})$, then $\boldsymbol{\Gamma_i^*} = \{\boldsymbol{X}: \boldsymbol{X} \cup \{\boldsymbol{p_i}\} \in \boldsymbol{\Gamma_i}\}$ is defined, where each $cl(\Gamma i^*)$ is a uniform access structure of rank m -1. The secret $K=(k_1,k_2,\ldots,k_m\}$, where each k_i, $1\leq i\leq m$ is taken randomly over $GF(q^{h(m-1)})$, which is considered as the space of the secret.

3) A polynomial f(x) of degree m.h(m -1)-1 with coefficients K is selected by a dealer to compute y_i=f(i-1)mod q, for i=1,…n.h(m -1). If one has no knowledge of any y_i, no information about the secret can be obtained.

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4) Random numbers $r_1, r_2, \ldots, r_n$ are also selected by the dealer over $GF(q^{h(m-1)})$. They presumed that, there exists a secret sharing scheme realizing $cl(\Gamma_i^*)$, such that, the secret is r_i+y_i and the share of participant p_j is $S_j(\Gamma_i^*)$, which is given by: $S_i = \langle R_i, S_i(\Gamma_1^*), \ldots, S_i(\Gamma_{i-1}^*), S_i(\Gamma_{i+1}^*), \ldots, S_i(\Gamma_n^*)\rangle$. The reconstruction of the secret is done when the authorized participants collect their share together.

ii. Al saidi et al. Scheme[17, 18]: It is summarized as follows:

1) The set of vertices $V=\{v_1,v_2,\ldots,v_n\}$ in graph G corresponds to the set of participants $P=\{p_1,p_2,\ldots,p_n\}$, while minimum access structure Γ_0 is represented by the minimum dominating set of vertices (*MID*)
2) The graph G is decomposed into n–subgraphs $G_i = (V_i, E_i)$, i=1,2,…,n, where $V_i = \{V \backslash N[v_i]\}$. The set Γ_0 is also decomposed into n Γ_i's where $\Gamma_i = \{MID \in \Gamma_0,\ where\ p_i \in MID\}$ and the set $\Gamma_i^* = \{X: X \cup \{p_i\} \in \Gamma_i\}$.
3) The coefficients of the polynomial $f(x) = (k_1x^{m-1} + k_2x^{m-2} + \cdots + k_m)$ are chosen randomly over $GF(q^{(m-1)!})$ and used to represent the secret $K=(k_1,k_2,\ldots,k_m\}$.
4) The secret K can be reconstructed by getting m or more y_i's, where y_i's are computed using $y_i=f(i)$ mod q, i=1,2,…,n, and the share for each participant p_i is calculated after selecting r random numbers $r_1,r_2,\ldots,r_n$ by the dealer such that: $S_i = \langle r_i, S_i(\Gamma_1^*), \ldots, S_i(\Gamma_{i-1}^*), S_i(\Gamma_{i+1}^*), \ldots, S_i(\Gamma_n^*)\rangle$. When the authorized participants pool their share together, the secret can be reconstructed.

Fuad et al. [14] proposed an optimal scheme based on minimum set of dominating path in cycle graph C_n. That scheme represented the minimum access structure Γ_0 by the minimum set of dominating paths in C_n. Based on this contribution, an efficient electronic voting protocol is proposed in this work.

3. E – Voting system

The electoral process is considered as one of the important and sensitive operations that attract many researchers to work on. The proposed system for E-voting focus on choosing the access structure Γ_0 to achieve high security. For this purpose, a secret sharing scheme based on minimum path dominating set of a graph C_n is used, and an algorithm that works for m candidate and n voters is designed also which is works on a bitwise-pattern representation votes.

The access structure Γ_0 is a set consist of n dominating paths each of length (n-3) for more details see [14]. The electoral process using the proposed system can be summarized in the following algorithm in three steps:

Algorithm 1: The proposed E-voting system

Input: m=the number of candidates, n=the number of voters, Input $k \in GF(p^2)$

Output: The number of votes for each candidate

Step1: key generation:

1-Take the number of candidates m and the number of voters n

2- Find Γ_0 directly using theorem 2 in [14] after representing each candidate as a vertex in graph C_m, so we have m sets each has (m-3) candidates.

All computation is done over $GF(p)$, where p is a prime, $p \geq m$

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Step2: The decomposition:

1- Construct the code for each candidate, which is represented as bitwise pattern according to the number of voters and the number of candidates.
2- Chose a value of k over $GF(p^2)$ to encoding the votes.
3- Construct the polynomial $f(x) = kx + v_i$, where v_i is the value of the vote.

Step3: The reconstruction:

1- Compute the shares for all candidates by $y_{ij}=f(j)$ for each voter, then send them to the collection center (CC).
2- Find the sum SCC_j of collection center, then apply Lagrange interpolation on any dominating set belongs to Γ_0 to obtain $f(x)$.
3- The number of votes for each candidate is founded from the constant term of the polynomial $f(x)$.

Example:

Let m=5 and n=7 so the related graph will be C_5 shown in Figure 1, then apply path dominating algorithm to find $P_1=\{e_1,e_2\}$, $P_2=\{e_2,e_3\}$, $P_3=\{e_3,e_4\}$, $P_4=\{e_4,e_5\}$, $P_5=\{e_5,e_6\}$

Hence Γ_0= $=\{P_1,P_2,\ldots,P_5\}$

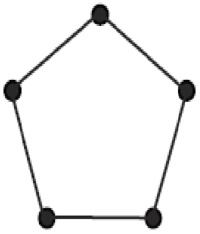

Figure 1: The cycle Graph C_5

Let $k = 13 \in GF(p^2)$, where $p \geq m$.
$f(x) = kx + v_i = 13x + v_i$, where v_i is the value of the vote for candidate i. Since we have 7 candidates, then we need at least 3 bit to represent code for each candidate. The 21 bit vote pattern $a_{20}a_{19}a_{18} \ldots a_3a_2a_1a_0$ is initially set to 0. When a voter votes for candidate 1, bit a_0 is set to 1, similarly for candidate 2, bit a_3 is set to 1 and so on for candidate 7, bit a_{18} is set to 1.

$$y_{i,j} = f(j)\,, CC_j = y_{ij}.$$

Now if the first voter votes for candidate 1, then $v_1 = 1$.

$$y_{1,1} = f(1) = 13(1) + 1 = 14 \Rightarrow CC_1 = 14.$$
$$y_{1,2} = f(2) = 13(2) + 1 = 27 \Rightarrow CC_2 = 27.$$
$$y_{1,3} = f(3) = 13(3) + 1 = 40 \Rightarrow CC_3 = 40.$$
$$y_{1,4} = f(4) = 13(4) + 1 = 53 \Rightarrow CC_4 = 53.$$
$$y_{1,5} = f(5) = 13(5) + 1 = 66 \Rightarrow CC_5 = 66.$$

If voter 2 votes for candidate 3, then $v_2 = 64$, and $y_{2,1}$ to $y_{2,5}$ are computed

If voter 3 votes for candidate 1, then $v_3 = 1$, and, $y_{3,1}$ to $y_{3,5}$ are computed.

If voter 4 votes for candidate 2, then $v_4 = 8$, and $y_{4,1}$ to $y_{4,5}$ are computed.

If voter 5 votes for candidate 3, then $v_5 = 64$, and $y_{5,1}$ to $y_{5,5}$ are computed.

If voter 6 votes for candidate 3, then $v_6 = 64$, and $y_{6,1}$ to $y_{6,5}$ are computed.

If voter 7 votes for candidate 4, then $v_7 = 512$, and $y_{7,1}$ to $y_{7,5}$ are computed.

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Now, to find the sum of all collection centers which is denoted by SCC_j, we have:

$SCC_1 = 805$, $SCC_2 = 896$, $SCC_3 = 987$, $SCC_4 = 1078$, $SCC_5 = 1169$.

Let, the qualified subset from Γ_0 is $A = \{e_1, e_2\}$. Then by applying Lagrange interpolation on the set A, we have the following polynomial

$$f(x) = \frac{805(x-2)}{(1-2)} + \frac{896(x-1)}{(2-1)}$$
$$= 91x + 714$$

Decoding the constant term 714 in binary, we obtain, 000 001 011 001 010. Each 3 bit represents the vote's number for the candidates respectively.

4. Implementation and analysis

The implementation of the algorithm is done in matlab. A flowchart for algorithm 1 is introduced in Figure 2. Table 1 gives some details about the construction of the code for each candidate and the calculation of theirs values v_i. Table 2 shows the shares generation for each voter. All results are based on example 1. In Table 3, shows comparison explains the running time required to election with different number of voters and same number of candidates:

Table 1: Representation of votes

Candidate	**Candidate's code (Bitwise representation)**	**Vote's value (v_i)**
1	000 000 000 000 001	2^0=1
2	000 000 000 001 000	2^3=8
3	000 000 001 000 000	2^6=64
4	000 001 000 000 000	2^9=512
5	001 000 000 000 000	2^{12}=4096

Table 2: Share Generation

Collection center	*CC 1*	*CC 2*	*CC 3*	*CC4*	*CC5*
Voter1	14	27	40	53	66
Voter2	77	90	103	116	129
Voter3	14	27	40	53	66
Voter4	21	34	47	60	73
Voter5	77	90	103	116	129
Voter6	77	90	103	116	129
Voter7	525	538	551	564	577
SCC	805	896	987	1078	1169

Table 3: Running time (1)

candidates	voters	Running time
5	7	0:0:001
5	10	0:0:001
5	50	0:0:002
5	100	0:0:002

Table 3: Running time (2)

candidates	voters	Running time
5	50	0:0:001
10	50	0:0:001
20	50	0:0:002
30	50	

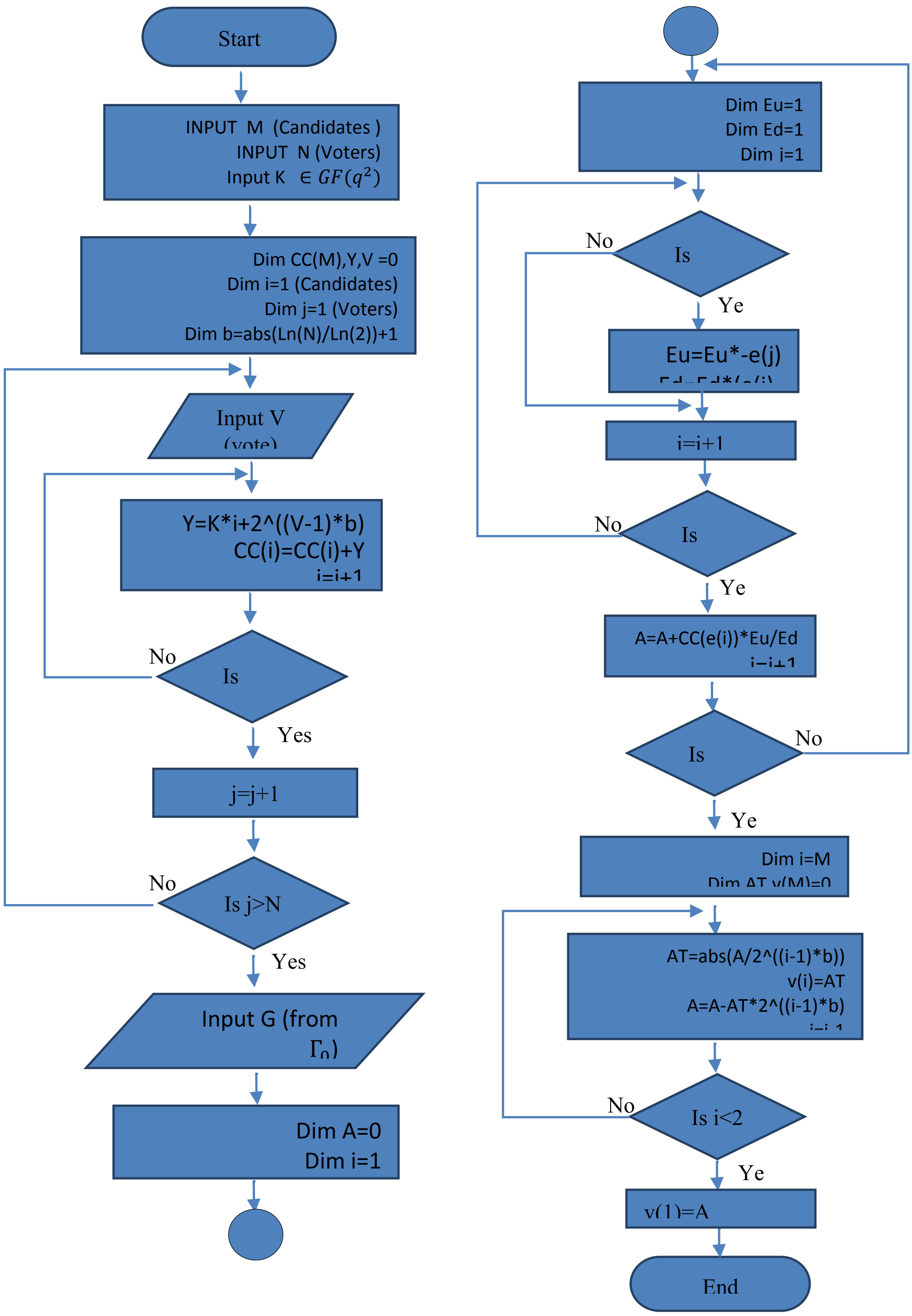

Figure 2: E-voting flowchart

5. Conclusion

In this paper the new E- voting system depends on secret sharing scheme is proposed, where a set of dominating paths in special graph is used to share the votes among a set of candidates. The efficiency of the system depends on the number of voters, as well as, it is active when the number of candidates is not large that decrease the running time and cost. To provide more security, the shares can be sent to the collection center using different secure channels.

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بروتوكول تصويت الكتروني جديد باستخدام نظام مشاركة السرية بالاعتماد على مجموعة الدروب المهيمنة

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المستخلص :
في بروتوكولات التصويت الإلكتروني السري نحن دائما بحاجة إلى نظام آمن. وقد ثبت أن نظام مشاركة السرية يوفر حماية جيدة من خلال توزيع مفتاح الإدخال على عدد من المشاركين ثم إعادة بناءه من خلال عملية آمنة. في هذا البحث تم استخدام مجموعة الدروب المهيمنة في بيان معين لمشاركة الأصوات بين مجموعة من المرشحين، بحيث يمثل كل مرشح حافة في ذلك البيان. استناداً على هذا البروتوكول الجديد برهنّا انه مأمون وسري. وقد تم اثبات كفاءة البروتوكول الجديد من خلال عامل الوقت والكلفة.

الكلمات المفتاحية: انظمة التصويت الألكتروني . نظام مشاركة السرية ، مجموعة الدروب المهيمنة.

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On the(G. n)_ Tupled fixed point theorems in fuzzy metric spacea

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Abstract

The porpose of this paper is to introduce a new concepts of (G. n)-tupled fixed point and (G. n)- tupled coincidence point. And, to study the existence of tupled fixed (coincidence) point for any type of mappings. We will also establish some convergence theorems to a unique (G. n) – tupled fixed (coincidence) point in the complete fuzzy metric spaces.

Keywords: fuzzy metric spaces, continuous t – norm , fixed point, upper semi – continuous, equicontinuous.

Subject classification: 46S40 .

1. Introduction

In [1] , Kamosil and Mchalek introduced the concept of fuzzy metric spaces(F. M. S). the existence of fixed points for mappings in fuzzy metric spaces studied by Gregri and Sapea [2], Mihet [3].The Fixed point theory for contractive mappings in fuzzy metric spaces is associated the fixed point theory for the same type of mappings in probabilistic metric space of menger type see, Qlu and Hong[4], Hong and Peng[5], Mohudine and Alotibietal [6], Wang [7], Hong [8], Sadtietal [9] ,[10] and many others. Zhand and Xiao[11] and Hu[12] introduced a coupled fixed point theorem for. In this paper, we introduce the concepts of (G. n) – tupled fixed (coincidence) point and we establish the (G. n)– tupled fixed (coincidence) point theorems in fuzzy metric spac es .

Now, we recall the following:

Definition (1.1) [3]

A binary operation $\aleph : [0,1]^2 \to [0,1]$ is called a continuous t– norm if the following conditions are satisfy:

i. $\aleph$ is a associative and commutative.
ii. $a\aleph 1 = a \quad \forall a \in [0,1]$.
iii. $a\aleph b \le c\aleph d$ whenever $a \le c \ \& \ b \le d$, $\forall\, a, b, c, d \in [0,1]$.
iv. $\aleph$ is continuous.

And denoted by (c. t. n)

Definition (1.2)[4]

A triple $(F, \mathcal{G}, \aleph)$ is called fuzzy metric space (F. M. S) if $X \neq \emptyset$, $\aleph$ is continuous t - norm and $\mathcal{G}: F \times F \times (0, \infty) \to [0,1]$ is a fuzzy set the satisfying the following conditions.

i. $\mathcal{G}_{(x,y,t)} > 0$
ii. $\mathcal{G}_{(x,y,t)} = 1 \ iff \ x = y$
iii. $\mathcal{G}_{(x,y,t)} = \mathcal{G}_{(y,x,t)}$
iv. $\mathcal{G}_{(x,y,.)}: (0, \infty) \to [0,1]$ is continuous.
v. $\mathcal{G}_{(x,z,t+s)} \ge \mathcal{G}_{(x,y,t)} \aleph \mathcal{G}_{(y,z,s)} \ \forall\, t, s > 0$."

Now, we will add the condition $\lim_{t\to\infty} \mathcal{G}_{(x,y,t)} = 1 \quad \forall x, y \in F$.

Lemma (1.3) [3]

In any fuzzy metric space $(F, \mathcal{G}, \aleph)$, where $\aleph$ is (c. t. n)If there exits $\Delta \in$ ℃ such that $\mathcal{G}_{(x,y,\emptyset(t))} \le \mathcal{G}_{(x,y,t)}$, $\forall\, t > 0$ then $x = y$.

Definition(1.4) [9]

For any $v \in [0,1]$, the sequence $< \aleph^n v >^{\infty}{}_{n=1}$ be defined by:
$\aleph^1 v = v \ and \ \aleph^n v = (\aleph^{n-1} v)\aleph v$. Then a t – norm $\aleph$ is said to be (c. t. n)of $\mathcal{H} - \mathcal{T}$ype if the sequence $< \aleph^n v >^{\infty}{}_{n=1}$ is equicontinuous at $v = 1$.

Definition (1.5) [13]

Let $(F, \mathcal{G}, \aleph)$ be a(F. M. S) then

i. A sequence in (v_n) in X is said to be convergent to a point $v \in X$ if $\lim_{t\to\infty} \mathcal{G}_{(v_n,v,t)} = 1$ for all $t > 0$.
ii. A sequence in (v_n) in X is called a Cauchy sequence if for each $0 < \varepsilon < 1 \ and \ t > 0$,there exists a positive integer n_0 such that $\mathcal{G}_{x_n,v_m,t)} > 1 - \varepsilon$ for each $n, m \ge n_0$."

Now we will give the concept of $(G.n)$_ tupled fixed(coincidence) point.

Definition (1.6)

Let $Z_1, Z_2, \dots\dots, Z_n : F^n \to F$ are mappings. Any element $(x_1, x_2, \dots\dots, x_n) \in F^n$ is called a $(G.n)$_ tupled fixed point of this mappings if

$$Z_1\left(\left(Z_2(\dots\dots(Z_{n(\ x_1, x_2, \dots, x_n\ )})\dots..)\right)\right) = x_1$$
$$Z_1\left(\left(Z_2(\dots\dots(Z_{n\ (x_2, x_3, \dots, x_1)}\ )\dots..)\right)\right) = x_2$$
$$\vdots$$
$$Z_1\left(\left(Z_2(\dots\dots(Z_{n(\ x_n, x_1, \dots, x_{n-1}\ )})\dots..)\right)\right) = x_n.$$

Definition (1.7)

Let $Z_1, Z_2, \dots\dots, Z_n : F^n \to F$ and $E_1, E_2, \dots\dots, E_n : F \to F$ are mappings. Any element $(x_1, x_2, \dots\dots, x_n) \in F^n$ is called $(G.n)$ – tupled coincidence point of this mapping if

$$Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots..\right)\right) = E_1\left(E_2\left(\dots\dots\left(E_{n(x_1)}\right)\dots..\right)\right)$$
$$Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_2,x_3,\dots,x_1)}\right)\dots..\right)\right) = E_1\left(E_2\left(\dots\dots\left(E_{n(x_2)}\right)\dots..\right)\right)$$
$$\vdots$$
$$\mathcal{F}_1\left(\mathcal{F}_2\left(\dots\dots\left(\mathcal{F}_{n(x_n,x_1,\dots,x_{n-1})}\right)\dots..\right)\right) = \mathcal{P}_1\left(\mathcal{P}_2\left(\dots\dots\left(\mathcal{P}_{n(x_n)}\right)\dots..\right)\right).$$

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In this paper ,we consider ℃ is the set of all functions $\Delta: [0,\infty) \rightarrow [0,\infty)$ such that:
Δ is increasing function.
Δ is upper semi – continuous .
$\sum_{n=0}^{\infty} \Delta^n{}_{(t)} < \infty$; $\forall t > 0$ where $\Delta^{n+1}{}_{(t)} = \Delta(\Delta^n{}_{(t)})$, $n \in N$.

2.Main Results

Now, we establish the convergence theorems to a unique $(G.n)$ – tupled fixed point as follows :

Theorem (2.1): Let $Z_1, Z_2, \ldots\ldots, Z_n: F^n \rightarrow F$ and let $(F, \mathcal{G}, \aleph)$ be a complete (F.M.S) such that $\aleph$ (c.t.n) of $\mathcal{H} - \mathcal{T}ype$ Suppose that $\Delta \in$ ℃ satisfying:

$$\mathcal{G}[Z_1\left(\left(Z_2(\ldots\ldots(Z)\ldots..)\right)\right),$$
$$Z_1\left(\left(Z_2(\ldots\ldots(Z)\ldots..)\right)\right), \Delta_{(t)}] \geq$$
$$\mathcal{G}[x_1, y_1, t]\aleph\mathcal{G}[x_2, y_2, t]\aleph\ldots\ldots\aleph\mathcal{G}[x_n, y_n, t] \qquad (2.1)$$

where $t > 0$ and $x_i, y_i \in F$, $\forall i = 1,2, \ldots\ldots, n$

If F containing $Z_1\left(\left(Z_2(\ldots\ldots(Z_{n(x^n)})\ldots..)\right)\right)$.

Then there exists a unique $(G.n)$ – tupled fixed point of compose these mappings.

Proof:

Suppose that $x_0{}^1, x_0{}^2, \ldots\ldots, x_1{}^n \in F$, since F containing $Z_1\left(\left(Z_2(\ldots\ldots(Z_{n(x^n)})\ldots..)\right)\right)$, that there exists $x_1{}^1, x_1{}^2, \ldots\ldots, x_1{}^n \in F$ such that

$$x_1{}^1 = Z_1\left(\left(Z_2\left(\ldots\ldots\left(Z_{n(x_0{}^1, x_0{}^2, \ldots, x_0{}^n)}\right)\ldots..\right)\right)\right),$$

$$x_1{}^2 = Z_1\left(\left(Z_2\left(\ldots\ldots\left(Z_{n(x_0{}^2, x_0{}^3, \ldots, x_0{}^n, x_0{}^1)}\right)\ldots..\right)\right)\right)$$

$$\vdots$$

$$x_1{}^n = Z_1\left(\left(Z_2\left(\ldots\ldots\left(Z_{n(x_0{}^n, x_0{}^1, \ldots, x_0{}^{n-1})}\right)\ldots..\right)\right)\right)$$

Also, $x_2{}^1 = Z_1\left(\left(Z_2\left(\ldots\ldots\left(Z_{n(x_1{}^1, x_1{}^2, \ldots, x_1{}^n)}\right)\ldots..\right)\right)\right)$,

$$x_2{}^2 = Z_1\left(\left(Z_2\left(\ldots\ldots\left(Z_{n(x_1{}^2, x_1{}^3, \ldots, x_1{}^n, x_1{}^1)}\right)\ldots..\right)\right)\right)$$

$$\vdots$$

$$x_2{}^n = Z_1\left(Z_2\left(\left(\ldots\ldots\left(Z_{n(x_1{}^n, x_1{}^1, \ldots, x_1{}^{n-1})}\right)\right)\ldots..\right)\right)$$

In general, we can construct the sequences, $< x_k{}^1 >, < x_k{}^2 >, \ldots$, and $< x_k{}^n >$ as

$$x_k{}^1 = Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_{k-1}{}^1, x_{k-1}{}^2, \ldots, x_{k-1}{}^n)}\right)..\right)\right)\right),$$

$$x_k{}^2 = Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_{k-1}{}^2, x_{k-1}{}^3, \ldots, x_1{}^n, x_{k-1}{}^1)}\right)\ldots\right)\right)\right)$$

$$\vdots$$

$$x_k{}^n = Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_{k-1}{}^n, x_{k-1}{}^1, \ldots, x_{k-1}{}^{n-1})}\right)\ldots\right)\right)\right)$$

We want to show that the above sequences are Cauchy sequences in $(F, \mathcal{G}, \aleph)$. Since $\aleph$ is (c.t.n) of $\mathcal{H} - \mathcal{T}ype$ then we have, $\forall$ ʎ > 0 $\exists\, \mu > 0$ such that:
(1-μ) ℵ (1-μ) ℵ … ℵ (1-μ) ≥ 1 − ʎ, $\forall n \in N$.
On other hand, for all $x, y \in F$, $\mathcal{G}(x, y, .)$ is continuous and $\lim_{t\to\infty}(x, y, t) = 1$ then there exists $t_\circ > 0$ such that:

$$\mathcal{G}[x_0{}^1, x_1{}^1, t_0] \geq 1 - \mu, \mathcal{G}[x_0{}^2, x_1{}^2, t_0] \geq 1 - \mu, \ldots\ldots, \mathcal{G}[x_0{}^n, x_1{}^n, t_0] \geq 1-\mu \qquad (2.2)$$

By using (2.1), we get:

- $\mathcal{G}[x_1{}^1, x_2{}^1, \Delta_{(t_0)}] =$

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_0{}^1, x_0{}^2, \ldots, x_0{}^n)}\right)\ldots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_1{}^1, x_1{}^2, \ldots, x_1{}^n)}\right)\ldots\right)\right)\right), \Delta_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]\aleph\mathcal{G}[x_0{}^2, x_1{}^2, t_0]\aleph\ldots\ldots\aleph\mathcal{G}[x_0{}^n, x_1{}^n, t_0]$$

Also,

- $\mathcal{G}[x_1{}^2, x_2{}^2, \Delta_{(t_0)}] =$

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_0{}^2, x_0{}^3, \ldots, x_0{}^n, x_0{}^1)}\right)\ldots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_1{}^2, x_1{}^3, \ldots, x_1{}^n, x_0{}^1)}\right)\ldots\right)\right)\right), \\ \Delta_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}[x_0{}^2, x_1{}^2, t_0]\aleph\mathcal{G}[x_0{}^3, x_1{}^3, t_0]\aleph\ldots\ldots * \mathcal{G}[x_0{}^n, x_1{}^n, t_0]$$

we continue this process in the same way

- $\mathcal{G}[x_1{}^n, x_2{}^n, \Delta_{(t_0)}] =$

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_0{}^n, x_0{}^1, \ldots, x_0{}^{n-1})}\right)\ldots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\ldots\left(Z_{n(x_1{}^n, x_1{}^1, \ldots, x_1{}^{n-1})}\right)\ldots\right)\right)\right), t_0 \end{bmatrix}$$

$$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]\aleph\mathcal{G}[x_0{}^1, x_1{}^1, t_0]\aleph\ldots\ldots\aleph\mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]$$

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As the sameway
and by using above inequalities

- $\mathcal{G}[x_2{}^1, x_3{}^1, \Delta^2{}_{(t_0)}] =$

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_1{}^1,x_1{}^2,\dots,x_1{}^n)}\right)\dots\right)\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_2{}^1,x_2{}^2,\dots,x_2{}^n)}\right)\dots\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}[x_1{}^1, x_2{}^1, \Delta_{(t_0)}] \aleph \mathcal{G}[x_1{}^2, x_2{}^2, \Delta_{(t_0)}] \aleph \dots \aleph \mathcal{G}[x_1{}^n, x_2{}^n, \Delta_{(t_0)}]$$
$$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^n \aleph \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^n \aleph \dots\dots \aleph \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^n$$

And,

- $\mathcal{G}[x_2{}^2, x_3{}^2, \Delta^2{}_{(t_0)}] =$

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_1{}^2,x_1{}^3,\dots,x_1{}^n,x_1{}^1)}\right)\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_2{}^2,x_2{}^3,\dots,x_2{}^n,x_2{}^1)}\right)\dots\right)\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}[x_1{}^2, x_2{}^2, \Delta_{(t_0)}] \aleph \mathcal{G}[x_1{}^3, x_2{}^3, \Delta_{(t_0)}] \aleph \dots \aleph \mathcal{G}[x_1{}^n, x_2{}^n, \Delta_{(t_0)}]$$
$$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^n \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^n \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^n$$

Continue this process, we get

- $\mathcal{G}[x_2{}^n, x_3{}^n, \Delta^2{}_{(t_0)}] =$

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_2{}^n,x_2{}^1,\dots,x_2{}^{n-1})}\right)\dots\right)\right)\right) \\ , Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_3{}^n,x_3{}^1,\dots,x_3{}^n)}\right)\dots\right)\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}[x_2{}^n, x_3{}^n, \Delta_{(t_0)}] \aleph \mathcal{G}[x_2{}^1, x_3{}^1, \Delta_{(t_0)}] \aleph \dots \aleph \mathcal{G}[x_2{}^{n-1}, x_3{}^{n-1}, \Delta_{(t_0)}]$$
$$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^n \aleph \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^n \aleph \dots\dots \aleph \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^n$$

Similarly

- $\mathcal{G}[x_k{}^1, x_{k+1}{}^1, \Delta^k{}_{(t_0)}] =$

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)\dots\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_k{}^1,x_k{}^2,\dots,x_k{}^n)}\right)\dots\right)\right)\right), \Delta^k{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}[x_{k-1}{}^1, x_k{}^1, \Delta^{k-1}{}_{(t_0)}] \aleph \mathcal{G}[x_{k-1}{}^2, x_k{}^2, \Delta^{k-1}{}_{(t_0)}] \aleph \dots \aleph \mathcal{G}[x_{k-1}{}^n, x_k{}^n, \Delta^{k-1}{}_{(t_0)}]$$
$$\vdots$$
$$\geq \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}} \aleph \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^{n^{k-1}} \aleph \dots\dots \aleph \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{k-1}}$$

Also,

- $\mathcal{G}[x_k{}^2, x_{k+1}{}^2, \Delta^k{}_{(t_0)}] =$

$$\mathcal{G}[Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_{k-1}{}^n,x_{k-1}{}^1)}\right)\dots\right)\right)\right), Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_k{}^2,x_k{}^3,\dots,x_k{}^n,x_k{}^1)}\right)\dots\right)\right)\right), \Delta^k{}_{(t_0)}]$$

$$\geq \mathcal{G}[x_{k-1}{}^2, x_k{}^2, \Delta^{k-1}{}_{(t_0)}] * \mathcal{G}[x_{k-1}{}^3, x_k{}^3, \Delta^{k-1}{}_{(t_0)}] * \dots * \mathcal{G}[x_{k-1}{}^1, x_k{}^1, \Delta^{k-1}{}_{(t_0)}]$$
$$\vdots$$
$$\geq \mathcal{G}[x_0{}^2, x_1{}^2, t_0]^{n^{k-1}} \aleph \mathcal{G}[x_0{}^3, x_1{}^3, t_0]^{n^{k-1}} \aleph \dots\dots \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}}$$

Continue this process, as the same way we get

$$\mathcal{G}[x_k{}^n, x_{k+1}{}^n, \Delta^k{}_{(t_0)}] \geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{k-1}} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}} \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^{k-1}}$$

Now, by using above inequalities and for each $n_0 \leq n < m$, we have

$$\mathcal{G}\left[x_k{}^n, x_m{}^n, \sum_{k=n_0}^{\infty} \Delta^k{}_{(t_0)}\right] \geq \mathcal{G}\left[x_k{}^n, x_m{}^n, \sum_{k=n_0}^{m-1} \Delta^{k+1}{}_{(t_0)}\right] \geq$$
$$\mathcal{G}[x_k{}^n, x_{k+1}{}^n, \Delta^k{}_{(t_0)}] * \mathcal{G}[x_{k+1}{}^n, x_{k+2}{}^n, \Delta^{k+1}{}_{(t_0)}] * \dots * \mathcal{G}[x_{m-1}{}^n, x_m{}^n, \Delta^{m-1}{}_{(t_0)}]$$
$$\vdots$$
$$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{k-1}} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{k-1}} \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^{k-1}} \aleph$$
$$\mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^k} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^k} \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^k} \aleph \dots \aleph$$
$$\mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{n^{m-2}} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{n^{m-2}} \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{n^{m-2}}$$

Let $l = \max\{n^{k-1}, n^k, n^{m-2}\}$

$$\geq \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^l \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^l \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^l \aleph$$
$$> \mathcal{G}[x_0{}^n, x_1{}^n, t_0]^{ml} \aleph \mathcal{G}[x_0{}^1, x_1{}^1, t_0]^{ml} \aleph \dots\dots \aleph \mathcal{G}[x_0{}^{n-1}, x_1{}^{n-1}, t_0]^{ml}$$

≥(1-μ) ℵ (1-μ)ℵ …… ℵ (1-μ) ≥(1-ʎ)

And hence, $\mathcal{G}[(x_k{}^n, x_m{}^n, t)] >$ (1-ʎ)

So, $< x_k{}^n >$ is Cauchy sequence. As the same way, we get $< x_k{}^1 >, < x_k{}^2 >$ and $< x_k{}^{n-1} >$ are Cauchy sequences .Since F is complete then there exists $a_1, a_2, \dots\dots, a_n \in X$ such that $\lim_{k\to\infty} x_k{}^1 =$

$$\lim_{k\to\infty} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,\dots,x_{k-1}{}^n)}\right)\dots\right)\right)\right) \to a_1$$

$$\lim_{k\to\infty} x_k{}^2 =$$

$$\lim_{k\to\infty} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,\dots,x_{k-1}{}^1)}\right)\dots\right)\right)\right) \to a_2$$

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$\lim_{k\to\infty} x_k{}^n =$
$\lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^n,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right) \longrightarrow a_n$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)\dots\right)\right) \\ ,Z_1\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right),\Delta_{(t)}\end{bmatrix}$$
$\geq \mathcal{G}[x_{k-1}{}^1,a_1,t]\aleph\mathcal{G}[x_{k-1}{}^2,a_2,t]\aleph\dots\aleph\mathcal{G}[x_{k-1}{}^n,a_n,t]$

As $n\to\infty$ and by continuity of $\mathcal{G}$, we get

$$\mathcal{G}\left[a_1,Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)\right),\Delta_{(t)}\right] = 1$$

Also,

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_{k-1}{}^1)}\right)\dots\right)\right)\right) \\ ,Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)\right),\Delta_{(t)}\end{bmatrix}$$
$\geq \mathcal{G}[x_{k-1}{}^2,a_2,t]\aleph\mathcal{G}[x_{k-1}{}^3,a_3,t]\aleph\dots\aleph\mathcal{G}[x_{k-1}{}^1,a_1,t]$

As, $n\to\infty$

$$\mathcal{G}\left[a_2,Z_1\left(Z_2\left(\dots\left(Z_{n(a_2,a_3,\dots,a_n)}\right)\dots\right)\right),\Delta_{(t)}\right] = 1$$

Continuity

$$\mathcal{G}\begin{bmatrix} (x_{k-1}{}^n,x_{k-1}{}^1,\dots,x_{k-1}{}^{n-1}), \\ Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right)\right),\Delta_{(t)}\end{bmatrix}$$
$\geq \mathcal{G}[x_{k-1}{}^n,a_n,t]\aleph\mathcal{G}[x_{k-1}{}^1,a_1,t]\aleph\dots\aleph$
$\mathcal{G}[x_{k-1}{}^{n-1},a_{n-1},t]$

As $n\to\infty$,we get

$$\mathcal{G}\left[a_n,Z_1\left(\left(\left(Z_2\left(\dots\left(Z_{n(a_n,a_1,\dots,a_{n-1})}\right)\dots\right)\right)\right)\right),\Delta_{(t)}\right]$$
$= 1$. And hens,

$a_1 = Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_1,a_2,\dots,a_n)}\right)\dots\right)\right)\right)$,

$a_2 = Z_1\left(\left(Z_2\left(\dots\left(Z_{n(a_2,a_3,\dots,a_1)}\right)\dots\right)\right)\right)$, ……. ,

$a_n = Z_1\left(Z_2\left(\dots\left(Z_{n(a_n,\dots,a_{n-1})}\right)\dots\right)\right)$

Therefore,
$(a_1,a_2,\dots\dots,a_n)$ is $(G.n)$ – tupled fixed point of compose the mappings of $Z_1,Z_2,\dots\dots,Z_n$.

Corollary(2.2)

Let $(X,\mathcal{G},*)$ be a(F.M.S) .Under the same assumptions of theorem(2.1) but

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots..\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots..\right)\right)\right),kt\end{bmatrix}$$
$\geq \mathcal{G}[x_1,y_1,t]*\mathcal{G}[x_2,y_2,t]*\dots\dots*\mathcal{G}[x_n,y_n,t]$

where $k\in(0,1)$, $t>0$ and $x_i,y_i\in F$, $\forall i = 1,2,\dots\dots,n$.Then there exists a unique $(G.n)$ – tupled fixed point of compose the mappings $Z_1,Z_2,\dots\dots,Z_n$.

Corollary(2.3)

Let $(F,\mathcal{G},\aleph)$ be a (F.M.S)Under the same assumptions of theorem(2.1) but

$$M\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots..\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots..\right)\right)\right),\Delta(t)\end{bmatrix}$$
$\geq \mathcal{G}[x_1,y_1,t]^{a_1}\aleph$
$\mathcal{G}[x_2,y_2,t]^{a_2}\aleph$ …… $\aleph$ $\mathcal{G}[x_n,y_n,t]^{a_n}$

where $\sum_{i=1}^{n} a_i \leq 1$, $t>0$ and x_i, $y_i\in F$ $\forall i = 1,2,\dots\dots,n$.Then there exists a unique $(G.n)$ – tupled fixed point of compose the mappings $Z_1,Z_2,\dots\dots,Z_n$.

Corollary(2.4)

Let $(F,\mathcal{G},\aleph)$ be a (F.M.S).Under the same assumptions of theorem(2.1) but

$$\mathcal{G}\begin{bmatrix} Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots..\right)\right)\right), \\ Z_1\left(\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots..\right)\right)\right),kt\end{bmatrix}$$
$\geq \mathcal{G}[x_1,y_1,t]^{a_1}\aleph\mathcal{G}[x_2,y_2,t]^{a_2}\aleph\dots\dots\aleph\mathcal{G}[x_n,y_n,t]^{a_n}$

where $\sum_{i=1}^{n} a_i \leq 1$, $k\in(0,1)$, and $x_i,y_i\in F$, $\forall i = 1,2,\dots\dots,n$.Then there exists a unique $(G.n)$ – tupled fixed of compose the mappings $Z_1,Z_2,\dots\dots,Z_n$.

Theorem (2.5)

Let $(F,\mathcal{G},\aleph)$ be a fuzzy metric space and A, B are two families of mappings such that $A = \{Z_1,Z_2,\dots\dots,Z_n: X^n\to X\}$, $B = \{E_1,E_2,\dots\dots,E_n: F\to F\}$. Suppose that $\Delta\in$ ℃ satisfying

$$\mathcal{G}[Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1,x_2,\dots,x_n)}\right)\dots..\right)\right), Z_1\left(Z_2\left(\dots\dots\left(Z_{n(y_1,y_2,\dots,y_n)}\right)\dots..\right)\right),\Delta_{(t)}]$$
$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1)}\right)\dots..\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(y_1)}\right)\dots..\right)\right),t\end{bmatrix}\aleph$$
$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2)}\right)\dots..\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(y_2)}\right)\dots..\right)\right),t\end{bmatrix}\aleph\dots\aleph$$
$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_n)}\right)\dots..\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(y_n)}\right)\dots..\right)\right),t\end{bmatrix}$$
(2.3)

where $t>0$ and $x_i,y_i\in F$ $\forall i = 1,2,\dots\dots,n$

If $E_1\left(E_2\left(\dots\dots\left(E_{n(X)}\right)\dots..\right)\right)$ is complete subspace of F containing $Z_1(Z_2(\dots\dots(Z)\dots..))$.Then there exists a unique $(G.n)$ – tupled coincidence fixed point of compose the mappings of $A$and B.

Proof:

Suppose that, $x_0{}^1, x_0{}^2, \dots\dots, x_1{}^n \in F$, since $E_1\left(E_2\left(\dots\dots\left(E_{n(x)}\right)\dots\dots\right)\right)$ containing $Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x^n)}\right)\dots\dots\right)\right)$, that there exists $x_1{}^1, x_1{}^2, \dots\dots, x_1{}^n \in F$ such that

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\dots\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^1, x_0{}^2, \dots, x_0{}^n)}\right)\dots\dots\right)\right)$$

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\dots\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^2, x_0{}^3, \dots, x_0{}^n, x_0{}^1)}\right)\dots\dots\right)\right)$$

$$\vdots$$

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\dots\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_0{}^n, x_0{}^1, \dots, x_0{}^{n-1})}\right)\dots\dots\right)\right)$$

Also, $E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^1)}\right)\dots\dots\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^1, x_1{}^2, \dots, x_1{}^n)}\right)\dots\dots\right)\right)$

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^2)}\right)\dots\dots\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^2, x_1{}^3, \dots, x_1{}^n, x_1{}^1)}\right)\dots\dots\right)\right)$$

$$\vdots$$

$$E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^n)}\right)\dots\dots\right)\right) = Z_1\left(Z_2\left(\dots\dots\left(Z_{n(x_1{}^n, x_1{}^1, \dots, x_1{}^{n-1})}\right)\dots\dots\right)\right)$$

In general, we can construct the sequences, $< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\dots\right)\right) >$, $< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\dots\right)\right) >, \dots,$ and $< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\dots\right)\right) >$ as follows

$$E_1\left(E_2\left(\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right) = Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1, x_{k-1}{}^2, \dots, x_{k-1}{}^n)}\right)\dots\right)\right)$$

$$E_1\left(E_2\left(\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right) = Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2, x_{k-1}{}^3, \dots, x_1{}^n, x_{k-1}{}^1)}\right)\dots\right)\right)$$

$$\vdots$$

$$E_1\left(E_2\left(\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right) = Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^n, x_{k-1}{}^1, \dots, x_{k-1}{}^{n-1})}\right)\dots\right)\right)$$

Now, we want to show that the above sequences are Cauchy sequences in $(F, M, \aleph)$, since $\aleph$ is t – norm of H – type, this implies $\forall\ \Lambda > 0\ \exists\ \mu > 0$ such that $(1-\mu)\aleph(1-\mu)\aleph \dots \aleph(1-\mu) \geq 1 - \Lambda$, $\forall n \in N$. on other hand. For all $x, y \in X$, $M(x, y, .)$ is continuous and $\lim_{t\to\infty}(x, y, t) = 1$ then there exists $t_\circ > 0$ such that.

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix} \geq 1 - \mu$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix} \geq 1 - \mu$$

$$\vdots$$

(2.4)

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix} \geq 1 - \mu$$

By using (2.3), we get

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^1)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_0{}^1, x_0{}^2, \dots, x_0{}^n)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_1{}^1, x_1{}^2, \dots, x_1{}^n)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}$$

$$*$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}$$

Also,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^2)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_0{}^2, x_0{}^3, \dots, x_0{}^n, x_0{}^1)}\right)\dots\right)\right) \\ , Z_1\left(Z_2\left(\dots\left(Z_{n(x_1{}^2, x_1{}^3, \dots, x_1{}^n, x_0{}^1)}\right)\dots\right)\right), \aleph_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^3)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}$$

we continue this process in the same way

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^n)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_0{}^n, x_0{}^1,\dots,x_0{}^{n-1})}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_1{}^n, x_1{}^1,\dots,x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}$$

As the same way and by using above inequalities,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_3{}^1)}\right)\dots\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_1{}^1, x_1{}^2,\dots,x_1{}^n)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_2{}^1, x_2{}^2,\dots,x_2{}^n)}\right)\dots\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^1)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^2)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^n)}\right)\dots\right)\right), \emptyset_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^n \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix}^n \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^n$$

- $$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_3{}^2)}\right)\dots\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_1{}^2, x_1{}^3,\dots,x_1{}^n, x_1{}^1)}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_2{}^2, x_2{}^3,\dots,x_2{}^n, x_1{}^1)}\right)\dots\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^2)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^3)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^n)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^n \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^n \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^n$$

Continue this process, we get

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_3{}^n)}\right)\dots\right)\right), \Delta^2{}_{(t_0)} \end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_2{}^n, x_2{}^1,\dots,x_2{}^{n-1})}\right)\dots\right)\right), \\ Z_1\left(Z_2\left(\dots\left(Z_{n(x_3{}^n, x_3{}^1,\dots,x_3{}^n)}\right)\dots\right)\right), \aleph^2{}_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_3{}^n)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_3{}^1)}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_2{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_3{}^{n-1})}\right)\dots\right)\right), \Delta_{(t_0)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^n \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0 \end{bmatrix}^n \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^n$$

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Similarly

- $$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+1}{}^1)}\right)\dots\right)\right), \Delta^k{}_{(t_0)}\end{bmatrix} =$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z\left(\dots\left(Z_{n(x_{k-1}{}^1, x_{k-1}{}^2, \dots, x_{k-1}{}^n)}\right)\dots\right)\right), \\ Z_1(Z_2(\dots(Z)\dots)), \Delta^k{}_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)}\end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)}\end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_{k-1}{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots)\right), \Delta^{k-1}{}_{(t_0)}\end{bmatrix}$$

$$\vdots$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots)\right), t_0\end{bmatrix}^{n^{k-1}}$$

Also

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k+1}{}^2)}\right)\dots\right)\right), \Delta^k{}_{(t_0)}\end{bmatrix} =$$

$$\mathcal{G}[Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2, x_{k-1}{}^3, \dots, x_{k-1}{}^n, x_{k-1}{}^1)}\right)\dots\right)\right),$$
$$Z_1\left(Z_2\left(\dots\left(Z_{n(x_k{}^2, x_k{}^3, \dots, x_k{}^n, x_k{}^1)}\right)\dots\right)\right), \Delta^k{}_{(t_0)}]$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)}\end{bmatrix}$$

$$* \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^3)}\right)\dots\right)\right), \\ \Delta^{k-1}{}_{(t_0)}\end{bmatrix} * \dots *$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right), \Delta^{k-1}{}_{(t_0)}\end{bmatrix}$$

$$\vdots$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^2)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^3)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}}$$

Continue this process, as the same way we get

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_{k+1}{}^n)}\right)\dots)\right), \Delta^k{}_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0\end{bmatrix}^{n^{k-1}}$$

Now, by using above inequalities and for each $n_0 \leq n < m$, we have

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots)\right), \sum_{k=n_0}^{\infty} \Delta^k{}_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots)\right), \sum_{k=n_0}^{m-1} \Delta^{k+1}{}_{(t_0)}\end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_{k+1}{}^n)}\right)\dots)\right), \Delta^k{}_{(t_0)}\end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_{k+1}{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_{k+2}{}^n)}\right)\dots)\right), \aleph^{k+1}{}_{(t_0)}\end{bmatrix} \aleph \dots$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_{m-1}{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots)\right), \emptyset^{m-1}{}_{(t_0)}\end{bmatrix}$$

$$\vdots$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots)\right), \\ E_1\left(E_2(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots)\right), t_0\end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k-1}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{k}} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{m-2}} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{m-2}} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^{n^{m-2}}$$

Let $l = \max\{n^{k-1}, n^k, n^{m-2}\}$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{l} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{l} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{l} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{l} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^{l}$$

$$> \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^n)}\right)\dots\right)\right), t_0 \end{bmatrix}^{ml} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^1)}\right)\dots\right)\right), t_0 \end{bmatrix}^{ml} \aleph \dots\dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_0{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_1{}^{n-1})}\right)\dots\right)\right), t_0 \end{bmatrix}^{ml}$$

$\geq (1-\mu)\aleph(1-\mu)\aleph \dots\dots \aleph(1-\mu) \geq (1-\lambda)$

And hence,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right), \\ E_1\left(E_2\left(\dots\dots\left(E_{n(x_m{}^n)}\right)\dots\right), t\right) \end{bmatrix} > (1-\lambda)$$

So, $< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^n)}\right)\dots\right)\right) >$ is Cauchy sequence.

As the same way, we get

$< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right) >$,

$< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right) >$ and

$< E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^{n-1})}\right)\dots\right)\right) >$ are Cauchy sequences

Now, to prove that the mappings in A and B have $(G.n)$ – tuplet coincidence fixed point.

Since $E_1\left(E_2\left(\dots\dots\left(E_{n(X)}\right)\dots\right)\right)$ is complete subspace of X then there exists $x_1, x_2, \dots\dots, x_n \in E_1\left(E_2\left(\dots\dots\left(E_{n(X)}\right)\dots\right)\right)$ and $a_1, a_2, \dots\dots, a_n \in X$ such that

$$\lim_{k\to\infty} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^1)}\right)\dots\right)\right)$$
$$= \lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,\dots,x_{k-1}{}^n)}\right)\dots\right)\right)$$
$$\longrightarrow E_1\left(E_2\left(\dots\dots\left(E_{n(a_1)}\right)\dots\right)\right) = x_1$$

$$\lim_{k\to\infty} E_1\left(E_2\left(\dots\dots\left(E_{n(x_k{}^2)}\right)\dots\right)\right)$$
$$= \lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,\dots,x_{k-1}{}^1)}\right)\dots\right)\right)$$
$$\to E_1\left(E_2\left(\dots\dots\left(E_{n(a_2)}\right)\dots\right)\right) = x_2$$

$\vdots$

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$$\lim_{k\to\infty} E_1\left(E_2(\dots\dots(E_{n(x_k{}^n)})\dots)\right)$$

$$= \lim_{k\to\infty} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^n,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right) \to E\left(E_2(\dots(g_{n(a_n)})\dots)\right) = x_n$$

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^1,x_{k-1}{}^2,\dots,x_{k-1}{}^n)}\right)\dots\right)\right), \\ Z_1\left(Z_2(\dots(Z_{n(a_1,a_2,\dots,a_n)})\dots)\right), \Delta_{(t)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_1)})\dots)\right), t \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_2)})\dots)\right), t \end{bmatrix} \aleph \dots$$

$$\aleph\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(x_{k-1}{}^n)})\dots)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_n)})\dots)\right), t \end{bmatrix}$$

As $n \to \infty$ and by continuity of M, we get $\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(a_1)})\dots)\right), \\ Z_1\left(Z_2(\dots(Z_{n(a_1,a_2,\dots,a_n)})\dots)\right), \Delta_{(t)} \end{bmatrix} = 1$

Also,

$$\mathcal{G}\begin{bmatrix} Z_1\left(Z_2\left(\dots\left(Z_{n(x_{k-1}{}^2,x_{k-1}{}^3,\dots,x_{k-1}{}^1)}\right)\dots\right)\right), \\ Z_1\left(Z_2(\dots(Z_{n(a_1,a_2,\dots,a_n)})\dots)\right), \Delta_{(t)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^2)}\right)\dots\right)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_2)})\dots)\right), t \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^3)}\right)\dots\right)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_3)})\dots)\right), t \end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_1)})\dots)\right), t \end{bmatrix}$$

As $n \to \infty$,

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(a_2)})\dots)\right), \\ Z_1\left(Z_2(\dots(Z_{n(a_2,a_3,\dots,a_n)})\dots)\right), \Delta_{(t)} \end{bmatrix} = 1$$

Continuity

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^n,x_{k-1}{}^1,\dots,x_{k-1}{}^{n-1})}\right)\dots\right)\right), \\ Z_1\left(Z_2(\dots(Z_{n(a_n,a_1,\dots,a_{n-1})})\dots)\right), \Delta_{(t)} \end{bmatrix}$$

$$\geq \mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(x_{k-1}{}^n)})\dots)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_n)})\dots)\right), t \end{bmatrix} \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^1)}\right)\dots\right)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_1)})\dots)\right), t \end{bmatrix} \aleph \dots \aleph$$

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2\left(\dots\dots\left(E_{n(x_{k-1}{}^{n-1})}\right)\dots\right)\right), \\ E_1\left(E_2(\dots\dots(E_{n(a_{n-1})})\dots)\right), t \end{bmatrix}$$

As $n \to \infty$, we get.

$$\mathcal{G}\begin{bmatrix} E_1\left(E_2(\dots\dots(E_{n(a_n)})\dots)\right), \\ Z_1\left(Z_2\left(\dots(Z_{n(a_n,a_1,\dots,a_{n-1})})\dots\right)\right), \emptyset_{(t)} \end{bmatrix} = 1$$

$$\Rightarrow E_1\left(E_2(\dots\dots(E_{n(a_1)})\dots)\right)$$

$$= Z_1\left(Z_2(\dots(Z_{n(a_1,a_2,\dots,a_n)})\dots)\right) = x_1$$

$$E_1\left(E_2(\dots\dots(E_{n(a_2)})\dots)\right)$$

$$= Z_1\left(Z_2(\dots(Z_{n(a_2,a_3,\dots,a_1)})\dots)\right) = x_2$$

$$\vdots$$

$$E_1\left(E_2(\dots\dots(E_{n(a_n)})\dots)\right)$$

$$= Z_1\left(Z_2(\dots(Z_{n(a_n,\dots,a_{n-1})})\dots)\right) = x_n$$

Therefore, $(a_1, a_2, \dots\dots, a_n)$ is $(G.n)$ – tupled coincidence point of compose the mappings of A and B.

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حول نظريات النقطة الصامدة الثلاثية نوع $(G.n)$ في فضاءات مترية ضبابية

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قسم الرياضيات

المستخلص :

الهدف من هذا البحث هو لتقديم مفهومين جديدين هما النقطة الصامدة الثلاثية والنقطة المتطابقة الثلاثية نوع$(G.n)$ ولدراسة الوجود للنقطة الصامدة (المتطابقة) الثلاثية لاي نوع من التطبيقات. ايضا سننشئ نظريات التقارب الى نقطة صامدة (متطابقة) ثلاثية نوع$(G.n)$ وحيدة في الفضاءات المترية الضبابية الكاملة .

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Analysis of Heat Transfer on Peristaltic Transport of Powell- Eyring Fluid in an Inclined Tapered Symmetric Channel with Hall and Ohm's Heating Influences

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Abstract

In this paper, we present an analysis of heat transfer on peristaltic flow for Powell- Eyring fluid in an inclined symmetric tapered channel is discussed. Hall effect, velocity, thermal slip conditions, and Ohm's heating are taken into consideration. The governing equations for the balance of mass, momentum and energy are modeled, and then simplified by holding the consideration of long wavelength and low Renolds number approximation. Graphical results are given to analyze the behavior of the parameters emerging in the problem. Effect of Hall parameter and Hartman number on velocity axial have opposite characteristics.

Key words: Heat Transfer, Hall Effect, Ohm's Heating, Powell- Eyring Fluid, Tapered Channel.

Subject Classification: 80Axx, 76Dxx, 76Txx.

1. Introduction

Peristaltic transport in recent times has collected a considerable attention due to its application in physiological, engineering, and biological systems. The peristalsis in general refers to sequential longitudinal and circular contractions of sinusoidal induced peristaltic waves that propagate along the channel that contains the fluid. Particularly this mechanism is the basis of many muscle tubes such as gastrointestinal tract, fallopian tube, bile duct, ureter and esophagus tube etc. Moreover it has a key role in many industrial applications like transport of sanitary fluid, blood pumps in heart lung machines, corrosive fluids transport [1,2] . As well as non- Newtonian fluid is considered to be more suitable than the Newtonian fluids in many industrial and physiological processes [3-5]. Varieties of non-Newtonian fluid models (exhibiting different rheological effects) are available and amongst those is the Powell- Eyring fluid [6]. Even this model is mathematically more complex, but deserves more consideration because of its distinct advantages over the non- Newtonian fluid models. Recently, numerous effective researches have been presented for the peristaltic flow of Powell- Eyring fluid under the assumptions of large wavelength and low Renolds number and with different flow conditions (see refs.[7-9]).The heat transfer refers to the exchange of the thermal energy among the different components of the system. However its rate depends on the temperature of various compartment and physical properties of the flow medium. The study of the influence of heat transfer on non -Newtonian fluids has become important in last years. This importance is due to numerous of industrial processes. Examples are food processing biochemical operations and transport in polymers, biomedical engineering, micro fabrication technologies [5]. Besides this the phenomenon of peristalsis with heat transfer has a wide spread applications in mechanical engineering, physiological processes as oxygenation and hemodialysis, diffusion of chemical impurities. In view of the above considerable applications many researches have been reported the heat transfer analysis in peristaltic motion for different non-Newtonian fluid and flow geometries. Ramesh and Devakar in [10] investigated the effect of heat transfer on the peristaltic transport of MHD second grade model through porous medium in an inclined symmetric channel. Hayat et al in [11] studied the heat transfer analysis in peristaltic flow of Prandtl- Eyring fluid through a curved channel. Veerakrishna et al in.[12] discussed the heat transfer with peristaltic flow of Williamson fluid under the effect of inclined magnetic field in a symmetric planar channel. Ghash in [13] adopted a heat transfer of Newtonian fluid through a rotating channel under the impact of Hall effect and transfer magnetic field. Hina in [8] explored the combined influences of slip and MHD on peristaltic motion of Eyring-Powell fluid with taken into consideration heat/ mass transfer. Abbasi et al.in [14] addressed the heat transfer in hydromagnetic peristaltic flow of variable viscosity fluid through porous medium. Also, Hina et al.in [9] investigates the heat transfer in the peristaltic transport of Powell- Eyring fluid inside a curved channel and its application in biomedicine.

The Hall current effect is considerable when Hall parameter is high. This can be achieved in a strong magnetic field. The Hall current has a remarkable effect on Lorentz force term. This effect is employed in numerous aspects like astrophysical, meteorological, MHD power generations, plasma flow problem and centrifugal machines. Motivated by such fact the influence of the Hall current on the flow can be seen via studies [5,13,15,16].Another considerable aspect in peristalsis is the non-uniform (tapered) flow configuration. This configuration is noticed in most practical applications like physiological body organs, small blood vessels, intestines, lymphatic vessels and ducts afferents. Blood flow suspension for Walter's B fluid model through tapered stenosed arteries discussed by Akbar in [17]. Influence of variable viscosity on MHD peristaltic flow of Pseudoplastic fluid model through a tapered symmetric channel studied by [18]. Further recent literature can be seen via refs. [3,19,20]. This work addresses heat transfer analysis of peristaltic flow of conducting Powell- Eyring model in an inclined tapered symmetric channel under the effect of Hall current. The mathematical model formulation is subjected to a transverse magnetic field presence. Besides this the energy equation is modeled by taking Joule (Ohm's) heating impact into account. The two-dimensional equations of motion and energy are simplified by adopting lubrication approach. The governing equation is carried out by utilizing long wavelength approximation and low Renolds number assumptions. The series solution of the stream function, axial velocity, heat transfer and temperature distribution have been obtained by employing the perturbation technique. The major behavior of Hartman number, Hall parameter, Froude number, Renold number, inclination of angle, phase difference, non-uniform parameter and Powell- Eyring fluid parameters on velocity profile, temperature, heat transfer and the peristaltic flow are analyzed in detail graphically.

2. Problem Mathematical description

Assume the peristaltic transport of an incompressible Powell- Eyring fluid in two-dimensional tapered symmetric channel of thickness $2d$. The channel is taken inclined at angle α to the horizontal axis. The fluid is electrically conducting in the presence of a strong applied magnetic field $B=(0,0,\beta_0)$. The magnetic Renolds number is taken a very low approximation hence the induced magnetic field is neglected. Symmetry in the flow achieved by the peristaltic waves with different amplitude and phases moving with a constant speed c and wavelength λ along the walls of the channel. Hall and Ohm's heating effects are taken into account.

The structure of walls geometry are described as:

$$Y_1 = H_1(X,\bar{t}\,) = d + \bar{m}_1 X + a\sin(\tfrac{2\pi}{\lambda}\,(X - c\bar{t}\,) + \phi) \qquad (1)$$

$$Y_2 = H_2(X,\bar{t}\,) = d + \bar{m}_1 X + b\sin(\tfrac{2\pi}{\lambda}\,(X - c\bar{t}\,)) \qquad (2)$$

In which Y_1, Y_2 are the lower and upper wall respectively, a, b are the amplitudes of waves along the lower and upper walls, $\bar{m}_1 (\ll 1)$ the non- uniform parameter.

The Cartesian coordinates (X, Y) in such a way that wave propagates in X- direction and Y – axis is taken normal to it. $\phi \in [0, \pi]$ the phase difference. Moreover $\phi = 0$ corresponds to symmetric channel with waves out of phase, and $\phi = \pi$ describes the waves in phase. Further a, b, d and ϕ satisfy the inequality so that the walls still parallel.

$$a^2 + b^2 + 2abd\cos\emptyset \leq (2d)^2 \quad (3)$$

The fluid is obeying the Powell- Eyring model and the Cauchy stress tensor $\bar{\tau}$ of it is given as follows

$$\bar{\tau} = -\bar{P}I + \bar{S} \quad (4)$$

$$\bar{S} = \left[\mu + \frac{1}{\beta\dot{\gamma}} \sinh^{-1}\left(\frac{\dot{\gamma}}{c_1}\right)\right] A_1 \quad (5)$$

$$\dot{\gamma} = \sqrt{\frac{1}{2}\, tra({A_1}^2)} \quad (6)$$

$$A_1 = \nabla\vec{V} + (\nabla\vec{V})^T \quad (7)$$

Where $\bar{S}$ express the extra stress tensor, I the identity tensor $\nabla = (\partial X, \partial Y, 0)$ the gradient vector, β, c_1 the material parameters of Powell-Eyring fluid, and μ the dynamic viscosity. The term $\sinh^{-1}$ can be approximated as

$$\sinh^{-1}\left(\frac{\dot{\gamma}}{c_1}\right) = \frac{\dot{\gamma}}{c_1} - \frac{\dot{\gamma}^3}{6{c_1}^3}, \left|\frac{\dot{\gamma}^5}{{c_1}^5}\right| \ll 1 \quad (8)$$

Applying the generalized Ohm's law, including the Hall current we get

$$\vec{J} = \sigma[\vec{V} \times \vec{B} - \frac{1}{en}(\vec{J} \times \vec{B})] \quad (9)$$

$$\vec{J} \times \vec{B} = \left(\frac{-\sigma{\beta_0}^2(U - mV)}{1+m^2} i - \frac{\sigma{\beta_0}^2(V + mU)}{1+m^2} j\right) \quad (10)$$

In which $\vec{J}$ characterize the current density vector, $\vec{V} = (U, V, 0)$ the velocity field, σ the electrical conductivity, n the number density of electron, e the electric charge, β_0 the magnetic field strength and $\left(m = \frac{\sigma\beta_0}{en}\right)$ the Hall parameter.

The balance of mass, momentum and temperature are given respectively below

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (11)$$

$$\rho\left(\frac{\partial U}{\partial \bar{t}} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = \frac{\partial \bar{P}}{\partial X} + \frac{\partial \bar{S}_{XX}}{\partial X} + \frac{\partial \bar{S}_{XY}}{\partial Y} - \frac{\sigma{\beta_0}^2(U - mV)}{1+m^2} + \rho g \sin\alpha \quad (12)$$

$$\rho\left(\frac{\partial V}{\partial \bar{t}} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = \frac{\partial \bar{P}}{\partial Y} + \frac{\partial \bar{S}_{YX}}{\partial X} + \frac{\partial \bar{S}_{YY}}{\partial Y} - \frac{\sigma{\beta_0}^2(V + mU)}{1+m^2} + \rho g \cos\alpha \quad (13)$$

and

$$\rho c_P\left(\frac{\partial T}{\partial \bar{t}} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right) = k\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + (\bar{S}_{YY} - \bar{S}_{XX})\frac{\partial V}{\partial Y} + \bar{S}_{XY}\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right) + \frac{\sigma{\beta_0}^2(U^2 + V^2)}{1+m^2} \quad (14)$$

Where

$\bar{t}, \rho, \bar{P}, \vec{J}, g, c_P, T, k, \sigma, \vec{J} \times \vec{B}, \frac{\vec{J}.\vec{J}}{\sigma}, (U, V, 0)$ are time, fluid density, pressure, current density, the gravity, the specific heat, temperature, thermal conductivity, the electrical conductivity, the component of Lorentz force, Joule heating component, and the velocity components corresponding to the laboratory frames $(X, Y, \bar{t})$,also $\bar{S}_{ij}$ represents the components of stress tensor and $(i = X, j = Y)$.

The components of extra stress tensor of Powell- Eyring defined by Eq.(5) are listed below

$$\bar{S}_{XX} = 2\left(\mu + \frac{1}{\beta c_1}\right)U_X - \frac{1}{3\beta {c_1}^3}[2{U_X}^2 + (U_Y + V_X)^2 + 2{V_Y}^2]U_X \quad (15)$$

$$\bar{S}_{XY} = \left(\mu + \frac{1}{\beta c_1}\right)(V_X + U_Y) - \frac{1}{6\beta {c_1}^3}\left[2{U_X}^2 + (U_Y + V_X)^2 + 2{V_Y}^2\right](V_X + U_Y) \quad (16)$$

$$\bar{S}_{YY} = 2\left(\mu + \frac{1}{\beta c_1}\right)V_Y - \frac{1}{3\beta {c_1}^3}\left[2{U_X}^2 + (U_Y + V_X)^2 + 2{V_Y}^2\right]V_Y \quad (17)$$

The corresponding boundary conditions are listed below

The slip conditions for velocity and temperature at the walls are

$$U \pm \gamma \bar{S}_{XY} = 0 \quad \text{at } Y = H_1 \text{ and } Y = H_2 \quad (18)$$

$$T \pm \beta_1 \frac{\partial T}{\partial Y} = T_0 \quad \text{at } Y = H_1 \text{ and } Y = H_2 \quad (19)$$

Flexible walls are given as

$$\left[-\tau\frac{\partial^3}{\partial X^3} + m_2\frac{\partial^3}{\partial X\partial \bar{t}^2} + d'\frac{\partial^2}{\partial \bar{t}\partial X}\right]Y = \frac{\partial \bar{S}_{XX}}{\partial X} + \frac{\partial \bar{S}_{XY}}{\partial Y} - \rho\left(\frac{\partial U}{\partial \bar{t}} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) - \frac{\sigma{\beta_0}^2(U - mV)}{1+m^2} + \rho g \sin\alpha \quad , Y = H_1 \text{ and } Y = H_2 \quad (20)$$

In which $T_0, \tau, m_2, d', \gamma, \beta_1$ are the temperature at the upper and lower walls, the elastic tension, the mass per unit area, the coefficient of viscous damping, the velocity slip parameter, and the thermal slip parameter respectively.

The dimensionless quantities are defined as

$$x = \frac{X}{\lambda}, y = \frac{Y}{d}, u = \frac{U}{c}, v = \frac{V}{c}, h_1 = \frac{H_1}{d}$$
$$h_2 = \frac{H_2}{d}, p = \frac{d^2\bar{p}}{\lambda\mu c}, \delta = \frac{d}{\lambda}, \gamma^* = \frac{\gamma}{d}, \beta_1^* = \frac{\beta_1}{d}$$
$$S = \frac{d\bar{S}(X)}{\mu c}, R1 = \frac{\rho c d}{\mu}, W = \frac{1}{\mu\beta c_1}, A = \frac{W}{6}\left(\frac{c}{c_1 d}\right)^2$$
$$\theta = \frac{T - T_0}{T_0}, E_1 = \frac{-\tau d^3}{\lambda^3\mu c}, E_2 = \frac{m_2 c d^3}{\lambda^3\mu c}, E_3 = \frac{d' d^3}{\lambda^2\mu}$$
$$Pr = \frac{\mu c_P}{k}, H = \beta_0 d\sqrt{\frac{\sigma}{\mu}}, E_c = \frac{c^2}{c_P T_0}, Br = E_c Pr,$$
$$a^* = \frac{a}{d}, b^* = \frac{b}{d}, \quad m_1 = \frac{\overline{m}_1\lambda}{d}, Fr = \frac{c}{\sqrt{gd}} \quad (21)$$

Where $\delta, E_1, E_2, E_3, R1, Pr, H, Ec, Br, Fr, m_1, P, h_1, h_2, x, y, u, v, A, W, \gamma^*, \beta_1^*$ are the wave number, the wall elastance parameter, the mass per unit area parameter, the wall damping parameter, the Renolds number, the Prandtl number, Hartman number, Eckret number, Brinkman number, Froude number, the dimensionless non- uniform parameter, the dimensionless pressure, the dimensionless lower wall surface, upper wall surface, components of the dimensionless coordinates, axial velocity, transverse component of velocity, the Eyring-Powell fluid dimensionless parameters ,the dimensionless velocity and thermal slip parameters respectively. Note that asterisks will be omitted for simplicity.

The stream function $\psi(x, y, t)$ and its connection with velocity components is scripted below

$$u = \frac{\partial\psi}{\partial y} \quad and \quad v = -\delta\frac{\partial\psi}{\partial x} \quad (22)$$

Substituting Eq.(21) into Eqs.(11) – (20) and make use of Eq.(22), note that the mass balance represented by Eq. (11) is identically satisfied, yields

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$$\delta R1\left(\psi_{ty}+\psi_y\psi_{xy}-\delta\psi_x\psi_{xy}\right)=-P_x+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\frac{H^2}{(1+m^2)}\left(\psi_y+\delta m\psi_x\right)+\frac{R1}{(Fr)^2}\,sin\alpha \qquad (23)$$

$$\delta^2 R1\left(-\delta^2\psi_{tx}-\delta^2\psi_y\psi_{xx}+\delta^3\psi_x\psi_{xy}\right)=P_y+\delta\frac{\partial S_{yy}}{\partial y}+\delta^2\frac{\partial S_{yx}}{\partial x}-\frac{H^2\delta}{(1+m^2)}\left(-\delta\psi_x+m\psi_y\right)+\frac{R1}{(Fr)^2}\,cos\alpha \qquad (24)$$

$$PrR1\left(\delta\frac{\partial\theta}{\partial t}+\delta\psi_y\frac{\partial\theta}{\partial x}-\delta\psi_x\frac{\partial\theta}{\partial y}\right)=\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+BrS_{xy}\left(\psi_{yy}-\delta^2\psi_{xx}\right)+\frac{H^2Br}{(1+m^2)}\left((\psi_y)^2+\delta^2(\psi_x)^2\right)+\delta Br\left(S_{xx}-S_{yy}\right)\psi_{xy} \qquad (25)$$

The dimensionless of stress components can be given by

$$S_{xx}=\delta(1+W)\psi_{xy}-\delta A\left[\delta^2\psi_{yx}^2+\frac{1}{4}\left(\psi_{yy}-\delta^2\psi_{xx}\right)^2+\delta^2\psi_{xy}^2\right]\psi_{xy} \qquad (26)$$

$$S_{xy}=(1+W)\left(-\delta^2\psi_{xx}+\psi_{yy}\right)-\left[4A\delta^2\left(-\delta^2\psi_{xx}+\psi_{yy}\right)\psi_{xy}^2+A\left(-\delta^2\psi_{xx}+\psi_{yy}\right)^3+2A\delta^2\psi_{yx}^2\left(-\delta^2\psi_{xx}+\psi_{yy}\right)\right] \qquad (27)$$

$$S_{yy}=-\delta(1+W)\psi_{xy}+4A\delta\left[\delta^2\psi_{xy}^2+\left(\psi_{yy}-\delta^2\psi_{xx}\right)^2+\delta^2\psi_{xy}^2\right]\psi_{xy} \qquad (28)$$

The corresponding dimensionless boundary conditions are listed below

$$\left.\begin{array}{c}\psi_y-\gamma\left((1+W)\psi_{yy}-A\psi_{yy}^3\right)=0\\ \theta-\beta_1\theta_y=0\end{array}\right\}$$

at $y=h_1$ (29)

$$\left.\begin{array}{c}\psi_y+\gamma\left((1+W)\psi_{yy}-A\psi_{yy}^3\right)=0\\ \theta+\beta_1\theta_y=0\end{array}\right\}$$

at $y=h_2$ (30)

and

$$\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}\right]y=\frac{\partial S_{xy}}{\partial y}-\frac{H^2\psi_y}{1+m^2}+\frac{R1}{(Fr)^2}sin\alpha \quad \text{at } y=h_1\ ,y=h_2 \qquad (31)$$

The dimensionless forms of lower and upper walls are

$$h_1(x)=-(1+m_1x+a\sin(2\pi(x-t)+\phi)) \qquad (32)$$

$$h_2(x)=\left(1+m_1x+b\sin\left(2\pi(x-t)\right)\right) \qquad (33)$$

Employing authentic long wavelength and low Renolds number approximations we obtain

$$-P_x+\frac{\partial S_{xy}}{\partial y}-\frac{H^2\psi_y}{1+m^2}=0 \qquad (34)$$

$$P_y=0 \qquad (35)$$

$$\frac{\partial^2\theta}{\partial y^2}+BrS_{xy}\psi_{yy}+\frac{H^2Br}{(1+m^2)}(\psi_y)^2=0 \qquad (36)$$

Eq.(35) indicates that the pressure is independent of the non- dimensional coordinate y . Differentiate Eq. (34) w. r. t y and make use of Eq. (35), the equations which govern the problem are

$$\frac{\partial^2 S_{xy}}{\partial y^2}-\frac{H^2}{(1+m^2)}\psi_{yy}=0 \qquad (37)$$

$$\frac{\partial^2\theta}{\partial y^2}+BrS_{xy}\psi_{yy}+\frac{H^2Br}{(1+m^2)}(\psi_y)^2=0 \qquad (38)$$

$$\left.\begin{array}{c}\psi_y-\gamma\left((1+W)\psi_{yy}-A\psi_{yy}^3\right)=0\\ \theta-\beta_1\theta_y=0\end{array}\right\}$$

at $y=h_1$ (39)

$$\left.\begin{array}{c}\psi_y+\gamma\left((1+W)\psi_{yy}-A\psi_{yy}^3\right)=0\\ \theta+\beta_1\theta_y=0\end{array}\right\}$$

at $y=h_2$ (40)

and

$$\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}\right]y=\frac{\partial S_{xy}}{\partial y}-\frac{H^2\psi_y}{1+m^2}+\frac{R1}{(Fr)^2}sin\alpha \quad \text{at } y=h_1\ ,y=h_2 \qquad (41)$$

Heat transfer coefficient at the lower wall is defined as

$$Z = \frac{\partial h_1}{\partial x}\, \theta_y(h_1) \tag{42}$$

3. Solution Technique

Now using the perturbation technique for small Eyring- Powell parameter A by expanding the flow quantities in a power series of A in such manner

$$\psi = \psi_0 + A\,\psi_1 + \cdots \tag{43}$$

$$\theta = \theta_0 + A\,\theta_1 + \cdots \tag{44}$$

Substituting Eqs.(44), (45) into Eqs.(37) - (42) and then comparing the coefficients of same power of A up to the first order we obtain the following two system.

3.1. Zeroth order system

The general form of zeroth- order system is

$$(1+W)\psi_{0yyyy} - \frac{H^2}{(1+m^2)}\psi_{0yy} = 0 \tag{45}$$

$$\theta_{0yy} + Br(1+W)\left(\psi_{0yy}\right)^2 + \frac{H^2 Br}{(1+m^2)}(\psi_{0y})^2 = 0 \tag{46}$$

with the respective boundary conditions

$$\left.\begin{aligned} \psi_{0y} - \gamma(1+W)\psi_{0yy} = 0 \\ \theta_0 - \beta_1\theta_{0y} = 0 \end{aligned}\right\} \text{at } y = h_1 \tag{47}$$

$$\left.\begin{aligned} \psi_{0y} + \gamma(1+W)\psi_{0yy} = 0 \\ \theta_0 + \beta_1\theta_{0y} = 0 \end{aligned}\right\} \text{at } y = h_2 \tag{48}$$

$$\left[E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x\partial t^2} + E_3\frac{\partial^2}{\partial t\partial x}\right]y = (1+W)\psi_{0yyyy} - \frac{H^2\psi_{0y}}{1+m^2} + \frac{R1}{(Fr)^2}sin\alpha$$

$$\text{at } y = h_1 \text{ and } y = h_2 \tag{49}$$

By solving the above systems for ψ_0 and θ_0, the final form solution for the zeroth order are

$$\psi_0 = \frac{1}{N_1^2}\left(e^{N_1 y}c_1 + e^{-N_1 y}c_2\right) + c_3 + c_4 y \tag{50}$$

$$\theta_0 = -Br(1+W)\left(\frac{1}{2N_1^2}\left(c_2^2 e^{-2N_1 y} + c_1^2 e^{2N_1 y}\right) + \frac{1}{N_1}\left(-2c_2c_4e^{-N_1 y} + 2c_1c_4e^{N_1 y}\right) + \frac{c_4^2N_1^2y^2}{2}\right) + b_1 + yb_2 \tag{51}$$

3.2. The First Order System

The first order system has the form

$$(1+W)\psi_{1yyyy} - \frac{H^2}{(1+m^2)}\psi_{1yy} - \frac{\partial^2}{\partial y^2}\left(\psi_{0yy}\right)^3 = 0 \tag{52}$$

$$\theta_{1yy} + Br\left(2(1+W)\psi_{0yy}\psi_{1yy} - \left(\psi_{0yy}\right)^4\right) + \frac{2H^2Br}{(1+m^2)}\psi_{0y}\psi_{1y} = 0 \tag{53}$$

Along with the appropriate boundary condition

$$\left.\begin{aligned} \psi_{1y} - \gamma\left((1+W)\psi_{1yy} - \left(\psi_{0yy}\right)^3\right) = 0 \\ \theta_1 - \beta_1\theta_{1y} = 0 \end{aligned}\right\} \text{at } y = h_1 \tag{54}$$

$$\left.\begin{aligned} \psi_{1y} + \gamma\left((1+W)\psi_{1yy} - \left(\psi_{0yy}\right)^3\right) = 0 \\ \theta_1 + \beta_1\theta_{1y} = 0 \end{aligned}\right\} \text{at } y = h_2 \tag{55}$$

$$(1+W)\psi_{1yyy} - \frac{\partial}{\partial y}\left(\psi_{0yy}\right)^3 - \frac{H^2}{(1+m^2)}\psi_{1y} = 0 \quad \text{at } y = h_1 \text{ and } y = h_2 \tag{56}$$

Solving the above system for first order the final calculated expressions for ψ_1 and θ_1 are

$$\psi_1 = \frac{1}{8(1+W)N_1^2}\, e^{-3N_1 y}(L_1 + L_2) + c_7 + c_8 y \tag{57}$$

$$\theta_1 = \left(\frac{-Br}{24N_1^2}\right)e^{-4N_1 y}\,(L_3 + L_4 + L_5) + b_3 + b_4 y \tag{58}$$

Where

$$N_1 = \frac{H}{\sqrt{(1+W)(1+m^2)}}, \quad W \geq 0$$

$$L_1 = \left(c_2^3 + 6c_2c_1^2e^{4N_1 y}(-5+2N_1 y) - 6c_2^2c_1e^{2N_1 y}(5+2N_1 y)\right)$$

$$L_2 = e^{2N_1 y}\left(c_1^3e^{4N_1 y} + 8(1+W)\left(e^{2N_1 y}\,c_5 + c_6\right)\right)$$

$$L_3 = 3c_2^4 + 36c_1c_2^2c_4N_1e^{3N_1 y}\,(7+2N_1 y) - c_2^3e^{N_1 y}\left(2c_4N_1 + 3c_1e^{N_1 y}(29+12N_1 y)\right)$$

$$L_4 = e^{3N_1 y}\left(3c_1^4e^{5N_1 y} + 2c_1^3c_4e^{4N_1 y}N_1 + 24c_1e^{2N_1 y}(1+W)\left(c_5e^{N_1 y} + 2c_8N_1\right) + 24c_4N_1(1+W)\left(-2c_6 + 2c_5e^{2N_1 y} + c_8e^{N_1 y}y^2N_1^3\right)\right)$$

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$$L_5 = 3c_2 e^{2N_1 y}\left(8c_6(1+W) + e^{N_1 y}\left(-16c_8(1+W)N_1 + 12{c_1}^2 c_4 e^{2N_1 y} N_1(-7+2N_1 y) + {c_1}^3 e^{3N_1 y}(-29+12N_1 y)\right)\right)$$

4. Results and Discussions

This section transmits the graphical description of various parameters on the outcomes flow quantities. The variation of streamlines, axial velocity, temperature, and heat transfer coefficient are demonstrate and analyzed graphically.

4.1. Velocity Distribution

The development of axial velocity in response to involved parameters has been recorded for fixed values ($x = 0.2,\ t = 0.1$) through the Figs. (1 to 7). All the profiles are parabolic about the channel's width due to zero displacement at the boundaries. Fig.(1) inspects the impact of wall properties parameters E_1 , E_2 and E_3 on velocity profile. It is noticed that for ascending values of E_1 , E_2 and E_3 the axial velocity tends to increase. The influence of fluid parameters A on the velocity profile is portrayed in Fig. (2,). It is seen that the velocity is increasing function for parameter (A) The effect of Hartman number (H) is sketched in Fig.(3). Reduction behavior for $u(y)$ is recognized as (H) increases. This outcome is due to the Lorentz force which opposes the fluid motion and hence reduces the velocity profile. Fig. (4) is plotted to study the effect of Hall parameter (m) on the velocity profile. One can observed that the velocity increases as the Hall parameter increases this due to the fact that the Hall effect balances the resistive influence of applied magnetic field. Fig. (5) has been sketched to notice the behavior of velocity profile upon various values of channel phase angle (ϕ). It is observed that the velocity increases in the entire tapered channel for ascending values of (ϕ). Fig. (6) is drawn to analyze the impact of amplitude of lower wall (a) on the velocity profile. This figure indicates that an increase in (a) the axial velocity increases near the lower wall for $-1.2 \le y \le -0.95$. However the velocity decreases in the rest of the channel $-0.8 \le y \le 1.2$ with higher values of (a). The velocity profile upon the non- uniform parameter (m_1) is plotted in Fig. (7). We observed that the velocity value increases as the magnitude of non- uniform parameter (m_1) increases

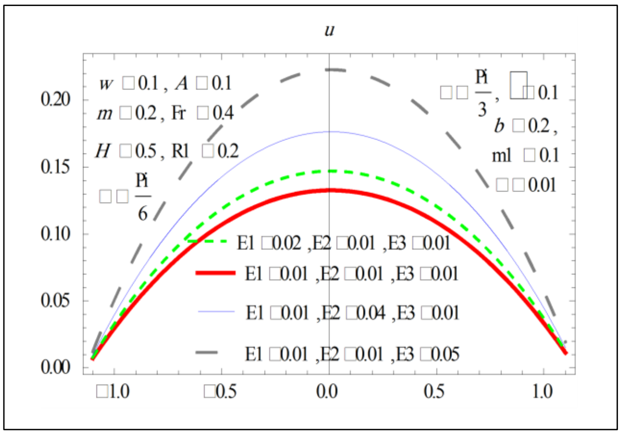

Fig.(1) Velocity Profile with Wall properties

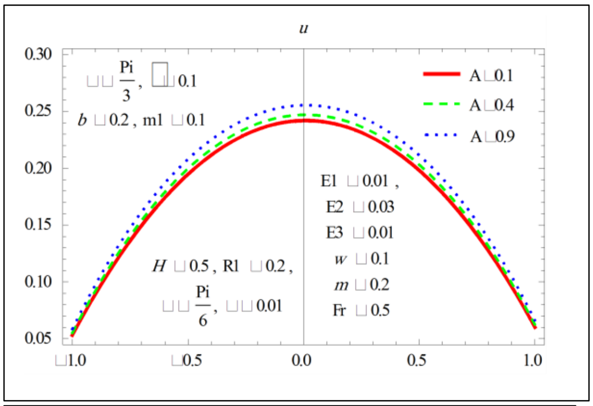

Fig.(2) Velocity Profile Via variation of A

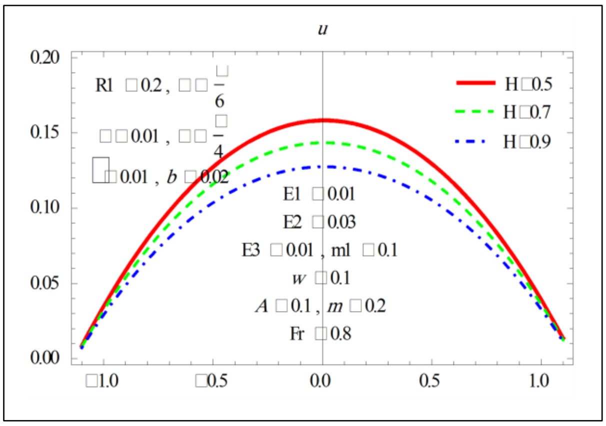

Fig.(3) Velocity Profile Via variation of H

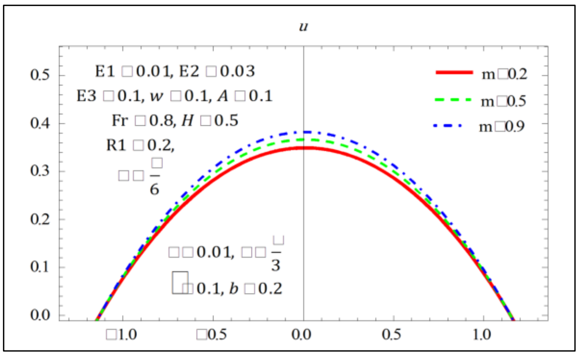

Fig.(4) Velocity Profile Via variation of m

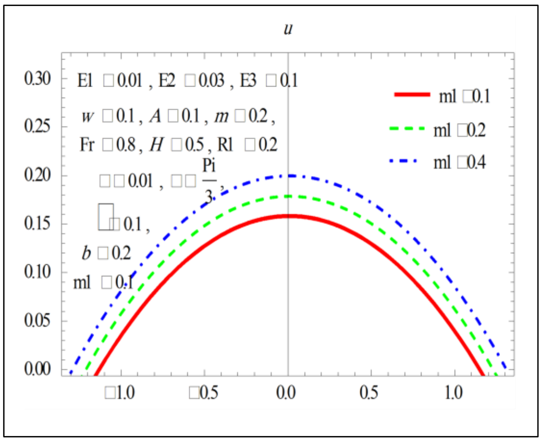

Fig.(7) Velocity Profile Via m_1

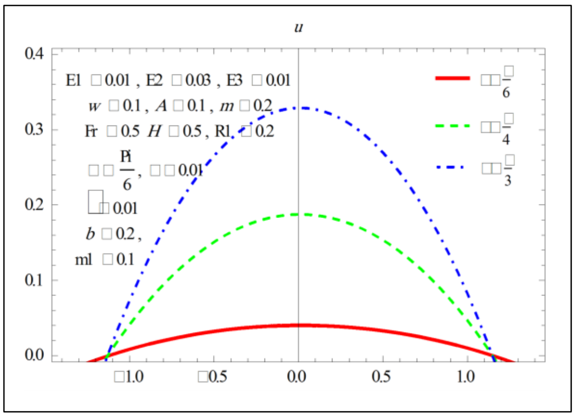

Fig.(5) Velocity Profile Via ϕ

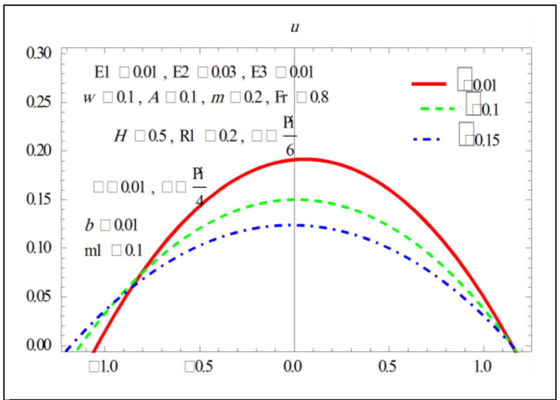

Fig.(6) Velocity Profile Via α

4.2. Temperature Distribution

The quantitative effects of different emerging parameters on temperature distribution $\theta(y)$ are observed physically at fixed values $(x = 0.2, t = 0.1)$via Figs. (8 - 13) . It is analyzed through the figures that $\theta(y)$ attains a maximum value near the central part of the channel.. The variation in temperature with an increase in material fluid parameter A is discussed in Fig.(8). Since the temperature is defined as average kinetic energy of molecules hence ascending values of A causes a risen in θ values. Figs.(9 and 10) is devoted to explain the influence of variation of both channel inclination (α) and phase angle (ϕ).It is observed that θ is rising with an increase in (α) magnitude but opposite attitude is recorded for (ϕ). Higher values of Brinkman number (Br) leads the temperature distribution to grow up see Fig.(11). However the temperature profile decreases for larger values of Froude number (Fr) through Fig.(12). The impact of Hall parameter (m) is captured in Fig.(13).

It is revealed that $\theta(y)$ increases with the increase of (m) near the lower and upper walls for specific at $-1.2 \leq y \leq -0.4 \cup 0.6 \leq y \leq 1.2$ whereas decline in temperature is noticed with growing in Hall parameter at the central part of the channel.

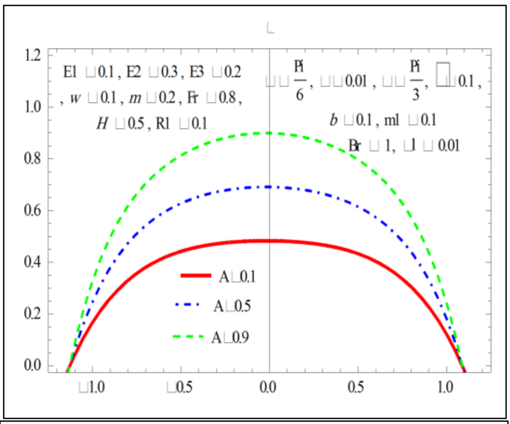

Fig. (8) Temperature profile for variation of A

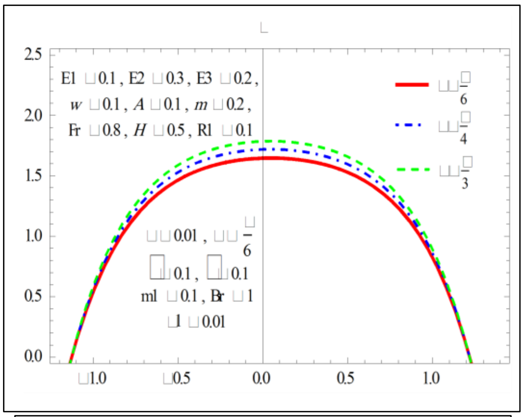

Fig.(9) Temperature Profile Via α

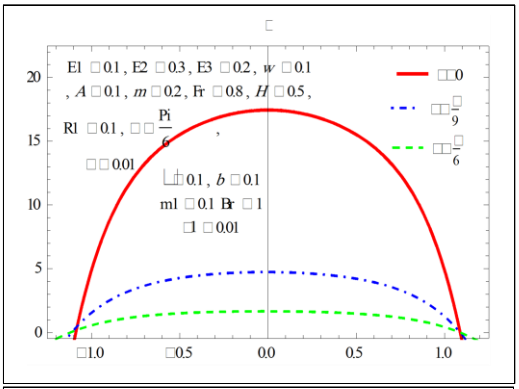

Fig.(10) Temperature Profile Via ϕ

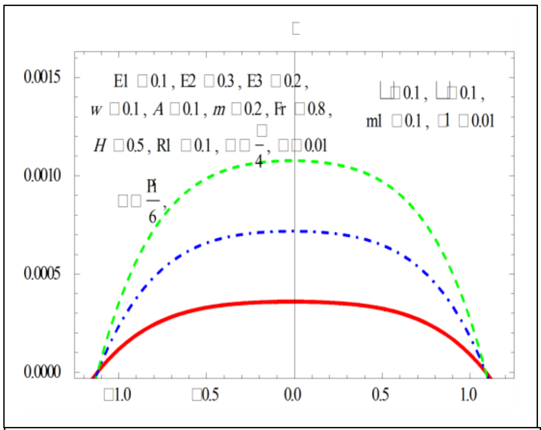

Fig.(11) Temperature Profile Via variation of Br

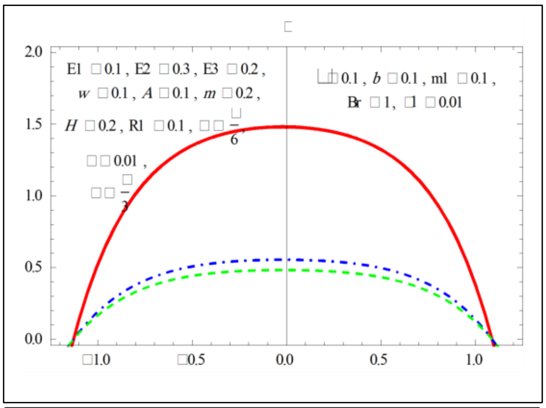

Fig.(12) Temperature Profile Via variation of Fr

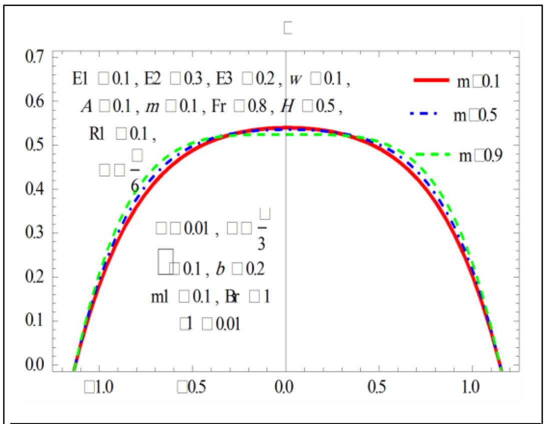

Fig.(18) Temperature profile for m

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4.3. Heat Transfer Rate

This subsection considered the development of heat transfer rate $Z(x)$ upon intricate parameters. The magnitude of non-dimensional heat transfer is given at the low wall by applying the inequality $Z(x) = \frac{\partial h_1}{\partial x}\theta_y|_{y=h_1}$ for fixed values$(y = 0.2, t = 0.1)$. Figs.(14- 20) shows that the peristaltic waves along the walls produce oscillatory behavior of $Z(x)$. Heat transfer rate (x) . From Fig.(14) we illustrate that the rate of heat transfer has increasing function as the material fluid parameter A becomes larger. Enhancement in heat transfer rate $Z(x)$ is noticed upon increasing both the Brinkman number Br and Hartman parameter via Figs.(15)&(16). According to Figs.(17) & Fig.(18) we observed that the channel inclination α, and Froude number have mixed behavior on $Z(x)$. Fig.(19) study the influence of upper wall amplitude b on rate of heat transfer. It shows that $Z(x)$ increases for ascending values of b. The impact of Hall parameter m on the rate of heat transfer is depicted through the Fig.(20) . Reduction in $Z(x)$ is obtained through the channel with growing up of Hall parameter m.

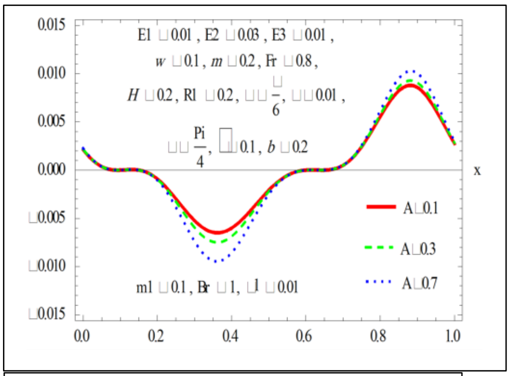

Fig.(14) $Z(x)$ for variation of A

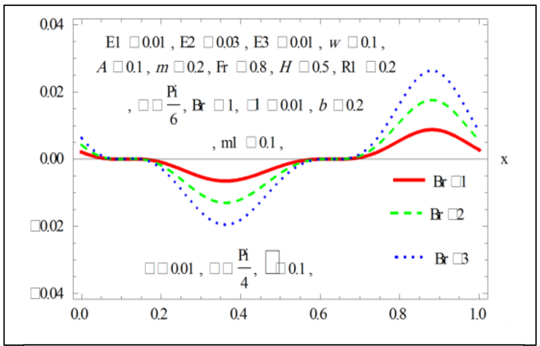

Fig.(15) $Z(x)$ for variation of Br

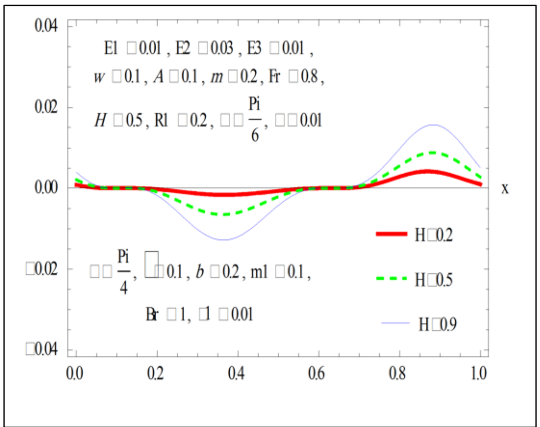

Fig.(16) $Z(x)$ for variation of H

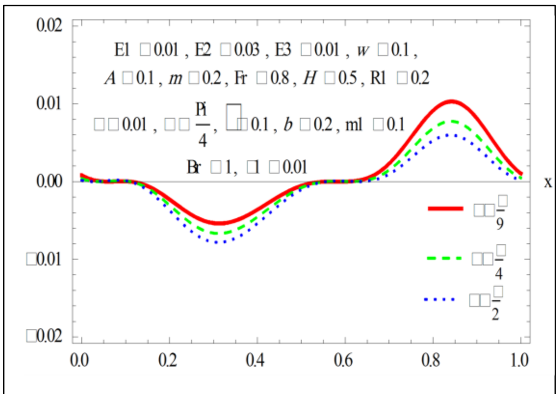

Fig.(17) $Z(x)$ for variation of α

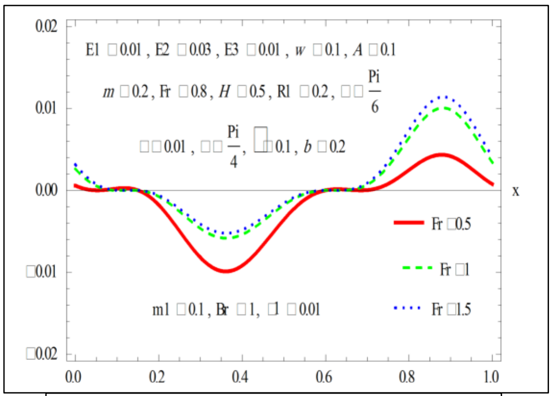

Fig.(18) $Z(x)$ for variation of Fr

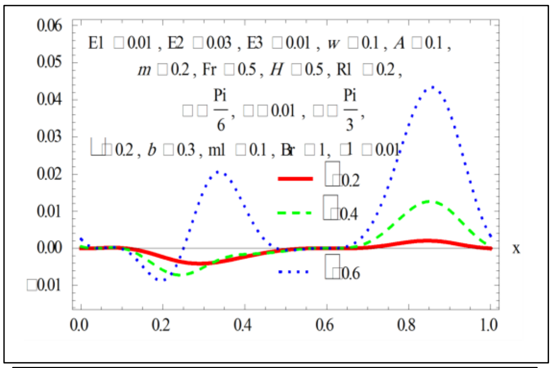

Fig.(19) $Z(x)$ for variation of b

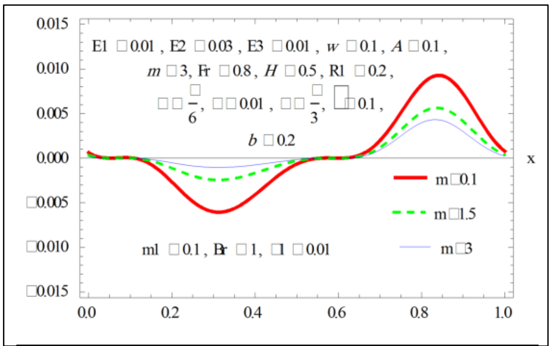

Fig.(20) $Z(x)$ for variation of m

4.4. Trapping Phenomena

In this part, the impact of various effective parameters on stream lines for fixed time $(t = 0.1)$ are furnished in Figs. (21- 28). The interesting phenomenon in peristaltic motion is trapping which refers to the formation of a bolus of fluid that circulates internally and moves with the peristaltic wave at the celerity of waves. The sketches declare that bolus appears near both lower and upper walls in all conditions. It can be visualize from Fig.(21) that when the parameters of wall: rigidity E_1 and tension E_2 increase the size of the bolus increases whilst the number of the trapped bolus remains same. However for ascending values of wall mass parameter E_3 no changes observed on both the size and number of bolus. The impact of material fluid parameter W on the stream lines pattern gained via Fig. (22). The size of trapped bolus extremely decreases whilst the number of bolus increases nears the upper wall and decreases toward the lower wall for larger values of W. It can be scrutinize from Fig.(23) that due to influence of Hall parameter m the size of bolus enhances . The impact of Hartman number H is studied through Fig. (24). It is noticed that larger values of H the bolus becomes smaller. This result is familiar since the Lorentz force opposite the fluid flow and hence reduces the fluid velocity. Fig.(25) is drawn to examine the effect of angle inclination α on streamlines patterns. Enhancement of α enlarge the size of the trapped bolus. Fig.(26) depicts the behavior of the upper wall amplitude b on streamlines. Considerable impact for b on trapped bolus in formulation and size can be noticed. We found that for larger values of b the bolus becomes larger and its form absolutely changes. Fig.(27) revealed that the trapped bolus decreases in size and numbers for higher values of non-uniform channel parameter m_1 . it can be analyzed from Fig.(28) that for higher values of Froude number Fr the size of the trapped bolus decreases and more bolus create near the upper wall opposite situation is seen near the lower wall.

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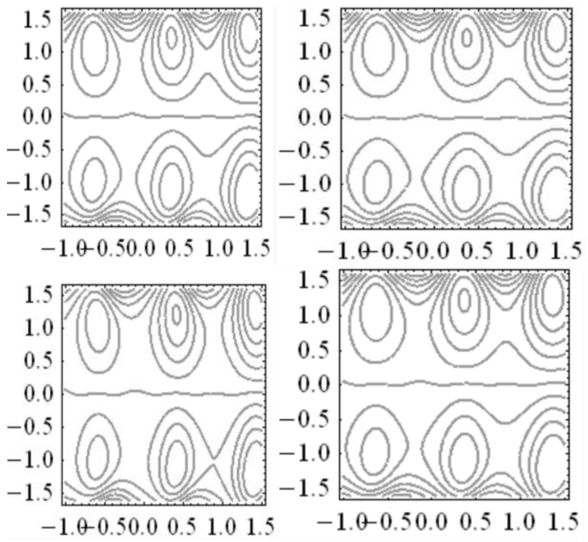
(a) (b)



(c) (d)

Fig.(21) Streamlines for $\left\{w = 0.1, A = 0.1, m = 0.2, \mathrm{Fr} = 0.1, H = 0.5, \mathrm{R1} = 0.2, \alpha = \frac{\pi}{3}, \gamma = 0.01, \phi = \frac{\pi}{6}, \mathrm{a} = 0.1, \mathrm{b} = 0.2, \mathrm{m}_1 = 0.1\right\}$(a) $(\mathrm{E1} = 0.02, \mathrm{E2} = 0.01, \mathrm{E3} = 0.01)$ (b) $(\mathrm{E1} = 0.04, \mathrm{E2} = 0.01, \mathrm{E3} = 0.01)$ (c) $(\mathrm{E1} = 0.02, \mathrm{E2} = 0.09, \mathrm{E3} = 0.01)$ (d) $(\mathrm{E1} = 0.02, \mathrm{E2} = 0.01, \mathrm{E3} = 0.05)$

(a) (b)

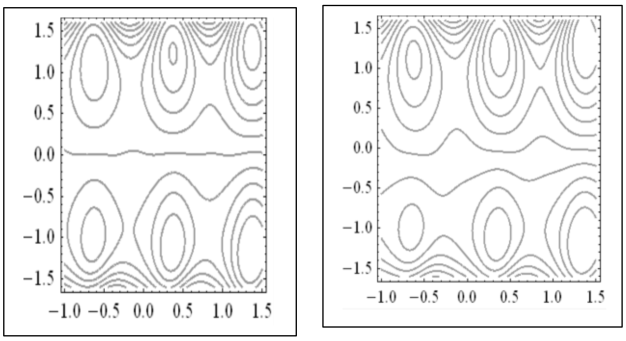


Fig.(22) Streamlines for $\left\{\mathrm{E1} = 0.02, \mathrm{E2} = 0.01, \mathrm{E3} = 0.01, A = 0.1, m = 0.2, \mathrm{Fr} = 0.1, H = 0.5, \mathrm{R1} = 0.2, \alpha = \frac{\pi}{3}, \gamma = 0.01, \phi = \frac{\pi}{6}, \mathrm{a} = 0.1, \mathrm{b} = 0.2, \mathrm{m}_1 = 0.1\right\}$ (a)$\mathrm{W} = 0.1$ (b) $\mathrm{W} = 0.5$

(a) (b)

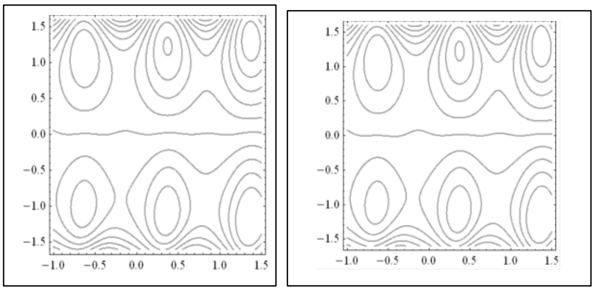


Fig.(23) Streamlines for $\mathrm{E1} = 0.02, \mathrm{E2} = 0.01, \mathrm{E3} = 0.01, W = 0.1, A = 0.1, \mathrm{Fr} = 0.1, H = 0.5, \mathrm{R1} = 0.2, \alpha = \frac{\pi}{3}, \gamma = 0.01, \phi = \frac{\pi}{6}, \mathrm{a} = 0.1, \mathrm{b} = 0.2, \mathrm{m}_1 = 0.1\}$ (a)$\mathrm{m} = 0.2$ (b)$\mathrm{m} = 0.6$

(a) (b)

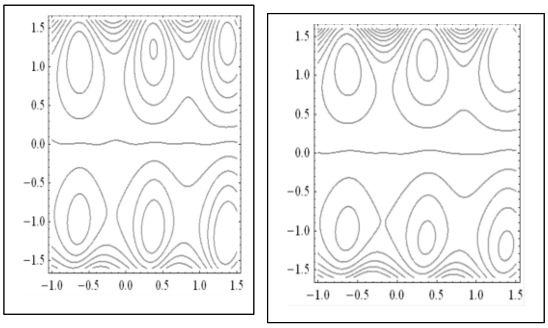


Fig.(24) Streamlines for $\mathrm{E1} = 0.02, \mathrm{E2} = 0.01, \mathrm{E3} = 0.01, W = 0.1, A = 0.1, \mathrm{Fr} = 0.1, m = 0.2, \mathrm{R1} = 0.2, \alpha = \frac{\pi}{3}, \gamma = 0.01, \phi = \frac{\pi}{6}, \mathrm{a} = 0.1, \mathrm{b} = 0.2, \mathrm{m}_1 = 0.1\}$ (a)$\mathrm{H} = 0.2$ (b)$\mathrm{H} = 0.8$

(a) (b)

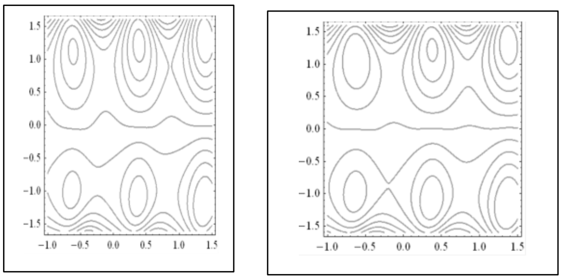


Fig.(25) Streamlines for $\{\mathrm{E1} = 0.02, \mathrm{E2} = 0.01, \mathrm{E3} = 0.01, W = 0.1, A = 0.1, \mathrm{Fr} = 0.1, m = 0.2, \mathrm{R1} = 0.2, H = 0.5, \gamma = 0.01, \phi = \frac{\pi}{6}, \mathrm{a} = 0.1, \mathrm{b} = 0.2, \mathrm{m}_1 = 0.1\}$ (a) $\alpha = \frac{\pi}{6}$ (b)$\alpha = \frac{\pi}{4}$

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(a) (b)

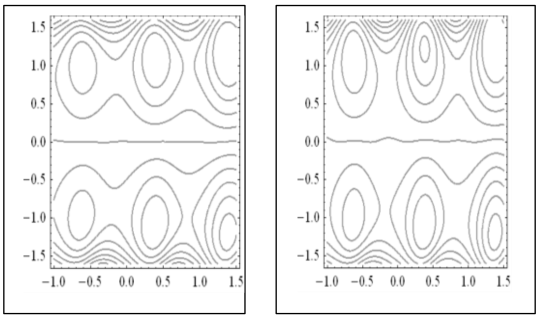

Fig.(26) Streamlines for $\{E1 = 0.02, E2 = 0.01, E3 = 0.01, W = 0.1, A = 0.1, Fr = 0.1, m = 0.2, R1 = 0.2, H = 0.5, \gamma = 0.01, \alpha = \frac{\pi}{3}, \emptyset = \frac{\pi}{6}, a = 0.1, m_1 = 0.1\}$ (a) $b = 0.1$ (b) $b = 0.2$

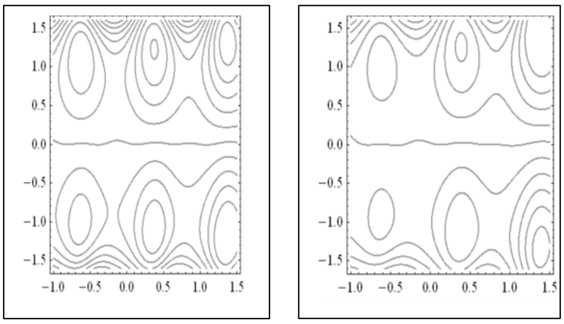

Fig.(27) Streamlines for $\{E1 = 0.02, E2 = 0.01, E3 = 0.01, W = 0.1, A = 0.1, Fr = 0.1, m = 0.2, R1 = 0.2, H = 0.5, \gamma = 0.01, \alpha = \frac{\pi}{3}, \emptyset = \frac{\pi}{6}, a = 0.1, b = 0.2\}$ (a) $m_1 = 0.1$ (b)$m_1 = 0.2$

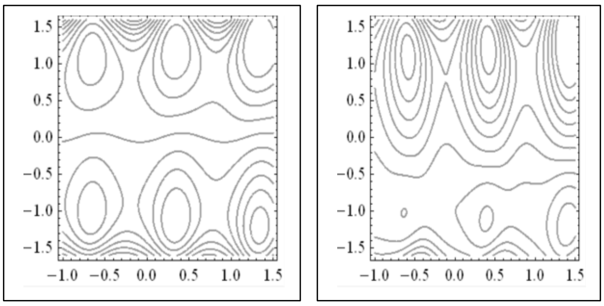

Fig.(28) Streamlines for $\{E1 = 0.02, E2 = 0.01, E3 = 0.01, W = 0.1, A = 0.1, m_1 = 0.1, m = 0.2, R1 = 0.2, H = 0.5, \gamma = 0.01, \alpha = \frac{\pi}{3}, \emptyset = \frac{\pi}{6}, a = 0.1, b = 0.2\}$(a) $Fr = 0.1$ (b)$Fr = 0.2$

5. Concluding Remarks

Peristaltic motion of Powell- Eyring fluid flowing during an inclined tapered symmetric geometry is analyzed. Hall and Ohm's heating effects are taken into account as well as heat transfer phenomenon is discussed. Analytical solutions have been determined using perturbation technique. Major considerations of the analysis have been listed below:

1. Velocity profile shows a parabolic behavior in nature. Furthermore it has an increasing function due to wall parameters E_1 , and E_2, phase difference angle ϕ , Hall parameter m, and non- uniform channel parameter m_1. Whilst a decreasing behavior is recognized via wall parameterE_3, Hartman parameter$H$.
2. The impacts of Powell- Eyring fluid parameters A and W on the velocity field and temperature profile are qualitatively opposite.
3. The temperature profile exhibits an increasing function when Brinkman Br number, and inclination of the channel α increase whereas it decreases with Froude number Fr and phase difference angle ϕ.
4. Hartman number H diminishes the temperature distribution field while mixed results in temperature profile are seen upon Hall parameter m .
5. Heat transfer rate obey an oscillatory behavior for all embedded parameters.
6. Magnitude of heat transfer coefficient gives a mixed function for Froude number Fr , inclination of the channel α.
7. The trapped bolus size occurring in tapered a symmetric channel increasing with Hall parameter m while its size and numbers decrease with Hartman number H.
8. Formulation of fluid bolus is strongly dependent on the non- uniform channel parameter m_1.

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تحليل انتقال الحرارة للانتقال الموجي لمائع باول- ايرنك خلال قناة متناظرة مائلة ومدببة مع تأثيرات هول وأوم الحرارية

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المستخلص :

في هذا البحث، ناقشنا مبدأ انتقال الحرارة في الجريان الموجي لمائع باول- ايرنك ضمن قناة مدبب متماثل ومائل. اخذا بنظر الاعتبار تاثير هول، وشرطي الانزلاق للسرعة والحرارة وكذلك تاثير اوم. معادلات الحاكمة لموازنة الكتلة ،الحركة والطاقة تم صياغتها ومن ثم تبسيطها اعتمادا على تقريب كبر طول الموجة وصغر عدد رينولد. اخيرا النتائج الصورية اعطيت لتحليل سلوك المتغيرات الوسطية المستجدة في المسالة وقد لوحظ ان تاثير متغير هول وعدد هارتمن يمتلكان خصائص متعارضة على محور السرعة.

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Fixed Point Principles in General b −Metric Spaces and b −Menger Probabilistic spaces

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Abstract

In this work, three general principles for existence a fixed point and a common fixed point are proved in types of general metric spaces, which conclude the existence a fixed point of set−valued mapping in a general b −metric space , the existence of common fixed point of three commuting orbitally continuous $\chi-$ condensing mappings and a result of fixed point for set−valued condensing mapping defined on probabilistic bounded subset of $b-$ Menger probabilistic metric space.

Keywords: G_b −metric space**s,** Menger probabilistic, set−valued mappings, condensing mappings**,** fixed ponts.

Mathematics Subject Classification: 47H05, 47H09

1. Introduction

Aghajani et al. [1] introduced the notion of $\boldsymbol{G_b}$-metric space as a generalization of $\boldsymbol{b-}$ **metric** spaces. Results of $\boldsymbol{G_b}$ −metric about fixed points and its applications can be found in the research papers of Abed and Jabbar [1-3], Mustafa Khan, Arshad and Ahmad [5] and references there in.

Another generalization of metric spaces introduced by Menger [6] called probabilistic metric space. Many results on the existence of fixed points or its application in nonlinear equations in these spaces have been studied by many researchers (see e.g. [7]). the notion of a generalized probabilistic metric space or a PGM −space as a generalization of a PM −space and a G −metric space have been defined by Zhou et al. [8] .

And then, Zhuet al. [10] presented some fixed point theorems in generalized probabilistic metric spaces.

Here, there are two aims the first one is proving the existence of fixed points and common fixed points for set-valued (or single valued) condensing mappings in orbitally complete general b- metric. The second is to define a measure for probabilistic subset of a Menger general b- metric space and employ it to prove a fixed point theorem for condensing mapping.

In this paper, $\mathcal{M}$ is general $b-$ metric space and

$2^{\mathcal{M}} = \{C: \emptyset \neq C \subset \mathcal{M}\}$,

$CB(\mathcal{M}) = \{C: \emptyset \neq C \subset \mathcal{M}, C \text{ is closed and bounded}\}$,

$K(\mathcal{M}) = \{C: \emptyset \neq C \subset \mathcal{M}, C \text{ is compact}\}$,

Also, $\mathbb{R}^+$ will be denote to non-negative reals, $\mathbb{R}^*=[-\infty,\infty]$, $\boldsymbol{N}$ be positive integers, $\bar{A}$ be the closure of a set A and $\rightrightarrows$ be set-valued mapping .

2. Preliminaries

"Let $\mathcal{M}$ be a non-empty set and $\Lambda: \mathcal{M}^3 \to R^+$ be a function satisfying the following for u, v, w and a in $\mathcal{M}$:

$i - \Lambda(u,v,w) = 0 \; iff \; u = v = w$

$ii - \Lambda(u,u,v) > 0 , u \neq v$

$iii - \Lambda(u,u,v) \leq \Lambda(u,v,w), \; u \neq v$

$iv - \Lambda(u,v,w) = \Lambda(p\{u,v,w\})$, p is permutation

$v - \Lambda(u,v,w) \leq b[\Lambda(u,a,a) + \Lambda(a,v,w)], b \geq 1.$

Then the pair $(\mathcal{M},\Lambda)$ is called general b- metric space."[1]

"A general $b-$ metric space $\mathcal{M}$ is called symmetric if $\Lambda(u,v,v) = \Lambda(v,u,u)$. For $u_0 \in \mathcal{M}$,$r > 0$ the ball with center u_0 and radius r is $B_r(u_0,r) = \{y \in \mathcal{M}: \Lambda(u_0,v,v) < r\}$ and the family $\{B_{\gamma}(u,r): u \in \mathcal{M} ,r > 0\}$ is a base of a topology [12]. The diameter of a set $C \subseteq \mathcal{M}$ is $(A) = sup_{a,b,c \in C} \Lambda(a,b,c)$" . Also, the definition of generalized $b-$ metric implies that

Proposition 2.1: [1] "For all $u, v, w, a \in \mathcal{M}$, the following hold

(1) $\Lambda(u,v,w) \leq b[\Lambda(u,u,v) + \Lambda(u,u,w)]$

(2) $\Lambda(u,v,v) \leq 2b\,\Lambda(v,u,u)$

(3) $\Lambda(u,v,w) \leq b[\Lambda(u,a,w) + \Lambda(a,v,w)]$, $b \geq 1$"

From this Proposition, we have

(5) $\Lambda(u,v,w) \leq b\,\Lambda(u,a,a) + b^2\,\Lambda(v,a,a) + b^2\Lambda(w,a,a)$

"A sequence $\{u_n\} \subseteq \mathcal{M}$ is [1] or [2]

(1) Cauchy sequence if ,$\forall\, \varepsilon > 0$ $\exists n_0 \in \boldsymbol{N}$ such that $\forall m,n,i \geq n_0$, $\Lambda(u_n,u_m,u_i) < \varepsilon$

(2) Convergent to a point $u \in \mathcal{M}$if $\forall \epsilon > 0, \exists n_0 \in \boldsymbol{N}$ such that $\forall$ $n,m \geq n_0$, $\Lambda(u_n,u_m,u) < \varepsilon$.

A space $\mathcal{M}$ is called complete if every Cauchy sequence is convergent in $\mathcal{M}$." Throughout this paper $(\mathcal{M},\Lambda)$ denotes general b $-$metric space.

Proposition 2.2: [1] " Let $\{u_n\}$ be a sequence in $\mathcal{M}$, then $\{u_n\}$ is Cauchy iff $\forall\ \varepsilon > 0$, $\exists\ n_0 \in \mathbf{N}$ such that $\Lambda(u_n,u_m,u_m) < \varepsilon$, $\forall$ m, n $\geq n_0$."

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Proposition 2.3: [4] "$\{u_n\}$ is convergent to $u \Leftrightarrow G(u, u_n, u) \to 0 \Leftrightarrow G(u_n, u, u) \to 0 \Leftrightarrow G(u_n, u_m, u) \to 0$ as $n, m \to \infty$. The sequence $\{u_n\}$ is Cauchy $\Leftrightarrow$ G(u_n, u_m, u_m)→0 as n, m→∞."

Definition 2.4: Let ($\mathcal{M}$, Λ) and ($\mathcal{M}'$, Λ') be general b −metric spaces, and $T : \mathcal{M} \to \mathcal{M}'$ be a function. Then T is continuous at a point $a \in \mathcal{M}$iff for $\forall\ \varepsilon > 0$, $\exists \delta > 0$ such that $u, v \in \mathcal{M}$and $G(a, u, v) < \delta$ implies $\Lambda'(T(a), T(u), T(v)) < \varepsilon$. A function T is continuous on $\mathcal{M}$ iff it is continuous at $\forall a \in \mathcal{M}$.

Definition 2.5: Let $T: \mathcal{M} \to \mathcal{M}$ then

(1) The orbit of a point $u \in \mathcal{M}$ is the set $O(u) = \{u, Tu, T^2u, \ldots..\}$, and let $C \subseteq \mathcal{M}$the orbit of a set C by T is $O_{T(C)}$= { T^n(u) : $n = 0, 1, 2, \ldots, u \in C$ } .

(2) An orbit of a point z is said to be bounded if $\exists\ K$> 0 such that $\Lambda(u, v, w) \leq K\ \ \forall\, u, v, w \in O(z)$, K is called a bound of $O(z)$.

(3) $\mathcal{M}$is said to be $T-$ orbitally bounded if $\delta\,(O(z)) < \infty$ $\forall z \in \mathcal{M}$.

(4) $\mathcal{M}$is said to be $T-$ orbitally complete if every Cauchy sequence in $O(z)$ converges to a point in $\mathcal{M}$.

(5) T is a called orbitally continuous if for any $\{u_n\} \subseteq O(u)$ and $u_n \to u$ implies $Tu_n \to Tu$, $\forall u \in \mathcal{M}$.

(6) The point $u \in \mathcal{M}$ is called a fixed point of the set-valued mapping $T : \mathcal{M} \to 2^{\mathcal{M}}$ if $u \in Tu$ and u is fixed point of a single mapping $T : \mathcal{M} \to \mathcal{M}$ if $u = Tu$.

The orbit of u by two mappings S, T is, $O(u)_{ST} =: \{u, Su, TSu, STSu, \ldots\}$, when T, S are commuting then $O(u) = \{T^m S^n u : m, n = 0, 1, \ldots\}$. Analogous to the general Hausdorff distance in [11] we define the following

Defintion 2.6 The function $\boldsymbol{\Lambda}: [CB(\mathcal{M})]^3 \to \mathbb{R}^+$ is called general $b-$ Hausdorff distance if

$$\boldsymbol{\Lambda}(A, B, C) = max\{sup_{x\in A}\Lambda(x, B, C), sup_{x\in B}\Lambda(x, C, A), sup_{x\in C}\Lambda(x, A, B)\},$$

where,

$$\Lambda(x, B, C) = \Lambda(x, B) + \Lambda(B, C) + \Lambda(x, C),$$

$$\Lambda(x, B) = \inf\{\Lambda(x, y), y \in B\},$$

$$\Lambda(A, B) = inf\,\{\Lambda(a, b), a \in A, b \in B\}.$$

Directly, we obtain the following

Lemma 2.7: If $A, B \in CB(X)$ and $a \in A$, then $\forall \varepsilon > 0, \exists b \in B \ni$ $\Lambda(a, b, b) \leq \boldsymbol{\Lambda}(A, B, B) + \varepsilon$. And, if B is compact then $\Lambda(a, b, b) \leq \boldsymbol{\Lambda}(A, B, B)$.

Let C be a bounded subset of $\mathcal{M}$, the measure of non−compactness of C is

$$\chi(C) = inf\,\{r > 0: C \subset \bigcup_{i=1}^{n} C_i \text{ and } \delta(C_i) \leq r\}.$$

Clearly, χ satisfies the following:

i- $\chi(\emptyset) = 0$

ii- $\chi(C) = 0 \Leftrightarrow$ C is relatively compact

iii- $0 \le \chi(C) \le diam(C)$

iv- $C \subseteq D \Rightarrow \chi(C) \le \chi(D)$

v- $\chi(C + D) \le \chi(C) + \chi(D)$

vi- $\chi(C) = \chi(\overline{C})$

vii- $\chi(\cup C_i) = max\{\chi(C_i)\}$

A set-valued mapping $T: \mathcal{M} \rightrightarrows 2^{\mathcal{M}}$ is called χ- condensing if for any bounded set $C \subset \mathcal{M}$, $T(C)$ is bounded and $\chi(T(C)) < \chi(C)$, $\chi(C) > 0$.

The space of all probability distribution functions is $\Delta^+ = \{s: \mathbb{R}^* \to [0,1]: s \text{ is left} - \text{continuous, non decreasing } on\ \mathbb{R}, s(0) = 0, \ s(+\infty) = 1\}$ and D+ = $\{s \in \Delta^+ : \ell^- s(+\infty) = 1\}$.

Here,

$\ell^- s(a_0) = \ell^- s(a_0) = \lim_{a \to a_0^-} s(a)$.

The space Δ^+ is partially ordered by ordering

$s \le r$ iff $(a) \le r(a), \forall a \in \mathbb{R}$.

The maximal element for $\Delta+$ is the probability distribution function

$$h(a) = \begin{cases} 0, & if\ a \le 0 \\ 1, & if\ a > 0 \end{cases} \quad \dots \quad (1)$$

Definition 2.8: A mapping $\Delta : [0,1]^2 \to [0,1]$ is a continuous $t-$norm if Δ satisfies the following

(i) Δ is commutative and associative;

(ii) Δ is continuous;

(iii) $\Delta(a, 1) = a, \forall a \in [0,1]$;

(iv) $\Delta(a, b) \le \Delta(c, d)$, whenever $a \le c$ and $c \le d$, $a, b, c, d \in [0,1]$.

Definition 2.9: A Menger probabilistic $b-$metric space (briefly, $b-$ Menger space) is a triple $(\mathcal{M}, \Lambda, \Delta)$, where, $\emptyset \neq \mathcal{M}$ Δ is a continuous $t-$norm , and $\Lambda: \mathcal{M} \times \mathcal{M} \times \mathcal{M} \to D^+$ such that, if $\Lambda_{x,y,z}$ denotes the value of Λ at the triple (x, y, z) , the following conditions hold for all $x, y, z, a \in \mathcal{M}$ and $\forall\ t, s \ge 0$

(1) $\Lambda_{x,y,z}(t) = 1$, iff $x = y = z$;

(2) $\Lambda_{x,y,z}(t) < 1$ iff $x \neq y$;

(3) $\Lambda_{x,y,z}(t) = \Lambda_{y,z,x}(t) = \Lambda_{z,x,y}(t) = \dots$,

(4) $\Lambda_{x,y,z}(t + s) \ge \Delta(\Lambda_{x,a,a}(t), \Lambda_{a,y,z}(s))$, .

Definition 2.10: A $b-$ Menger space is called symmetric if $\Lambda_{x,y,y}(t) = \Lambda_{y,x,x}(t), \forall x, y \in \mathcal{M}$.

Definition 2.11: Let $(\mathcal{M}, \Lambda, \Delta)$ be a $b-$ Menger space.

i- A sequence $\{x_n\}$ in $\mathcal{M}$ is said to be

-Convergent to $x \in M$ if $\forall \varepsilon > 0$, $\lambda > 0$, $\exists N \in \boldsymbol{N}$ such that $\Lambda_{x,x_n,x_m}(\varepsilon) > 1 - \lambda$ whenever $m, n \ge N$.

-Cauchy sequence if, $\forall \varepsilon > 0$ and $\lambda > 0$, $\exists N \in \boldsymbol{N}$ such that $\Lambda_{x_n,x_m,x_l}(\varepsilon) > 1 - \lambda$ whenever $n, m, l \ge N$.

ii- $\mathcal{M}$ is complete if every Cauchy sequence in $\mathcal{M}$ converges to a point in $\mathcal{M}$.

iii- The strong $\lambda-$neighborhood of x is

$N_x(\lambda) = \{q \in \mathcal{M} : \Lambda_{x,q,q}(t)(\lambda) > 1 - \lambda\}$

and neighborhood system for $\mathcal{M}$ is the union $\cup_{x \in V} N_x$ where $N_x = \{N_x(\lambda) : \lambda > 0\}$.

iv- The (ϵ, λ) – topology in $\mathcal{M}$ is introduced by the family of neighborhoods given by

$U_v(\epsilon, \lambda) = \{u; T_{u,v,v}(\epsilon) > 1 - \lambda\}$.

If $t-$ norm Δ is continuous then $\mathcal{M}$ is a metrizable topology space, with respect to $(\epsilon,\lambda)-$ topology.

v- The probabilistic diameter of a subset A of $\mathcal{M}$ is
$D_C(t) = sup_{s<t} \; inf_{p,q,q \in A} T_{p,q,q}(s), t \in \mathbb{R}^+$,
and the set C is probabilistic bounded if and only if $sup_{t\in\mathbb{R}+} D_C(t) = 1$.

Analogously with Measure of non-compact set, we give

Definition 2.12: Let $\mathcal{M}$ be a $b-$ Menger space and $C \subset \mathcal{M}$ be a probabilistic bounded, the function $\chi_C : \mathbb{R} \to [0,1]$ is $\chi_C = \sup\{\epsilon > 0$, there is a finite family $\{C_j\}_{j\in J}$ in $\mathcal{M}$ such that $C = \bigcup_{j\in J} C_j$ and $D_{C_j}(u) \geq \epsilon, \forall j \in J\}$.

The χ_C function has the following properties:

i- $\chi_A(t) \geq D_A(t)$, $\forall t \in \mathbb{R}^+$

ii- $\emptyset \neq A \subset B \subset \mathcal{M} \Rightarrow \chi_A(t) \geq \chi_B(t)$, $\forall t \in \mathbb{R}^+$

iii- $\chi_{A\cup B}(t) = \{\chi_A(t), \chi_B(t)\}$, $\forall t \in \mathbb{R}^+ \chi_A(t) = \chi_{\bar{A}}(t)$, $t \in \mathbb{R}^+$

iv- $\chi_A = h \Leftrightarrow A$ is precompact, where $h(x)$ as in (1)

Definition 2.13: Let $\subset \mathcal{M}$, K is probabilistic bounded, $T : K \to 2^{\mathcal{M}}$ and $T(K)$ is probabilistic bounded subset of $\mathcal{M} \ni \forall B \subset K$ and
$\chi_{T(B)}(t) \leq \chi_B(t), \forall t > 0$
implies that B is pre-compact then T is called a condensing mapping on K w.r.t. χ.

3. Fixed Points in general $b-$ metric spaces

Let Φ denoted the class of all function $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ which satisfying the following conditions:

(1) φ is continuous ,
(2) φ is non- decreasing,
(3) $\varphi(t) < t. \forall t > 0$,and
(4) $\sum_{n=1}^{\infty} \varphi^n(t) < \infty$, $\forall t \in \mathbb{R}^+$.

Thus $\varphi^n(0) = 0$ for each n and $\lim_{n\to\infty} \varphi^n(t) = 0 \; \forall t > 0$.

Directly, we have the following Lemma

Lemma 3.1: If $\{u_n\}$ is a bounded sequence in $\mathcal{M}$ with constant bound K satisfying
$\Lambda(u_n, u_{n+1}, u_m) \leq \varphi^n(k), \forall m > n \in N$,
where $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ satisfying $\sum_{n=1}^{\infty} \varphi(t) < \infty, \forall t \in R^+$ then $\{u_n\}$ is Cauchy.

Theorem 3.2: Let $\mathcal{M}$ be a general $b-$ metric space and $T: \mathcal{M} \rightrightarrows K(\mathcal{M})$ be a set-valued mapping. if $\mathcal{M}$ is $T-$ orbitally complete and

$$\Lambda(Tx, Ty, z) \leq \varphi(\Lambda(x,y,z)) \quad \ldots (2)$$

$\forall x, y, z \in \mathcal{M}$ with $x \notin T(x), y \notin T(y)$

for all $x, y, z \in \mathcal{M}$ with $x \notin T(x), y \notin T(y)$, where $\varphi \in \Phi$. Then T has a fixed point.

Proof: Let $x_0 \in \mathcal{M}$, define $\{x_n\}$ by $x_{n+1} \in Tx_n$, $n \geq 0$

The proof is divided into three steps: we must prove that

- $\Lambda(x_n, x_{n+1}, x_m) \leq \varphi^n(L) \quad \ldots (3)$

for any $m > n$,
$L = \lim_{m\to\infty} \sum_{i=0}^{m} 2b^{m-i} \varphi^i(q)$,
$q = max\{\Lambda(x_0, x_1, x_0), \Lambda(x_0, x_1, x_1)\}$.

- $\{x_n\}$ in (2) is bounded .
- $\lim_{n\to\infty} x_n = u \in Tu$

For first step, from (2)

$\Lambda(x_n, x_{n+1}, x_m) \le \boldsymbol{\Lambda}(Tx_{n-1}, Tx_n , x_m)$
$\le \varphi ((x_{n-1}, x_n, x_m)), \ldots$ (4)
$n = 1,2, \ldots$

By induction,
$\Lambda (x_n, x_{n+1}, x_m) \le \varphi(\Lambda(x_{n-1}, x_n, x_m)) \le \varphi^2(\Lambda(x_{n-2}, x_{n-1}, x_m)) \ldots \le \varphi^n(\Lambda(x_0, x_1, x_m))$ … (5)

By using Proposition (2.1) , definition of general $b-$ metric and (5) we get

$\Lambda(x_0, x_1, x_m) \le b\, \Lambda (x_0, x_{m-1}, x_{m-1}) + b^2\, \Lambda (x_1, x_{m-1}, x_{m-1}) + b^2 \Lambda \left(x_m, x_{m-1}, x_{m-1}\right)$
$\le b\Lambda(x_0, x_1, x_{m-1}) + b^2 \varphi^{m-1} (\Lambda(x_0, x_1, x_0)) + b^2 \varphi^{m-1}(\Lambda(x_0, x_1, x_1))$

Since $q = max\{(x_0, x_1, x_0), \Lambda(x_0, x_1, x_1)\}$, then
$\Lambda (x_0, x_1, x_m) \le \sum_{i=0}^{m} 2b^{m-i} \varphi^i$ (q)
$\le L < \infty$

Substituting into (5) yield (3). For second step, for any integers $s \ge m \ge n$, there exists p and r such that
$\Lambda(x_n, x_m, x_s) = \Lambda(x_n , x_{n+p}, x_{n+r})$

By similar argument, we have $\Lambda (x_n, x_m, x_s) < \infty$. This showing the second step. Moreover,
Lemma (3.1) implies that $\{x_n\}$ is Cauchy, hence convergent to $u \in \mathcal{M}$.
For third step
$\Lambda(x_{n+1}, Tu, x_n) \le \boldsymbol{\Lambda}(Tx_n, Tu , x_n)$
$\le \varphi (\Lambda(x_n, u , x_n))$
$\Rightarrow (u, Tu , u) = 0$, as $n \to \infty$, and hence $u \in Tu$.

As a special case of the above theorem, we give

Corollary 3.3: Let $\mathcal{M}, \Lambda$ and T be in Theorem (3.2) such that

$\boldsymbol{\Lambda} (Tu, Tv, w) \le \lambda \Lambda (u, v , w)$, $u, v , w \in \mathcal{M}$ with $u \notin T(u)$, $u \notin T(u)$ … (6)

where $0 \le \lambda<1$. Then T has a fixed point.

For $\chi-$ condensing, we need the following lemma:

Lemma 3.4: Let $\mathcal{M}, \Lambda$ and $\mathcal{M} \rightrightarrows 2^{\mathcal{M}}$ be χ $-$condensing mapping. If $\mathcal{M}$ an $T-$ orbitally bounded and complete Then $\overline{O(u)}$ is compact, $\forall u \in \mathcal{M}$.

Proof: Let $u \in \mathcal{M}$ and $M = \{u_n\}$, where $u_n = T^n u$. Then

$$M = \{u\} \cup \{ Tu , T^2 u , \ldots\} = \{u\} \cup T(M)$$

If M is not pre-compact, then $\chi (M) = \chi (T(M)) < \chi (M)$, which is a contradiction.
Therefore, $\overline{M} = \overline{O(u)}$ is compact, since $\overline{M}$ is complete.

Theorem 3.5: Let $T : \mathcal{M} \rightrightarrows CB(\mathcal{M})$ be an orbitally continuous $\chi-$ condensing mapping on a $T-$ orbitally bounded complete general $b-$ metric space $\mathcal{M}$. If

$\boldsymbol{\Lambda}(Tx, Ty , z) < \Lambda(x, y , z)$, … (7)

$\forall x, y , z \in \mathcal{M}$, $x \notin T(x)$, $y \notin T(y)$
then T has a fixed point .

Proof: Suppose that x_0 in $\mathcal{M}$, then by Lemma(3.4) $M = \overline{O(x_0)}$ is compact. Since T is continuous on M then $\boldsymbol{\Lambda} (Tx, Ty , z)$ and $\Lambda(x, y , z)$ are bounded.
Define the well-defined
$S : M^3 \to R^+$ by $S(x, y, z) = \frac{\Lambda(Tx,Ty ,z)}{\Lambda(x,y ,z)}$ $\forall x, y , z \in M$

By the continuity of T, S is continuous. The compact of product sets is compact implies that M^3 is compact. So, S attains its maximum at $(u, v, w) \in M^3$. Call the value C from (3.1) , 0 < C < 1. By definition of S we get

$$\frac{\Lambda(Tx,Ty,z)}{\Lambda(x,y,z)} = S(x,y,z) \le S(u,v,w) = C$$

for all $x, y, z \in \mathcal{M}$ with $x \notin T(x), y \notin T(y)$, Now, the result follows from Corollary (3.3). This completes the proof.

Define

$$\delta(x,y,z) = \delta(O(x) \cup O(y) \cup O(z))$$

$$\bar{\delta}(x,y,z) = \delta(\overline{O(x)} \cup \overline{O(y)} \cup \overline{O(z)})$$

Theorem 3.6: : Let $T_i : \mathcal{M} \to \mathcal{M}$ are commuting orbitally continuous $\chi-$ condensing mappings , $i = 1,2,3$, such that

$\Lambda\ (T_1x, T_2y, T_3z\ ) < \bar{\delta}(x,y,z)$, $\forall\, x, y, z \in \mathcal{M}$ and $T_1x \neq T_2y \neq T_3z$ (8)

If $\mathcal{M}$ is orbitally bounded and complete. Then $\exists!\, u \in \mathcal{M}, T_i(u) = u, \forall i.$

Proof: Let $O_3(x) = \{ T_1^k\ T_2^m\, T_3^n : k, m, n = 0,1,2 \dots . \}$ be the orbit of x by T_1, T_2, T_3 .

Since, $\chi\ (O_{T_1}(x)) = max\, \{\chi(x)$, $\chi(O_{T_1}(T_1 x))\} = \chi\,(T_1\,(O_{T_1}(x)))$ and T_1 is condensing $\Rightarrow O_{T_1}(x)$ pre-compact. Similarly for $O_3(x)$;

$\chi\ (O_3(x)) = max\,\{\chi\ (O_{T_1}(x))$, $\chi\ (O_{T_2}(O_{T_1}(x)))$, $\chi\ (O_{T_3}(O_{T_2}(O_{T_1}(x)))\}$.

Therefore, the condition of condensing$\Rightarrow$ $O_3(x)$ is totally bounded, $\Rightarrow$ pre-compact.

Now, if $M_0 = \overline{O_3(x)}$, so, M_0 is compact . Clearly $T_i\ (M_0) \subset M_0$, $i = 1,2,3$. Now, let

$M_i = \cap_{n=1}^{\infty} T_i^n(M_{i-1})$, $i = 1, 2, 3$

Thus M_i is T_i-invariant, $i = 1, 2, 3$. The finite intersection property assures that M_i is non-empty compact subsets, $i = 1, 2, 3$.

Suppose $u \in M_1$, there exist $x_n \in T_1^{n-1}(M_0)$ such that $T_1(x_n) = u$, $n = 1,2, \dots$

Thus a subsequence, say also (x_n) converges to $v \in M_0$.

Since $\{x_{n+1}, x_{n+2}, \dots\} \subset T_1^n(M_0)$ and $T_1^n(M_0) \subset M_0 \Rightarrow v \in T_1^n(M_0)$, $n = 1,2, \dots$

We have $v \in M_1$ and $T_1(v) = u$ $\Rightarrow T_1(M_1) = M_1$.

The properties of M_2 and $T_1 \Rightarrow T_1(M_2) = M_2$, $T_1(M_3) = M_3$.

Similarly, $T_i\ (M_2) = M_2$, $T_i\ (M_3) = M_3$,
i =1, 2,3.

Now, We claim that M_3 singleton, and $M_3 = \{x\}$, then x is the singleton fixed point of T_1, T_2, T_3 .

If not, $\delta\ (M_3) > 0$, the compactness of $M_3 \Rightarrow \exists a, b, c \in M_3, a \neq b \neq c \ni \delta(M_3) = \Lambda\ (a, b, c)$. This implies that

$a \in T_1\ (a_1)$, $b \in T_2\ (b_1)$, $c \in T_3(c_1)$, for $a_1, b_1, c_1 \in M_3$, hence, by (8), we get

$$\overline{O_{T_1}(a_1)} \cup \overline{O_{T_2}(b_1)} \cup \overline{O_{T_3}(c_1)} \subset \overline{M_3} = M_3 ,$$

$$0 < \delta(M_3) = \Lambda\ (a, b, c) < \delta(\overline{O_{T_1}(a_1)} \cup \overline{O_{T_2}(b_1)} \cup \overline{O_{T_3}(c_1)}) \le \delta(M_3)$$

which is a contradiction. So x is unique.

Note that, it is possible to modify Theorem (3.6) for finite commuting continuous condensing mappings. Also, the composition of two compact (moreover, $\chi -$ condensing) mappings is compact ($\chi -$ condensing), implies that

Corollary 3.7: Let $\mathcal{M}$ be as theorem (3.6) and $T : \mathcal{M} \to \mathcal{M}$ be an orbitally continuous compact mapping such that
$\Lambda\,(T^r u,\, T^s v,\, T^t w) < \delta(u\,, v, w)$ … (9)

$\forall u\,, v, w \in \mathcal{M}$ with $\neq Tu$, $v \neq Tv\,, w \neq Tw$ and r , s , and t are fixed positive integers . Then T has a unique fixed point in $\mathcal{M}$.

Proof: Fix $T_1 = T^r$, $T_2 = T^s$, $T_3 = T^t$, and then apply Theorem (3.6).

Corollary 3.8: Let $\mathcal{M}$ and T be as Corollary (3.7) such that

$$\Lambda(Tu\,, Tv, Tw) < max \begin{Bmatrix} \Lambda(u\,,v,w), \Lambda(u\,,Tv\,,w), \\ \Lambda(v\,,Tv\,,w), \Lambda(u\,,Tv\,,w), \\ \Lambda(v, Tu\,,w) \end{Bmatrix} \quad \ldots (10)$$

$\forall u\,, v, w \in \mathcal{M}$ with $u \neq Tu\,, v \neq Tv, or\ w \neq Tw$. Then T has a unique fixed point p in $\mathcal{M}$. Moreover T is continuous at p.

Proof: The inequality (10) implies that $\Lambda(Tu\,, Tv, Tw) < \delta(u\,, v, w)$ and the existence and uniqueness of a fixed point p follow from corollary (3.7). For continuity, let $\{w_n\} \subset \mathcal{M}$ with $w_n \neq$ p f or each n and $\lim_{n\to\infty} w_n$ = p. From (10)

$$\Lambda(p\,,p,Tw_n) = \Lambda\,(Tp\,,Tp\,,Tw_n) < \Lambda\,(p, Tp\,, w_n) \ \ldots(11)$$

Taking the limit as $n \to \infty$ implies that T is continuous at p.

The following example shows that the condensing conditions in (8) and (9) are essential.

Example 3.9: Let $\mathcal{M} = \boldsymbol{N}$,

$$\Lambda(m.n.k) = \begin{cases} 0\,, & m = n = k \\ \frac{r+1}{n+1}, & n < m, k, r \text{ is any positive real number} \end{cases}$$

Then$(\mathcal{M}\,, \Lambda)$ is complete general $b -$metric with $b = 1$. Consider $T_1(n) = T_2(n) = T_3(n) = n + 1, \forall n$ which have no fixed point in $\mathcal{M}$. Easily, one checks that T_1, T_2, T_3 satisfy conditions (8), (9) except the condensing property, since. $\delta(\mathcal{M})$= $\delta(\mathcal{M}\backslash\{1\}) = r$.

Theorem 3.10: Let $T : \mathcal{M} \to \mathcal{M}$ be a $\chi -$ condensing orbitally continuous mapping and $\mathcal{M}$be a complete bounded general $b -$ metric space. Let $\in \mathcal{M}$. If (9) holds on $\overline{O(a)}$, then T has a unique fixed point $p \in \overline{O(a)}$, and $\lim_{n\to\infty} T_n x = p, \forall\, x \in \overline{O(a)}$.

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Proof: Lemma (3.4) and hypotheses $\Rightarrow T$ is compact. Now apply corollary (3.7).

Corollary 3.11: Let $\mathcal{M}$ as in Theorem (3.9) and $T : \mathcal{M} \to \mathcal{M}$ be a continuous χ −condensing mapping satisfying (10) for all $x, y, z \in \mathcal{M}$ with $x \neq Tx$, $y \neq Ty$, or $z \neq Tz$. Then T has a unique fixed point $p \in \mathcal{M}$.

Theorem 3.12: Let $T : \mathcal{M} \to \mathcal{M}$ be a mapping on a general b −metric space $\mathcal{M}$. Suppose that $\exists a \in \mathcal{M} \ni \overline{O(a)}$ is bounded and complete. Suppose that T is continuous and χ − condensing on $\overline{O(a)}$ and satisfies (7) ,$\forall x, y, z \in \overline{O(x)}$, and $x \neq Tx$, $y \neq Ty$, $z \neq Tz$. Then T has a fixed point in $\overline{O(a)}$.

Proof: If $\exists\, n \in \boldsymbol{N} \ni T^n(a) = T^{n+1}(a) \Rightarrow T$ has a fixed point in $\overline{O(a)}$ (since Lemma (3.4) implies that $\overline{O(x)}$ is compact). Assume that $T^n(a) \neq T^{n+1}(a)$, $\forall n$.Let u be an accumulation point of $\overline{O(a)}$ and $u \neq Tu$,.Then T satisfies condition (7) $\forall x, y, z \in \overline{O(a)}$. Therefore, by Corollary (3.7) , T has a unique fixed point $p \in \overline{O(a)}$. This contradicts the assumption that $u \neq Tu$. Hence $u = T\,u$, for some accumulation point $u \in \overline{O(a)}$.

4. Fixed Points in $b-$ Menger Spaces

Consider

$$\omega = \left\{ \begin{matrix} \omega : R^+ \to R^+ : \omega \text{ is continuous} \\ \omega(0) = 0, \\ \omega(a+b) \geq \omega(a) + \omega(b), \\ a, b \in \mathbb{R}^+ \end{matrix} \right\},$$

If Δ_f is an Archimedean t – norm with the additive generator f and Δ is $t-$ norm $\ni \Delta \geq \Delta_f$ then by

$\Lambda_{\omega_1, \omega_2, \omega_2}(p, q, q) = sup\,\{u; u \geq 0, \omega_1(u) \leq f \circ \Lambda_{p,q,q}(\omega_2(u)) p, q \in \mathcal{M}\ ; \omega_1, \omega_2 \in \omega\}$

a metric on $b-$ Menger space $(\mathcal{M}, \Lambda, \Delta)$ is defined and $\Lambda_{\omega_1, \omega_2, \omega_2}$ induces the (ϵ, λ) - topology.

Theorem 4.1 : Let $(\mathcal{M}, \Lambda, \Delta)$ be a $b-$ Menger space $T : \mathcal{M} \rightrightarrows CB(\mathcal{M})$be a closed mapping and $\emptyset \neq K \subset \mathcal{M}$ be a probabilistic bounded such that $T(K)$ is probabilistic bounded. If $\mathcal{M}$ is symmetric and:

a) there exist $\omega_1, \omega_2 \in \omega$ and $f: [0,1] \to [0, b]$is a decreasing function, $b > 0 \ni$ $inf_{x \in K}\ inf_{y \in Tx}\ sup\,\{u;\ u \geq 0, \omega_1(u) \leq f \circ \Lambda_{x,y,y}(\omega_2(u))\} = 0$.

b) T is $\chi-$ condensing on K.

Then T has a fixed point.

Proof: The condition (a) follows that $\forall n \in N, \exists x_n \in K$ and $y_n \in Tx_n$,

(1) $sup\,\{u;\ u > 0,\ \omega_1(u) \leq f \circ \Lambda_{x_n, y_n, y_n}(\omega_2(u))\} < 2^{-n}$.

and then,

(2) $\omega_1(2^{-n}) > f \circ \Lambda_{x_n, y_n, y_n}(\omega_2(2^{-n}))$

We shall prove that (2) implies that $\forall\epsilon > 0$, $\lim_{n\to\infty} \Lambda_{x_n, y_n, y_n}(\epsilon) = 1$ $\Rightarrow \forall\epsilon > 0$ and $\lambda \in (0,1)$,$\exists n_0(\epsilon,\lambda) \in N$ so that $\Lambda_{x_n, y_n, y_n}(\epsilon) > 1 - \lambda$, $\forall n \geq n_0(\epsilon,\lambda)$. Since ω_1 is continuous and $\omega_1(0) = 0$, $\exists n_0(b) \in \mathrm{N} \Rightarrow \omega_1(2^{-n}) < b$, $\forall n \geq n_0(b) \Rightarrow$ for $n \geq n_0(b)$, from (2) it follows that, $f^{-1}[\omega_1(2^{-n})] < \Lambda_{x_n, y_n, y_n}(\omega_2(2^{-n}))$

Let $n_1(\epsilon,\lambda) \in N$ such that $\omega_1(2^{-n}) < f(1\text{-}\lambda)$, $\omega_2(2^{-n}) < \epsilon$, $n \geq n_1(\epsilon,\lambda)$.

then

$\Lambda_{x_n, y_n, y_n}(\epsilon) \geq \Lambda_{x_n, y_n, y_n}(\omega_2(2^{-n})) > f^{-1}[\omega_1(2^{-n})] > 1\text{-}\lambda$

For every $n \geq n_2(\epsilon,\lambda,b) = max\{n_0(b), n_1(\epsilon,\lambda)\}$, which means that $\lim_{n\to\infty} \Lambda_{x_n, y_n, y_n}(\epsilon) = 1$.

From (b), we obtain that $\{x_n ; n \in N\}$ is compact $\Rightarrow$there exists $\{x_{n_k}\}_{k\in\mathrm{N}}$ a convergent sub-sequence. If $z = \lim_{k\to\infty} x_{n_k} \Rightarrow \lim_{k\to\infty} y_{n_k} = z$. Since $y_{n_k} \in T_{x_{n_k}} \Rightarrow z \in Tz$, by closeness of T.

For $u > 0$, we need to prove that $\chi_{\{x_n ; n\in N\}}(u) = \chi_{\{y_n ; n\in N\}}(u))$.

Let $\epsilon \in (0, u)$ and $\chi_{\{y_n ; n\in N\}}(u - \epsilon) > 0$. It is enough to prove that

$\chi_{\{y_n ; n\in N\}}(u - \epsilon) \leq \chi_{\{x_n ; n\in \mathrm{N}\}}(u)$.

Let $r < \chi_{\{y_n ; n\in N\}}(u - \epsilon) \Rightarrow$there exists $A_1, A_2, \dots, A_n \subset \mathcal{M}$ such that $\{y_n ; \mathrm{n} \in \mathbf{N}\} = \bigcup_{j=1}^{n} A_j$, $D_{A_j}(u - \epsilon) \geq r, \forall j \in \{1,2,\dots,n\}$.

Thus, $inf_{x,y\in A_j} \Lambda_{x,y,y}(u - \epsilon) > r$ and so $\Lambda_{x,y,y}(\mathrm{u} - \epsilon) > r$, $\forall x,y \in A_j$.

Let $p \in (0,r)$ and $q \in (0,1)$ such that $1 \geq u, w > 1 - q \Rightarrow \Delta(u, \Delta(r,w)) > r - p$.

Since $\Delta(1, \Delta(r,1)) = 1$and the mapping $(u,w) \to \Delta(u, \Delta(r,w))$ is continuous such a number q exists. For $j \in \{1,2,\dots,n\}$, $B_j = \{ z ; \Lambda_{z,y,y}(\frac{\epsilon}{4}) > 1 - q$, for some $y \in A_j \}$.

If $n_1(\epsilon,q) \in \boldsymbol{N}$ is such that $\Lambda_{x_n, y_n, y_n}(\frac{\epsilon}{4}) > 1 - q$, $\forall n \geq n_1(\epsilon,\mathrm{q})$

then, $\{x_n ; n \geq n_1(\epsilon,\mathrm{q})\} \subseteq \bigcup_{j=1}^{n} B_j$.

We shall prove that

$sup_{s<u}\ inf_{x,y\in B_j} \Lambda_{x,y,y}(s) \geq r - p$.

If $x \in B_j$ and $y \in B_j$, then there exists $x^* \in T_j$ and $y^* \in A$ so that $\Lambda_{x,x^*,x^*}(\frac{\epsilon}{4}) > 1 - q$, $\Lambda_{y,y^*,y^*}(\frac{\epsilon}{4}) > 1 - q$.

Since $\Lambda_{x^*,x^*,y^*}(u - \epsilon) \geq r$ we have that

$\Lambda_{x,y,y}(u - \frac{\epsilon}{2}) \geq \Delta(\Lambda_{x,x^\sim,x^\sim}(\frac{\epsilon}{4}), \Delta(\Lambda_{x^*,y^*,y^*}(u - \epsilon), \Lambda_{x^*,y,y}(\frac{\epsilon}{4})))$

$\geq \Delta(\Lambda_{x,x^*,x^*}(\frac{\epsilon}{4}), \Delta(r, \Lambda_{x^*,y,y}(\frac{\epsilon}{4}))) > r - P$

which implies

$sup_{s<u}\ inf_{x,y\in B_j} \Lambda_{x,y,y}(s) \geq r - P$.

and so

$\chi_{\{x_n ; n\geq n_1(\epsilon,q)\}}(u) \geq r - p$.

Since

$\chi_{\{x_n ; n\in N\}}(u) = \chi_{\{x_n ; n\geq n_1(\epsilon,q)\}}(\mathrm{u})$,

we obtain that $\chi_{\{x_n ; n\in N\}}(u \geq \mathrm{r})$.

then $\chi_{\{y_n ; n\in N\}}(u \leq \chi_{\{x_n ; n\in N\}}(u)$, $\forall u > 0$. Similarly, $\chi_{\{x_n ; n\in N\}}(u) \leq \chi_{\{y_n ; n\in N\}}(u)$.

So, we proved that

$\chi_{\{y_n ; n\in N\}}(u) = \chi_{\{x_n ; n\in N\}}(u)$, $\forall u > 0$.

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مبادئ للنقطة الصامدة في فضاءات - المترية المعممة وفضاءات - منكر المحتملة

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المستخلص

في هذا العمل تم البرهنة على ثلاثة مبادئ عامة حول ايجاد النقطة الصامدة النقطة الصامدة المشتركة في انماط من الفضاءات - المترية المعممة حيث تضمنت وجود نقطة صامدة لتطبيقات متعددة القيم في تعميم لفضاء -المتري و وجود نقطة صامدة مشتركة لثلاثة تطبيقات المكثفة المستمرة مساريا والمتبادلة وكذلك نتيجة للنقطة الصامدة للتطبيقات المكثفة المتعددة القيم المعرفة على مجموعة مقيدة محتملة جزئية من - فضاء منكر المحتملة.

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Fabrication of Substitution-Box Initiated Implementing Invertible Mapping and Improving its Competency of Confusion by Compliment's Mechanism

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Abstract
In this research article, an innovative strategy is exploited to design a nonlinear component Substitution Box (S-box). To achieve an objective, initially we pick out a one specific sort of primitive irreducible polynomial of degree 8 to generate elements of Galois field $GF(2^8)$. Furthermore, we established a precise category of invertible mapping through an employment of left action of invertible matrix having order of 2×2 on $GF(2^8)$ to generate elements of S-box. Moreover, to improve the confusion aptitude of erected S-box we exerted 1's and 2's compliment's technique for shuffling of an elements of S-box. Eventually, to inspect the capacity of designed S-box we bring into effective action of different procedures from literature such as strict avalanche criterion, nonlinearity, linear approximation probability, bit independence criterion and differential approximation probability.

Keywords*:* Invertible mapping, 1's compliment, 2's compliment, Cryptographic features.

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1. Introduction

Due to fast and latest developments in the field of information technology security of confidential information becomes very important. Different organizations and companies are needed the protection of their important information because the concealment of confidential data may be cause of collapse of whole organization or company. To overcome this type of problem a different encryption algorithms are introduced in literature such as Advanced Encryption Standard (AES) and Data Encryption Standard (DES) etc. AES and DES encryption algorithms utilized ***Substitution-box (S-box)*** to create confusion during the process of encryption so that an attacker cannot obtain confidential date easily due to confusion capability [1]. The recent symmetric cryptosystems relies on significant constituents known as S-box. This nonlinear constituent produces nonlinearity to increase confusion during encryption as well as increases the security for cryptosystems. Due to confusion creating ability S-box is most important component of AES algorithm and a number of cryptographers are showing their interests to improve confusion creating capability of S-box.

In this research proposal, we propose to enhance the confusion creating capacity of S-box as well as increasing its security against some differential and linear attacks. First of all we generated 256 elements of $GF(2^8)$ utilizing a specific type of primitive polynomial [2]-[3]. Secondly we constructed a transformed S-box after utilization of left action of 2×2 invertible matrix on elements of Galois field $GF(2^8)$. To enhance the confusion ability of transformed S-box we applied two different techniques of 1's compliment and 2's compliment [4]. To obtain revised S-box we applied compliment methods and altered the elements with corresponding values which are generated using primitive polynomial. The proposed methodology for the construction of transformed S-box and revised S-box is also graphically presented in Fig.1. Additionally, to observe the confusion ability and strength of transformed S-box and revised S-box we also critically analyzed these S-boxes for well-known cryptographic properties. In the end, we made comparison of transformed S-box and revised S-box with renowned S-boxes from literature such as Skipjack S-box [5], S_8 Liu J S-box [6], Hussain [7] and Residue Prime S-box [8].

In this research paper we arranged the whole work as follows: in section 2 we briefly described a technique used to generate elements of $GF(2^8)$ and also discussed about stepwise procedure to design transformed and revised S-boxes. Furthermore, mathematical model used for the construction of both S-boxes is completely discussed in section 2. Section 3 includes the assessment of constructed S-boxes and their comparison with renowned S-boxes after utilization of important cryptographic properties. Section 4 deals with the conclusion of research article.

2. Step-Wise Procedure and Mathematical Model for Proposed Method

To design the substitution box for better encryption, we have designed following procedure,

Step 1: First of all we generate elements of $GF(2^8)$ through the utilization of specific primitive polynomial $p(\varphi) = 1 + \varphi^2 + \varphi^3 + \varphi^4 + \varphi^8$ which implies that $1 + \varphi^2 + \varphi^3 + \varphi^4 + \varphi^8 = 0$. Then generated values of φ under modulo $p(\varphi)$ are listed in Table 1 in terms of φ.

Step 2: An invertible mapping $y(x) = (mx + n)/(rx + s)$ is designed after the application of left action of invertible matrix $\begin{pmatrix} m & n \\ r & s \end{pmatrix}$ $on\ GF(2^8), where\ m.s - n.r \neq 0\ and$ $m, n, r, s \in GF(2^8)$

Step 3: Since $m, n, r, s \in GF(2^8)$ then the values of m, n, r, s are varying from 0:255 under a certain condition that $m.s - n.r \neq 0$. For particular instances m=35, n=23, r=14 and s=9 then we have $y(x) = (35x + 23)/(14x + 9)$.

Step 4: Utilization of particular primitive polynomial to find elements of transformed S-box $y(0), y(1), y(2), \dots, y(255)$.

Step 5: To enhance the confusion ability of S-box we applied 1's compliment method.

Step 6: Lastly, we utilized 2's compliment technique to increase more confusion after the toggling of elements of S-box.

Step 7: In the end, elements are replaced with corresponding elements from Table 1 to get elements of revised S-box.

As we already know that there exist a number of invertible mappings in the field of mathematics but for better encryption power we must need a powerful invertible mapping. For this purpose, we firstly construct an invertible mapping through the utilization of a left action of a projective linear group such as

$$y: \begin{pmatrix} 35 & 23 \\ 14 & 9 \end{pmatrix} \times GF(2^8) \longrightarrow GF(2^8)$$

$$y(x) = \frac{35x + 23}{14x + 9}, \quad \forall\, x \in GF(2^8) \qquad (1)$$

The elements of transformed S-box are calculated using transformation presented in (1) and for these purpose the values of $x = 0{:}\,255$ are applied. The constructed elements of transformed S-box are indicated in 16×16 matrix presented in Table 2. Afterward, the elements of transformed S-box are converted into corresponding binary number system to enhance their confusion capability after utilization of 1's and 2's compliment methods.

2.1 Application of 1's Compliment Method

- **i.** At initial stage elements of transformed S-box are converted in 8-bits representation.
- **ii.** Utilization of '0'as MSB to complete 8-bits representation when numbers of bits are less than 8.
- **iii.** To find 1's compliment of selected element of S-box in 8-bits form, a number is subtracted from the binary number which consists of same binary digits which are all equal to 1.

2.2 Application of 2's Compliment Method

- **i.** Before application of 2's compliment technique elements of S-box must be converted into 8-bits form after utilization of '0' as MSB.
- **ii.** Apply 2's compliment method on elements obtained after application of 1's compliment.
- **iii.** To find 2's compliment of selected element of S-box in 8-bits form, flip all bits 0 into 1 and 1 into 0 from right side but without any change to the first 1.

After the application of compliment's techniques the elements of revised S-box are presented in Table 3 for further comparison.

3. Assessment of Transformed S-Box and Revised S-Box for Encryption Abilities

3.1 Comparison of Nonlinearity Analysis

Nonlinearity is most applicable property applied on S-boxes to analyze the confusion ability of S-boxes. The best value of nonlinearity for constructed S-boxes is equal to 120 for eight binary digits and this value can be calculated by using $N_f = \frac{2^n - 2^{n/2}}{2} = 120$, where n=number of bits [9]-[10]. Nonlinearity of revised S-box and other renowned S-boxes are calculated by utilization of analysis software [11].

Furthermore, analysis report of constructed S-boxes and S-boxes from literature is presented in Table 4 for comparison of confusion creating capacity. Nonlinearity behavior of compared S-boxes is also interpreted in Figure 2. Analysis of transformed and revised S-boxes indicates that compliment technique increases confusion capability from 103.25 to 105.25.

Also Table 4 and graphical interpretation of average nonlinearity represents that confusion capacity (105.25) of revised S-box is comparatively better than transformed S-box, Residue prime S-box (99.5), Husain's S-box (104.75) and S_8 Liu J S-box (104.875). Moreover, nonlinearity value is also very close to the value of Skipjack S-box (105.75).

Table 1: Generated Elements of $GF(2^8)$ Corresponding to Polynomial [1 0 1 1 1 0 0 0 1]

$GF(2^8)$	Binary Form	$GF(2^8)$	Binary Form
0	00000000	φ^7	00100110
φ	00010110	φ^8	01000101
φ^2	00110111	φ^9	01100010
φ^3	00010010	.	.
φ^4	00001000	.	.
φ^5	00110100	φ^{254}	10001110
φ^6	01000000	φ^{255}	00000001

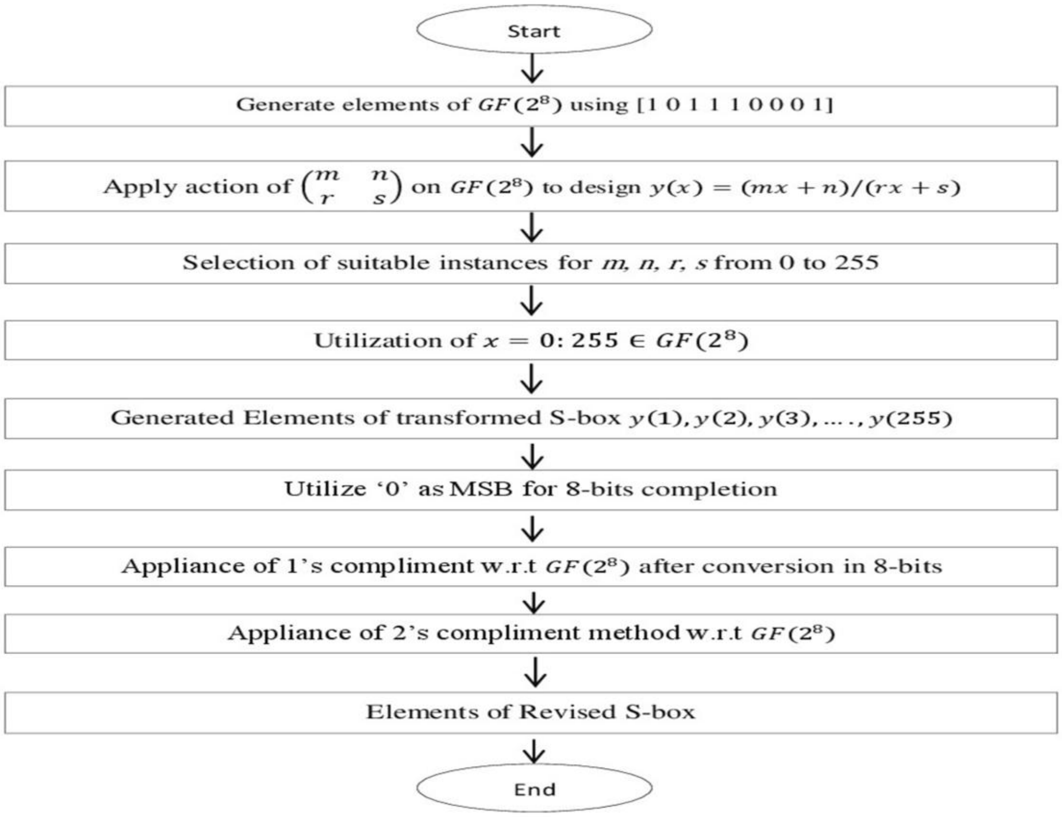

Fig. 1: Graphical overview of proposed scheme

3.2 Strict Avalanche Criterion (SAC) of Revised S-Box and Their Comparison Report

Davida and Kam [12] suggested the notion of completeness and moreover Feistel [13] proposed a concept of avalanche effect. A transformation will satisfy the concept of completeness if every bit of ciphertext depends on bits of plaintext. Additionally, transformation satisfies the condition of avalanche effect if 50% output binary digits are going to be changed due to change in a single input bit. According to definition an S-box must satisfy the condition of SAC if average value is equal to 0.5 [14].

Calculated minimum, maximum and average values of SAC of revised S-box and other S-boxes are presented in Table 5 for comparison. Comparison detail is also graphically indicated in Fig. 3 which shows that the average value of SAC of revised S-box is approximately close to 0.5 and comparatively better than all other S-boxes picked from literature for comparison.

3.3 Assessment of Bits Independence Criterion (BIC)

Bits independence criterion is well-known and desirable property was firstly introduced by

Table 2: Elements of Transformed S-Box Produced by Invertible Function

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	237	57	180	90	61	146	120	154	219	22	91	201	232	132	137	134
1	32	229	160	50	19	119	114	117	14	194	71	161	190	105	131	97
2	60	0	30	149	40	55	83	203	121	252	222	80	216	228	199	213
3	130	254	29	171	35	23	52	38	191	253	221	102	211	196	168	48
4	107	205	244	206	82	78	21	212	217	158	178	138	110	238	16	63
5	34	93	73	113	141	13	235	188	144	46	7	45	169	173	230	39
6	234	18	4	118	163	224	187	231	197	170	226	142	248	126	200	68
7	54	81	25	109	3	125	51	183	17	233	247	133	88	27	64	20
8	1	176	6	150	77	76	41	140	47	155	5	193	208	198	192	95
9	204	96	246	58	43	53	59	156	250	75	245	101	174	175	210	111
10	42	243	28	44	94	103	220	215	162	129	116	67	11	9	153	72
11	89	122	135	184	240	242	98	179	209	8	223	207	157	148	31	36
12	128	85	195	104	87	241	159	70	69	185	255	62	139	145	236	124
13	182	66	251	189	112	106	15	127	123	166	56	147	164	10	92	181
14	202	33	37	186	172	249	26	100	115	86	152	12	108	84	74	24
15	136	165	99	239	143	167	177	227	49	218	214	65	79	2	225	151

Table 3: Revised S-Box Erected By Application of Compliment Method

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	238	206	224	136	25	140	107	190	176	195	154	171	79	49	210	138
1	104	141	36	212	108	191	28	116	51	241	237	19	227	137	128	40
2	223	163	117	246	8	162	30	59	91	218	239	220	166	93	65	167
3	255	47	211	16	119	54	183	147	226	213	17	12	80	5	145	103
4	58	181	234	124	41	208	101	149	96	76	1	13	120	10	185	87
5	200	50	245	232	240	168	130	115	24	186	152	158	100	72	21	243
6	3	23	161	179	177	202	64	156	42	15	196	133	132	146	33	112
7	175	228	126	222	57	215	184	114	66	216	249	230	9	157	110	221
8	99	98	164	48	250	78	236	61	62	86	38	86	55	134	170	180
9	209	219	63	225	122	235	113	26	192	142	125	92	94	203	144	251
10	127	151	4	88	60	43	68	123	178	194	201	231	52	53	18	198
11	150	102	214	109	95	229	165	197	106	11	207	172	32	118	129	77
12	174	242	252	81	85	233	20	135	248	71	199	105	6	45	253	0
13	97	90	205	193	69	189	131	2	7	84	35	217	182	160	29	169
14	247	56	70	155	153	34	143	187	67	44	14	111	74	121	204	83
15	37	159	31	22	75	89	46	148	244	173	27	188	73	254	139	39

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Table 4: Nonlinearity Comparison of Revised S-Box with Renowned S-Boxes

S-boxes	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	Avg.
Revised S-box	102	108	106	104	106	104	106	106	105.25
Transformed S-box	104	102	96	106	106	106	106	106	103.25
Residue Prime	94	100	104	104	102	100	98	94	99.5
Skipjack S-box	104	104	108	108	108	104	104	106	105.75
Hussain	104	100	108	106	102	106	104	108	104.75
S_8 Liu J S-box	105	105	104	100	107	105	106	107	104.875

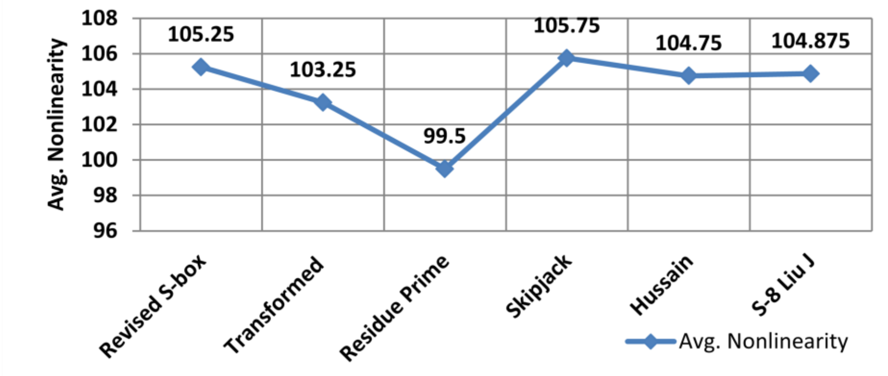

Fig. 2: Average Nonlinearity comparison

Table 5: Assessment of SAC of Revised S-Box and Their Comparison

S-boxes	Avg. value	Min. value	Max. value
Revised S-box	0.502197	0.40625	0.625
Transformed S-box	0.492432	0.40625	0.59375
Residue Prime	0.51	0.343	0.67
Skipjack S-box	0.53	0.39	0.59
Hussain	0.49	0.391	0.59
S_8 Liu J S-box	0.499	0.429	0.59

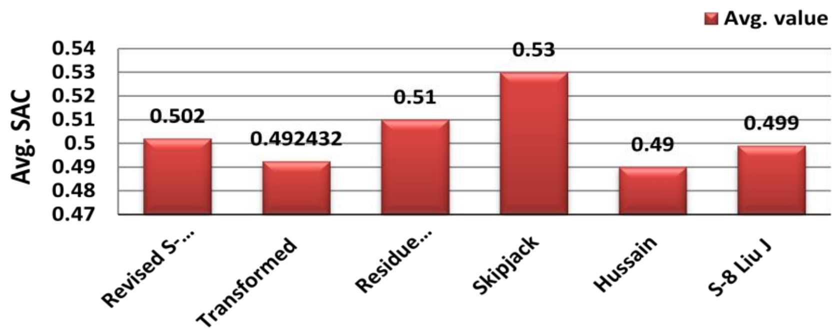

Fig. 3: Analysis comparison of average value of SAC

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Tavares and Webster [15]. This property analyzes the change of output binary digits when input binary digits of plaintext are complemented. Also we observe the independent of two output bits when one input bit is altered. The test of BIC is applied on nonlinearity of revised S-box, transformed S-box, residue prime, skipjack, Husain's S-box and S_8-Liu J S-boxes.

Table 6: Comparison of BIC of Nonlinearity for Renowned S-Boxes with Revised S-Box

S-boxes	Avg. value	Min. value
Revised S-box	**103.643**	**96**
Transformed S-box	**100.429**	**92**
Residue Prime	**101.71**	**94**
Skipjack S-box	**104.14**	**102**
Hussain	**105.071**	**100**
S_8 Liu J S-box	**104.786**	**99**

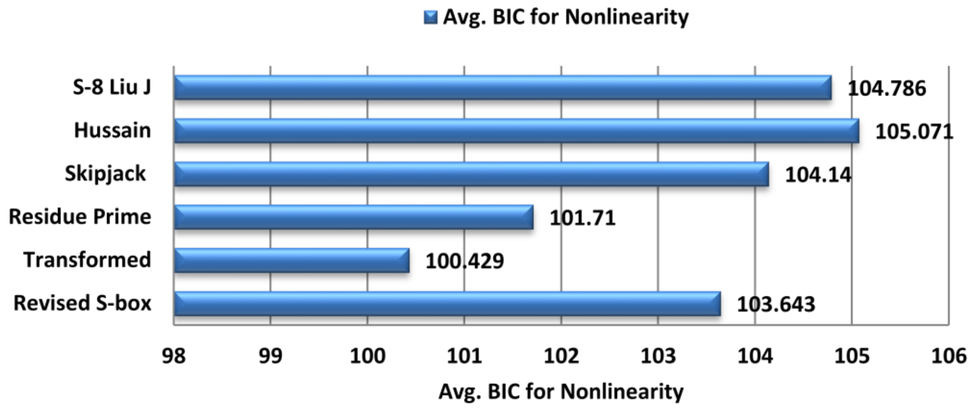

Fig. 4: Graphical Interpretation of BIC for Nonlinearity

3.4 Analysis of Differential Approximation Probability (DAP)

The analysis of differential approximation probability (DAP) is also most important property to observe the strength of S-boxes against some differential attacks. The differential probability value specifies the resistance of S-box against differential attacks. According to DAP property input differential Δx must uniquely map to Δy at output level to calculate differential pair $(\Delta x, \Delta y)$ such that:
Input differential: Δx
Output differential: $\Delta y = S(x) \oplus S(x \oplus \Delta x)$

Mathematically DAP is defined in [16] for 8-bits as follows,

$$DP_{(\Delta x \to \Delta y)} = \left[\frac{\#\{x \in X / S(x) \oplus S(x \oplus \Delta x) = \Delta y\}}{256}\right]$$

The proposed revised S-box and other S-boxes are analyzed for DAP test and results are described in Table 7. The minimum DAP value of S-box indicates that there are less chances of attacks. Graphical behavior of DAP from Fig. 5 shows that the probability value of revised S-box is superior to all other S-boxes except S_8 Liu J S-box because both have same probability value.

Table 7: Analysis Report of Differential Approximation Probability

S-boxes	Revised S-box	Transformed S-box	Residue Prime	Skipjack	Hussain	S_8 Liu J S-box
Max. DAP	0.0390625	0.492188	0.281	0.0468	0.125	0.0390

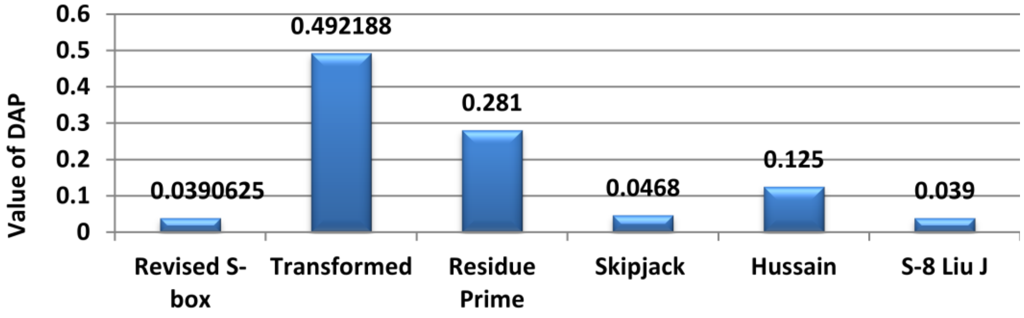

Fig. 5: Graphical performance of Maximum DAP

3.5 Behavior of S-Box against Linear Attacks

The linear approximation probability (LAP) is most important cryptographic property which is utilized to analyze the imbalance of an event. The LAP value represents the resistance of S-box against linear attacks. The value of LAP come close to probability of zero is considered as best value because it reduces the chances of linear attacks on ciphertext. The best supreme value of LAP is 0 because it indicates zero chances of attacks but only in rare cases it could happen. For input bits the mask Γm and for output bits the mask Γn are utilized. The definition of LAP for 8-bits is described in [17] as follows,

$$LP = \underset{\Gamma m, \Gamma n \neq 0}{Max} \left| \frac{Number\ of\ \{x \in X/x.\Gamma m = S(x).\Gamma n\}}{256} - \frac{1}{2} \right|$$

Analysis results of S-boxes are listed in Table 8 to make comparison. Maximum value of LP for all S-boxes are graphically interpreted in Fig. 6 and comparison shows that after application of compliment technique the LP value of revised S-box (0.132813) is better than transformed S-box (0.148438). Also resistance of revised S-box against linear attacks is identical to residue prime S-box and comparable with other S-boxes.

Table 8: Analysis and Comparison of Linear Approximation Probability

S-boxes	Revised S-box	Transformed S-box	Residue Prime	Skipjack S-box	Hussain S-box	S_8 Liu J S-box
Maximum value	162	166	162	156	160	159
Maximum LP	0.132813	0.148438	0.132	0.109	0.125	0.105

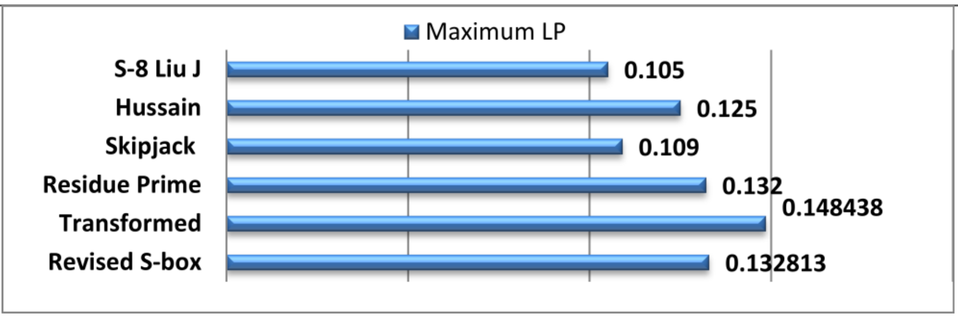

Fig. 6: Graphical presentation of maximum LP

4. Conclusion

This research proposal originated an innovative mechanism for the erection of Substitution box (S-box) with the assistance of implementation of invertible function. To acquire high encryption strength and confusion ability we employed 1's and 2's compliment technique on transformed S-box and derived Revised S-box. To inspect the information encryption capacity of Revised S-box, we perform comparison with eminent S-boxes from literature. We implement comparison of S-boxes through well-known cryptographic properties and investigation report indicates that fabrication of Revised S-box is superior to some S-boxes. The analysis behavior of Revised S-box represents that 1's and 2's compliment methodology is very reliable to increase encryption capabilities of S-boxes. Furthermore, property of differential probability specifies that Revised S-box can create greater resistance against differential attacks when compared with other S-boxes. Therefore, Revised S-box can be utilized in any encryption algorithm to protect confidential information.

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Intertwining approximation in space $L_{\psi,p}(I)$, $0<p<1$

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Abstract: In this paper, we find the degree of best approximation between a pair of a nearly intertwining polynomials, and a pair of a nearly intertwining splines to a non-negative function $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{j}_{\mathfrak{s}})$, in "$L_{\psi,p}$, $0<p<1$", we find the order of best a nearly intertwining approximation in the above terms.

Keywords: approximation, nearly intertwining, spline, modulus of smoothness.

Mathematics Subject Classification: 41A10,41A50 .

1.Introduction: The weighted quasi normed space $L_{\psi,p}(I), 0<p<1$ have form([6]) :

$$"L_{\psi,p}(I)=\{f, f: I\subset\Re\to\Re : \left(\int_I \left|\frac{f(x)}{\psi(x)}\right|^p dx\right)^{\frac{1}{p}}<\infty,\quad 0<p<1\}" .$$

And the (quasi) norm $(\|f\|_{L_{\psi,p}(I)}<\infty)$, where as always,

$$(\|f\|_{L_{\psi,p}(I)}=\left(\int_I \left|\frac{f(x)}{\psi(x)}\right|^p dx\right)^{\frac{1}{p}}) , \ x\in I$$

Let $\mathfrak{s}\geq 0$, so that $-b=\mathrm{j}_{\mathfrak{s}+1}<\mathrm{j}_{\mathfrak{s}}<\cdots<\mathrm{j}_1<\mathrm{j}_0=b$ for $\mathrm{j}_{\mathfrak{s}}\in \mathrm{j}_{\mathfrak{s}}$.

And we suppose $\Delta(\mathbb{j}_{\mathfrak{s}})$, are all set of non-negative functions f on I=$[-b,b]$, and we will write a function f which belongs to the same class $\Delta^0(\mathbb{j}_{\mathfrak{s}})$,is said to be copositive. Ration estimates of the approximation of the restriction are given in terms of $(\Delta_n(x)\omega_k^{\varphi}(f',\Delta_n(x))_p)$, in some inequality in this paper where $\Delta_n(x)=n^{-1}\sqrt{1-x^2}+n^{-2}$,and $c_1\Delta_n(x)\leq h_{\mathrm{j}}\leq c_2\Delta_n(x)$, ([6]). For $x\in \mathfrak{k}_{\mathrm{j}}=[\mathbb{X}_{\mathrm{j}+1},\mathbb{X}_{\mathrm{j}}]$,and c_1, c_2 are constants and $h_{\mathrm{j}}=|\mathfrak{k}_{\mathrm{j}}.|$ "Nearly intertwining approximation", in which intertwining points are allowed to shift by an amount no larger than $\Delta_n(\mathrm{j}_i)$, (using $\mathbb{j}_{\mathfrak{s}}^*$,instead of ($\mathbb{j}_{\mathfrak{s}}$), improves to the order of $(n^{-1}\omega_k^{\varphi}(f',n^{-1})_{\psi,p})$, for f .

2. Notations and Definitions:

Let $(\delta = min|j_{i+1} - j_i|, 0 \le i \le s)$ where $j_0 = -b$ and $j_{s+1} = b$, ([6]).

And let $I_i = [j_i^{(v)}, j_i^{(\mathcal{K}-1)}]$, and $\mathbb{J}_i = \left[\frac{j_i+j_i^{(v)}}{\mathcal{K}-1}, \frac{j_i+j_i^{(\mathcal{K}-1)}}{\mathcal{K}-1}\right]$.

Such that $j_i < \acute{j}_i < \cdots < j_i^{(\mathcal{K}-1)}$, $v = 1, \dots, \mathcal{K}-2$.

$(c_1 \hbar_j \le |I_i| = (\mathcal{K}-1)|\mathbb{J}_i| \le c_2 \hbar_j)$, where $c_i, i = 1,2$ positive number.

$$\mathbb{J}_s^* = \left\{j_i^{(v)}, v = 1, \dots, (\mathcal{K}-1) : \mathbb{X}_{j(i)+1} = \acute{j}_i < \cdots < j_i^{(\mathcal{K}-1)} = \mathbb{X}_{j(i)}, 0 < i \le s, j = 1, \dots, n\right\}$$

$\Delta^0(\mathbb{J}_s^*)$,setall functions $(f), \ni (-1)^{s-i} f(x) \ge 0$.We denote

$$(I_i \setminus \mathbb{J}_i = \left[j_i^{(v)}, \frac{j_i + j_i^{(v)}}{\mathcal{K}-1}\right) \cup \left(\frac{j_i + j_i^{(\mathcal{K}-1)}}{\mathcal{K}-1}, j_i^{(\mathcal{K}-1)}\right]).$$

Now we will write some important definitions in our work

$$\left((alm\Delta)^0(\mathbb{J}_s^*)\right) = \left\{f : \left((-1)^{s-i} f(x)\right) \ge 0, x \in I_i \setminus \mathbb{J}_i\right\}).$$

Let

$$\Delta_h^{\mathcal{K}}(f,x,I)_\psi = \Delta_h^{\mathcal{K}}((f,x)_\psi = \begin{cases} \sum_{i=0}^{\mathcal{K}} \binom{\mathcal{K}}{i} (-1)^{\mathcal{K}-i} \dfrac{f(x - \frac{\mathcal{K}h}{2} + ih))}{\psi\left(x + \frac{\mathcal{K}h}{2}\right)} & , \quad x \pm \dfrac{(\mathcal{K}h)}{2} \in I \\ 0 & , \quad o.w. \end{cases}.$$

The "Ditzian-Totik modulus of smoothness" of (f)([6]):

$$\omega_{\mathcal{K}}^{\varphi}(f,,\delta,I)_{\psi,p} = \sup_{0<h\le\delta} \left\|\Delta_{h\varphi(.)}^k(f,.)\right\|_{L_{\psi,p}(I))}.$$

The degree of "almost intertwining polynomial" of (f)([6]):

$$\widetilde{E_n}(f, alm\mathbb{J}_s)_{\psi,p} = inf\left\{\|p - f\|_{L_{\psi,p,}} + \|f - q\|_{L_{\psi,p,}} : p, q \in \prod_n, (-1)^{s-i}(p(x) - f(x)) \ge 0, (-1)^{s-i}((f(x) - q(x) \ge 0)\right\}.$$

The degree of "nearly intertwining polynomial approximation"of(f), with respect to$\mathbb{J}_s$([6]):

$$\widetilde{E_n}(f, nearly\mathbb{J}_s)_{\psi,p} = inf\left\{(\|P - Q\|_{L_{\psi,p,}}) : P, Q \in \prod_n, P(x) - f(x) \in \Delta^0(\mathbb{J}_s^*), f(x) - Q(x)\Delta^0(\mathbb{J}_s^*)\right\}.$$

From the definitions above we get for $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{J}_s)$([6]):

:

$\widetilde{E_n}(f, alm\mathbb{J}_s)_{\psi,p} \le \widetilde{E_n}(f, nearly\mathbb{J}_s)_{\psi,p} \le \widetilde{E_n}(f, \mathbb{J}_s)_{\psi,p,}$. …(1

3. Auxiliary Result:

In the following theorems we show that a"nearly intertwining approximation"by $\{p_1, p_2\} \subset \Pi_r$, a nearly intertwining polynomials has an order,$C|I_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |I_i|, I_i)_{\psi,p}$ and the generalization to $f \in L_{\psi,p} \cap \Delta^0(\mathbb{J}_s)$, which has an order $Cn^{-1}\omega_{\mathcal{K}}^{\varphi}(\acute{f}, n^{-1})_{\psi,p}$, also a nearly intertwining approximation by $\{S_1, S_2\}$, from the order (r), on the knot sequence$\{\mathbb{X}_j\}_{j=0}^n$, a "nearly intertwining splines" has an order $C|\mathbb{J}_{\mathcal{K}}|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathbb{J}_{\mathcal{K}}|, \mathbb{J}_{\mathcal{K}})_{\psi,p}$, where$C$, both of the above dependent on (p, κ).

Theorem ***(1):***

Let $f \in L_{\psi,p}(I_i) \cap \Delta^0(\mathbb{J}_s^*)$, and$\mathbb{J}_i \subset I_i$, be two sub intervals of I .Then there exist $j_1 \in \mathbb{J}_1$, and two polynomial p_1, p_2 of degree k $\ni \{p_1, p_2\}$a nearly intertwining pair for f $\ni p_2(x) \le f(x) \le p_1(x)$ onI_i, with respect to $\{j_1\}$, that satisfies:

$\widetilde{E_n}(f, nearly\mathbb{J}_s)_{\psi,p} \le C|I_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |I_i|I_i)_{\psi,p,}$. … (2

Where C, dependent on (p, κ).

Proof: It is known:

"Any inferable functions(h) on $(-\infty, \infty)$,

$$|\{\chi : M(h(\chi)) > \tau\}| \le c\tau^{-1}\int_{-\infty}^{\infty}|h(\chi)|, \tau > 0,$$

where $M(h(\chi)) = \sup|J|^{-1}\int_J |h(\chi)|$, is the "$Hardy - Little\ wood\ maximal\ operator$" [4])).

Let

$$F(x) = \begin{cases} \left(\frac{f}{\psi}\right)(\mathrm{j}_i^{(v)}) & ; \quad x < \mathrm{j}_i^{(v)} \\ \frac{1}{2}\left(\frac{f}{\psi}\right)(x) & ; \quad x \in \mathrm{I}_i \\ \left(\frac{f}{\psi}\right)(\mathrm{j}_i^{(\mathcal{K}-1)}) & ; \quad x > \mathrm{j}_i^{(\mathcal{K}-1)} \end{cases},$$

And $t = 2c|\mathbb{j}_i|^{-1}|\mathrm{I}_i|\int_{\mathrm{I}_i}|\acute{f}|$.

By using the above operator to F', we get: $|\{x: M(F(x))\acute{} > t\}| \le Ct^{-1}\int_{-\infty}^{\infty}|F(x)|$

$$= Ct^{-1}\int_{\mathrm{I}_i}|\acute{F}(x)| = \frac{C}{4}t^{-1}\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right| = \frac{1}{2}\frac{|\mathbb{j}_i|}{|\mathrm{I}_i|}\ ;\ 0 < p < 1.$$

Thus there exists $\mathrm{j}_i \in \mathbb{j}_i$, such that $M(F(\mathrm{j}_1))\acute{} \le t$.

Now when $\mathcal{K} > 1, v = 1, \dots, \mathcal{K}-2$, that is:

$$L_1(x) = f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1},$$

$$L_2(x) = f(\mathrm{j}_1) - t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}, \quad \mathcal{K} > 1, v = 1, \dots, \mathcal{K}-2$$

They will form an "intertwining pairs" of f on I_i with respect to $\{\mathrm{j}_i\}$. We have from $L_1(x)$, $\mathrm{j}_i \le x \le \mathrm{j}_i^{(\mathcal{K}-1)}$ it is easy from modified Chebyshev partition ([6]),and since $M\left(\acute{F}(\mathrm{j}_1)\right) \le t$, hence

$$\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} M(\acute{F})(\mathrm{j}_i) \le t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}, \text{ and}$$

$$f(\mathrm{j}_1) + \frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} M((\acute{F})(\mathrm{j}_1)) \le f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1},$$

Hence

$$f(\mathrm{j}_1) + \frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} M((\acute{F})(\mathrm{j}_1)) \le L_1(x),$$

For an error estimate of L_1 and L_2 we first note from I_i, $L_2(x) \le f(x) \le L_1(x)$

$$|L_1(x) - L_2(x)| = \left| f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} - f(\mathrm{j}_1) + t\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1} \right|$$

$$= 2t\left|\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}\right|$$

$$= 2\left|\frac{(\mathrm{j}_i^{(\mathcal{K}-1)} - \mathrm{j}_i^{(v)})}{\mathcal{K}-1}\right|\left(\frac{2C}{4}|\mathrm{j}_i|^{-1}|\mathrm{I}_i|\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right|\right)$$

$$\le C|\mathrm{I}_i|\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right|$$, where C dependents on $p, \mathcal{K}$.

By (Theorem (1.2.1),[6]):

$$\int_{\mathrm{I}_i}|L_1(x) - L_2(x)|^p \le C^p|\mathrm{I}_i|^p\int_{\mathrm{I}_i}\left|\left(\frac{f}{\psi}\right)'(x)\right|^p$$

And since $|\mathrm{I}_i| \le Ch_j$ and $h_j \approx \Delta_n(x)$, then we get from the above that :

$$\|L_1 - L_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C\Delta_n(x)\|\acute{f}\|_{L_{\psi,p}(\mathrm{I}_i)}, \quad \dots (3$$

Let P' be a "best polynomial approximation" to (f') on I_i, of degree $r-1$, $P = \int_{\mathrm{j}_i}^{x}\acute{P}(t)dt$,Since $\int_{\mathrm{j}_i}^{x}\acute{P}(t)dt > 0$,this implies that $P \in \Delta^0(\mathbb{j}_s^*)$, to prove (2) apply (3)to $f - P \in \Delta^0(\mathbb{j}_s^*)$, then $\{L_1, L_2\}$,a nearly intertwining pair for $f - \mathrm{P}$.

Define $p_i = L_i(x) + \mathrm{P}, i = 1,2$, obviously $p_1 - f, p_2 - f \in \Delta^0(\mathbb{j}_s^*)$, and $|\mathrm{j}_i^{(v)} - \mathrm{j}_i| \le \Delta_n(\mathrm{j}_i)$,and $\{p_1, p_2\}$, a "nearly intertwining pair" of polynomials of degree r for f on I_i, and

$$\|p_1 - p_2\|_{L_{\psi,p}(\mathrm{I}_i)} = \|L_1 - L_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C\Delta_n(x)\|\acute{f} - \acute{P}\|_{L_{\psi,p}(\mathrm{I}_i)},$$

by using (Theorem 2.3.2[6]) we get $\|p_1 - p_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C\Delta_n(x)\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|, \mathrm{I}_i)_{\psi,p}$,hence

$$\|p_1 - p_2\|_{L_{\psi,p}(\mathrm{I}_i)} \le C|\mathrm{I}_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|, \mathrm{I}_i)_{\psi,p}, \quad \dots (4$$

Now ,by using the definition of best nearly intertwining approximation by a pair $\{p_1, p_2\}$,we get

$$\widetilde{E_n}(f, nearly\mathbb{j}_s^*)_{\psi,p} \le\le C|\mathrm{I}_i|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathrm{I}_i|, \mathrm{I}_i)_{\psi,p}, \quad \dots (5$$

The result (5) can be generalized by using (Theorem 2.1.2[6]) when: $f \in L_{\psi,p}(I) \cap \Delta^0(\mathbb{j}_s)$, $I = [-b, b]$, specifically when the function is derived by a pair $\{p_1, p_2\}$, this implies that

$$\widetilde{E_n}(f, nearly\mathbb{j}_s)_{\psi,p} \le Cn^{-1}\omega_{\mathcal{K}}^{\varphi}(\acute{f}, n^{-1})_{\psi,p}, \quad \dots (6$$

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Also there exist a nearly intertwining pair of polynomial $\{p_1, p_2\} \subset \Pi_n$, satisfy

$|p_1(x) - p_2(x)| = |L_1(x) - L_2(x)| \leq C|\mathfrak{l}_i| \|\acute{f}\|_{L_{\psi,p}(\mathfrak{l}_i)}$,

and there for

$|p_1(x) - p_2(x)| \leq C\Delta_n(x)\omega_{\mathcal{K}}^{\varphi}(\acute{f}, \Delta_n(x))_{\psi,p}$.

Corollary: Suppose $f \in L_{\psi,p}(I) \cap \Delta^0(\mathfrak{j}_s)$, $0 < p < 1, k > 1$, there is a pair of " a nearly intertwining polynomials " $\{p_1, p_2\}$, of degree r to f with respect to $\{\mathfrak{j}_1\}$, satisfies:

i) $\widetilde{E_n}(f, nearly\mathfrak{j}_s)_{\psi,p} \leq Cn^{-1}\omega_{\mathcal{K}}(\acute{f}, n^{-1})_{\psi,p}$.

ii) $\widetilde{E_n}(f, almy\mathfrak{j}_s)_{\psi,p} \leq Cn^{-1}\omega_{\mathcal{K}}(\acute{f}, n^{-1})_{\psi,p}$.

Where C, dependents on p, $\mathcal{K}$.

Proof:

i) By using (Theorem 1.6.3 [6]), and the result (5) then the prove is complete.

ii) By the relationship (1) and the result (i) of this corollary we get the result.

Theorem(2): Suppose $f \in L_{\psi,p}(I) \cap \Delta^0(\mathfrak{j}_s)$, $\mathfrak{j}_s = \{\mathfrak{j}_i, i = 1, \dots, s : -b = \mathfrak{j}_0 < \mathfrak{j}_1 < \cdots < \mathfrak{j}_s < \mathfrak{j}_{s+1} = b\}$. Let $\{\mathbb{X}_j\}_{j=0}^{n}$, is a single knot sequence, there is a pair of "a nearly intertwining spline" $\{S_1, S_2\}$, of degree $r, r \geq 2$ on $\{\mathbb{X}_j\}_{j=0}^{n}$, for f with respect to $\mathfrak{j}_s$, satisfy :

$\|S_1 - S_2\|_{L_{\psi,p}(\mathfrak{j}_\kappa)} \leq C|\mathfrak{j}_\kappa|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathfrak{j}_\kappa|, \mathfrak{j}_\kappa)_{\psi,p}$.

Where C, dependents on p, $\mathcal{K}$.

Proof: Let there exist a polynomials $\{p_1, p_2\}$, of degree r, and interpolate f at k points at $\mathfrak{j}_c = [b - \mu|I|, b], 0 < \mu < \frac{1}{2}$, (Theorem(2.3.4):[6]). By using differentiated in (theorem(2.1.2)[6]) in case $b - \mu|I| < b - \frac{1}{2}\mu|I|$, and by the result (4) then there exist "a nearly intertwining pair of a polynomials "$\{p_1, p_2\} \subset \Pi_r$, for f at $\mathfrak{j}_c \ni$ "$p_2(x) \leq f(x) \leq p_1(x)$", $\forall x \in \mathfrak{j}_c$, hence

$\|p_1 - p_2\|_{L_{\psi,p}(\mathfrak{l}_i)} \leq C|\mathfrak{j}_c|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathfrak{j}_c|, \mathfrak{j}_c)_{\psi,p}$.

Now, we define $\overline{p_1}$ and $\overline{p_2}$ on $\mathfrak{j}_c$, by $\overline{p_1} = p_1$ and $\overline{p_2} = p_2$ if $(-1)^{s-i} > 0$, $i = 1, \dots, s$, and $\overline{p_1} = p_2$ and $\overline{p_2} = p_1$ if $(-1)^{s-i} < 0$. Hence "$((-1)^{s-i}(\overline{p_1(x)} - f(x)) \geq 0$, $(-1)^{s-i}(\overline{p_1(x)} - f(x)) \leq 0$", and

$\|\bar{p}_1 - \bar{p}_2\|_{L_{\psi,p}(\mathfrak{j}_c)} \leq C|\mathfrak{j}_c|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathfrak{j}_c|, \mathfrak{j}_c)_{\psi,p}$. … (3.7

Near thepoint $(-b + \mu|I|)$, we will construct different local polynomial. Specifically, we will approximate (f'), at $\mathfrak{j}_{\mathcal{A}} = [-b + \mu|I|, b - \mu|I|]$. From the above also there exist a pair of polynomial $\{P, Q\}$ of degree $< r - 1$, such that $Q(x) \leq f'(x) \leq P(x), \forall x \in \mathfrak{j}_{\mathcal{A}}$, then

$\|P - Q\|_{L_{\psi,p}(\mathfrak{j}_{\mathcal{A}})} \leq C|\mathfrak{j}_{\mathcal{A}}|\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathfrak{j}_{\mathcal{A}}|, \mathfrak{j}_{\mathcal{A}})_{\psi,p}$.

Let "$P^* = P$ and $Q^* = Q$", if$((-1)^{s-i} > 0)$, and "$P^* = Q$ and $Q^* = P$" if $((-1)^{s-i} \leq 0)$. Now to check that

$$\overline{P^*(x)} = \int_{-b+\mu|I|}^{x} P^*(t)dt + f(-b + \mu|I|),$$ and

$$\overline{Q^*(x)} = \int_{-b+\mu|I|}^{x} Q^*(t)dt + f(-b + \mu|I|),$$

satisfy the inequalities $((-1)^{s-i}(P^*(x) - f(x))\text{sgn}(x - (-b + \mu|I|)) \geq 0$, $((-1)^{s-i}(Q^*(x) - f(x))\text{sgn}(x - (-b + \mu|I|)) \leq 0$, hence

$$\|\bar{P}^* - \bar{Q}^*\|_{L_{\psi,p}(\mathfrak{j}_{\mathcal{A}})} = \left\| \int_{-b+\mu|I|}^{x} (P^*(t) - Q^*(t))dt \right\|_{L_{\psi,p}(\mathfrak{j}_{\mathcal{A}})},$$

$$= \left\| \int_{-b+\mu|I|}^{x} (P(t) - Q(t))dt \right\|_{L_{\psi,p}(\mathfrak{j}_{\mathcal{A}})}$$

$$\leq \left\| \int_{\mathfrak{j}_{\mathcal{A}}} (P(t) - Q(t)) \right\|_{L_{\psi,p}(\mathfrak{j}_{\mathcal{A}})},$$

$$\leq C|\mathfrak{j}_{\mathcal{A}}|\|P - Q\|_{L_{\psi,p}(\mathfrak{j}_{\mathcal{A}})},$$

$\leq C|\mathfrak{j}_{\mathcal{A}}|^2\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathfrak{j}_{\mathcal{A}}|, \mathfrak{j}_{\mathcal{A}})_{\psi,p}$, that is

$\|\bar{P}^* - \bar{Q}^*\|_{L_{\psi,p}(\mathfrak{j}_c)} \leq C|\mathfrak{j}_{\mathcal{A}}|^2\omega_{\mathcal{K}}^{\varphi}(\acute{f}, |\mathfrak{j}_{\mathcal{A}}|, \mathfrak{j}_{\mathcal{A}})_{\psi,p}$.

After the interlocking polynomials have been configured "intertwining",with the function f , which has a right approximation order, now we will merge these polynomials together in the form of "smooth spline approximants"(S_1, S_2) , on $\{\mathbb{X}_j\}_{j=0}^{n}$. If both $\mathfrak{J}_{\mathcal{B}}$ $=[b-\mu|I|, b-\frac{1}{2}\mu|I|]$, and $\mathfrak{J}_c$, are non-contaminated, then P_3 and P_2, overlap on $\mathfrak{J}_{\mathcal{B}}$, which contains (m) interior knots from $\{\mathbb{X}_j\}_{j=0}^{n}$. By "Beatsons Lemma "[4]), ∃ a splines $\bar{S}_i$, has order r at $\mathfrak{J}_c$,on $\{\mathbb{X}_j\}_{j=0}^{n}$,which are associated with a polynomials P_3 , P_2, in technique at points" $b-\mu|I|, b-\frac{1}{2}\mu|I|$ ", countinuously.

Furthermore, the draw of splines $\bar{S}_i$, it is located between the polynomials" P_3, P_2 ", accordingly:

" $sgn\,(P_3(x)-f(x)) = sgn\,(P_2(x)-f(x)) = sgn\,(\bar{S}_i(x)-f(x))$ " , $x\in\mathfrak{J}_c$.

By the same method, taking in to account the overlapping polynomials " P_1, P_4 "

" $sgn\,(P_4(x)-f(x)) = sgn\,(P_1(x)-f(x)) = sgn\,(S_i(x)-f(x))$ " ,

$x\in\mathfrak{J}_{\mathcal{T}}=[-b+\mu|I|, b-\frac{1}{2}\mu|I|]$.Then we get by ([1])that:

$$\int_{\mathfrak{J}_{\mathcal{T}}}|\bar{S}_i-S_i|^p \le 2^p\left(\int_{\mathfrak{J}_{\mathcal{T}}}|P_3-P_4|^p + \int_{\mathfrak{J}_{\mathcal{T}}}|P_1-P_2|^p\right) ,$$

by the inequality (7) on an interval $\mathfrak{J}_{\mathcal{T}}$, then:

$\|\bar{S}_i-S_i\|_{L_{\psi,p}(\mathfrak{J}_{\mathcal{T}})}\le C|\mathfrak{J}_{\mathcal{T}}|\omega_{\mathcal{K}}^{\varphi}(\acute{f},|\mathfrak{J}_{\mathcal{T}}|,\mathfrak{J}_{\mathcal{T}})_{\psi,p}\ldots$ (8

In the same way, we can overlapping a polynomial pieces which fall within the periods contaminated intervals. The Spline pieces $\bar{S}_i, S_i$, check the same guess above with a slightly larger interval of $\mathfrak{J}_{\mathcal{T}}$, on the right-hand side.

Now, we define the final spline S_1 over $\mathfrak{J}_c$ as follows:

If there is only one polynomial P_1 over $\mathfrak{J}_c$,then we set S_1 to P_1. If there are two polynomials overlapping on $\mathfrak{J}_c$,must be a combination spline $\bar{S}_i$, set S_1 to $\bar{S}_i$. We get from the above $S_1-f\in\Delta^0(\mathfrak{J}_s)$, on an interval $I=[-b,b]$.By the same method we set $S_2-f\in\Delta^0(\mathfrak{J}_s)$.Since the intervals " ($\mathfrak{J}_i$, $\left[\frac{(\mathrm{j}_i+\mathrm{j}_i^{(\mathcal{K}-1)})}{\mathcal{K}-1}, \tilde{a}\right]$, $\left[\tilde{b}, \frac{\mathrm{j}_i+\mathrm{j}_i^{(v)}}{\mathcal{K}-1}\right]$) ", in the partition o (I_i), where" $\tilde{a}=\frac{\mathrm{j}_i+\mathrm{j}_i^{(\mathcal{K}-1)}}{\mathcal{K}-1}+\mu|\mathfrak{J}_i|$ " and" $\tilde{b}=\frac{\mathrm{j}_i+\mathrm{j}_i^{(v)}}{\mathcal{K}-1}-\mu|\mathfrak{J}_i|$ ", (see Lemma (2.3.1)[6]),can be compared to size. And each interval $\mathfrak{J}_{\kappa}=[-b+\mu|I|, b]$,denote contain more than (m), such interval ((the value of (m) depends on the length of the original interval)).Therefore we can get the result from (7)and (8), that is

$\|S_1-S_2\|_{L_{\psi,p}(\mathfrak{J}_{\kappa})}\le C|\mathfrak{J}_{\kappa}|\omega_{\mathcal{K}}^{\varphi}(\acute{f},|\mathfrak{J}_{\kappa}|,\mathfrak{J}_{\kappa})_{\psi,p}$.

Where C , dependents on $p, \mathcal{K}$.

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التقريب المتشابك في الفضاء $L_{\psi,p}(I)$, $0 < p < 1$

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المستخلص:

في هذا البحث تم ايجاد درجة افضل تقريب لزوج من متعددات الحدود المتشابكة بين زوج من شرائح متعددات الحدود المتشابكة تقريبا للدالة المقيدة, $f \in L_{\psi,p}(I) \cap \Delta^0(\mathring{J}_s)$ ، في الفضاء $0 < p < 1$, $L_{\psi,p}$, هذا يعني ايجاد رتبة افضل تقريب متشابك تقريبا للمصطلحات اعلاه .

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On $\mathbb{P}-$ Hausdorff Topological Random Systems

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Abstract

We are taking aview of topologicall random systems which is introduce considered as a mixing between two fundamental branches of mathematics "topology" and "probability Theory". The concept of $\mathbb{P}-$ Hausdorff topological random system is studied and some properties of such system are given and proved.

Key words. Random set, Topological random system, $\mathbb{P}-$neighborhood, $\mathbb{P}-$limlit point**,** $\mathbb{P}-$closed set and $\mathbb{P}-$Hausdorff system.

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Introduction. The "topology" and "probability Theory" (specially random sets) are the important tools in the study of pure and applied mathematics. Therefore we mix here these two theories by define the topological random system. As first step of our study are taking aview of topologicall random system and define the concept of $\mathbb{P}-$ Hausdorff topological random system. Throughout our paper we state and prove some properties of $\mathbb{P}-$ Hausdorff topological random system. This work consist of three sections. In section 2 we state the definition of random set and some concepts related with probability theory.

In section 3, our new concept "the topological random system" is introduced and some concept in terms of probability concepts such as $\mathbb{P}-$neighborhood, $\mathbb{P}-$limlit point and $\mathbb{P}-$closed set are given. In Section 4 the concept of $\mathbb{P}-$Hausdorff system is introduced and some essential properties are proved.

Throughout this paper all probability space are complete (A" probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is said to be complete "[1,2,3] if for every $A \in \mathcal{F}$with $\mathbb{P}(A) = 0$,then $B \in \mathcal{F}$, for all subsets $B \subset A$) and every metric space is polish space (complete separable[4]).

2. Random Sets.

The origin of the recent concept of a random set energies as far back as the inspiring book by A.N. Kolmogorov [5](first published in 1933) where he arranged out the foundations of probability theory. In this section the definition of random set is given and some properties of such sets. Theset $A(\omega) := \{x \in X: (\omega, x) \in A\} \in \mathcal{B}$ is called the $\omega -$ section of A.

Let $A: \Omega \to \mathcal{B}(X), \omega \longmapsto A(\omega)$, beafunction whose values are subsets of X. Afunction is individually determined by its graph $\text{graph(A)} := \{(\omega, x) \in \Omega \times X: x \in A(\omega)\} \subset \Omega \times X$. Conversely,

each subset $A \subset \Omega \times X$ defines such a function via $\omega \longmapsto A(\omega)$.

Definition 2.1[1]: "Let (X, d) be a metric space which is considered a measurable space with Borel $\sigma-$ algebra $\mathcal{B}(X)$ and $(\Omega, \mathcal{F})$ be a measurable space and. The set-valued function $A: \Omega \to \mathcal{B}(X), \omega \longmapsto A(\omega)$, is said to be ***random set*** if for every $x \in X$ the function $\omega \longmapsto d(x, A(\omega))$is measurable. If $A(\omega)$is closed (compact) for all $\omega \in \Omega$, it is called a ***random closed(compact) set***".

<u>Proposition 2.2[1]:</u>" Let the set-valued function $\text{A}: \Omega \to \mathcal{B}(X)$ take values in the subspace of closed subsets of a Polish space X. Then:

(i)A is a random closed set if and only if for all open sets $U \subset X$ the set $\{\omega: A_\omega \cap U \neq \emptyset\}$ is measurable.

(ii) If A is a random closed set then $\text{graph(A)} \in \mathcal{F} \otimes \mathcal{B}$.

The property of A being a random closed set is thus slightly stronger than graph(A) being measurable and A_ω being closed.

For convenient, throughout this paper we adopt the following definition of random set" .

Definition 2.3 [1] Agreement $\text{A}: \Omega \to \mathcal{B}(X)$ beaset-valued function where X be "a topological space". Then A issaid to berandomclosed set if for every opensets $U \subset X$ theset $\{\omega: A_\omega \cap U \neq \emptyset\}$ ismeasurable. Thecomplement of randomclosedset is called randomopenset.

Examples [1]

(a) "The set $A = \{\zeta\}$ is an RCS where ζ is a random point in" .

(b) "The set $A = (-\infty, \zeta]$ is RCS on $X = \mathbb{R}^1$ if ζ is RV. Also the set $A = (-\infty, \zeta_1] \times (-\infty, \zeta_2] \dots \times (-\infty, \zeta_n]$ is RCS in $\mathbb{R}^n$ if $(\zeta_1, \dots, \zeta_n)$ is n −dimensional random vector".

Theorem 2.4[1]

(i) the closure of the complement of any closed random set is closed random set.

(ii) The closure of any random open set is closed random set.

(iii)The interior of any closed random set is open random set

(iv) the intersection of any two random set is random set.

For more detail about random set see[1] and [2].

3. Topological Random System.

In this section the new concept of topological random system is introduced. Also the concepts of, $\mathbb{P}$ −limlit point **and** $\mathbb{P}$ −closed set are introduced.

Definition 3.1 (Topological Random System) Let $(\Omega, \mathcal{F}, \mathbb{P})$be a completeprobabilityspace and(X, τ) be a topologicalspace (which is considered as measurable space withBorel $\sigma -$ algebra$\mathcal{B}(X)$.) The triple $(\Omega, X, \mathfrak{R})$is called topological random system (shortly, TRS), where $\mathfrak{R}$ is the collection of random sets in X.

Example 3.2 Agreement $(\Omega, \mathcal{F}, \mathbb{P})$ be a completeprobabilityspace, where $\Omega \coloneqq \{H, T\}$, $\mathcal{F} \coloneqq 2^{\Omega}$ and $\mathbb{P}(\{H\}) = \mathbb{P}(\{T\}) = \frac{1}{2}$ and let $\mathbb{R}$ be the set of all realnumbers endowed with the usualtopology(in this case $\mathbb{R}$ is polish space) . Define thecollection $\mathfrak{R} \coloneqq \{A, B, C, D\}$ of sub sets of $\mathbb{R}$,where

$$\mathrm{A}: \Omega \to \mathcal{B}(\mathbb{R}) \text{ , } \mathrm{A}(\omega) = \phi, \forall \omega \in \Omega$$

$$\mathrm{B}: \Omega \to \mathcal{B}(\mathbb{R}) \text{ , } \mathrm{B}(\omega) = [0, \infty), \forall \omega \in \Omega$$

$$\mathrm{C}: \Omega \to \mathcal{B}(\mathbb{R}) \text{ , } \mathrm{C}(\omega) = (-\infty, 0], \forall \omega \in \Omega$$

$$\mathrm{D}: \Omega \to \mathcal{B}(\mathbb{R}) \text{ , } \mathrm{D}(\omega) = \mathbb{R}, \forall \omega \in \Omega.$$

Thus $\mathfrak{R} \coloneqq \{A, B, C, D\}$ be the collection of randomsets in $\mathbb{R}$. (In fact is the collection of closedrandom set in $\mathbb{R}$). Hence the triple $(\Omega, \mathbb{R}, \mathfrak{R})$ is TRS**.**

Definition 3.3(sub-topological random system).The triple $(\Omega, Y, \mathfrak{R})$ is said to be sub-topological random system of $(\Omega, X, \mathfrak{R})$ if Y isasubspace (as a topologicalspace) of X and the intersection of each open randomset in X with Y is randomopen setin Y.

Definition 3.4 (Random Neighborhood). Let $(\Omega, X, \mathfrak{R})$ be an TRS and $x \in X$. A random neighborhood (shortly, RN) of x is a random set N suchthatthereexistsrandomopenset Uwiththepropertythat

$$\mathbb{P}\{\omega: x \in U(\omega) \subseteq N(\omega)\} = 1.$$

The collection $\mathfrak{N}_x$ denoted to all Rnhd of x and is called random neighborhood system (RNS) at x.

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Example 3.5 Consider $\mathbb{R}$ endowed with the" usual topology, and $(\Omega, \mathcal{F}, \mathbb{P})$ be any complete probability space". Let ζ_ω, $\omega \in \Omega$, be a real-valuedrandom process on Ω with continuous sample paths. Then $A = \{\omega: \zeta_\omega > 0\}$ is RN of each elements of itself.

Theorem 3.6 The RNS $\mathfrak{N}_x$ has the following properties.

[RN1] If $N \in \mathfrak{N}_x, \rightarrow \mathbb{P}\{\omega: x \in N(\omega)\} = 1$.

[RN2] If $N, M \in \mathfrak{N}_x, \rightarrow N \cap M \in \mathfrak{N}_x$.

[RN3] If $N \in \mathfrak{N}_x, \rightarrow \exists\ M \in \mathfrak{N}_x \ni N \in \mathfrak{N}_y$ for each $y \in M$.

[RN4] If $N \in \mathfrak{N}_x$ and $\mathbb{P}\{\omega: N(\omega) \subseteq M(\omega)\} = 1$, then $M \in \mathfrak{N}_x$.

[RN5] G is random open set if and only if G contains An RN of each of its points.

Proof.

[RN1]: Suppose that $N \in \mathfrak{N}_x$, then $\mathbb{P}\{\omega: x \in U(\omega) \subseteq N(\omega)\} = 1$. Let $\omega \in \{\omega: x \in U(\omega) \subseteq N(\omega)\}$, then $x \in U(\omega) \subseteq N(\omega)$,i.e., $x \in N(\omega)$. Thus $\omega \in \{\omega: x \in N(\omega)\}$. Therefore $\{\omega: x \in U(\omega) \subseteq N(\omega)\} \subseteq \{\omega: x \in N(\omega)\}$. Since $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete "probability space", then $\{\omega: x \in N(\omega)\} \in \mathcal{F}$. Now by properties of $\mathbb{P}$ we have

$$\mathbb{P}\{\omega: x \in U(\omega) \subseteq N(\omega)\} \leq \mathbb{P}\{\omega: x \in N(\omega)\}$$

or, equivalently $1 \leq \mathbb{P}\{\omega: x \in N(\omega)\}$. Hence $\mathbb{P}\{\omega: x \in N(\omega)\} = 1$.

[RN2]: Suppose that $N, M \in \mathfrak{N}_x$, thenthereexists two random opensets $U$and Vsuch that

$$\mathbb{P}\{\omega: x \in U(\omega) \subseteq N(\omega)\} = 1 = \mathbb{P}\{\omega: x \in V(\omega) \subseteq M(\omega)\}.$$

Clearly that

$$\{\omega: x \in U(\omega) \subseteq N(\omega)\} \cap \{\omega: x \in V(\omega) \subseteq M(\omega)\} \subseteq \{\omega: x \in U(\omega) \cap V(\omega) \subseteq N(\omega) \cap M(\omega)\}.$$

$$\mathbb{P}(\{\omega: x \in U(\omega) \subseteq N(\omega)\} \cap \{\omega: x \in V(\omega) \subseteq M(\omega)\})^c$$
$$= \mathbb{P}(\{\omega: x \in U(\omega) \subseteq N(\omega)\}^c \cup \{\omega: x \in V(\omega) \subseteq M(\omega)\}^c)$$
$$= \mathbb{P}\{\omega: x \in U(\omega) \subseteq N(\omega)\}^c + \mathbb{P}\{\omega: x \in V(\omega) \subseteq M(\omega)\}^c$$
$$-\mathbb{P}(\{\omega: x \in U(\omega) \subseteq N(\omega)\}^c \cap \{\omega: x \in V(\omega) \subseteq M(\omega)\}^c)$$
$$= 0 + 0 - \mathbb{P}(\{\omega: x \in U(\omega) \subseteq N(\omega)\}^c \cap \{\omega: x \in V(\omega) \subseteq M(\omega)\}^c).$$

Thus we must have

$$\mathbb{P}(\{\omega: x \in U(\omega) \subseteq N(\omega)\}^c \cap \{\omega: x \in V(\omega) \subseteq M(\omega)\}^c) = 0,$$

and consequently

$$\mathbb{P}(\{\omega: x \in U(\omega) \subseteq N(\omega)\} \cap \{\omega: x \in V(\omega) \subseteq M(\omega)\})^c = 0$$

So, by completeness of $(\Omega, \mathcal{F}, \mathbb{P})$ we have

$$\{\omega: x \in U(\omega) \cap V(\omega) \subseteq N(\omega) \cap M(\omega)\}^c \in \mathcal{F}.$$

Hence

$$\{\omega: x \in U(\omega) \cap V(\omega) \subseteq N(\omega) \cap M(\omega)\} \in \mathcal{F}.$$

Therefore

$$\mathbb{P}\{\omega: x \in U(\omega) \cap V(\omega) \subseteq N(\omega) \cap M(\omega)\} = 1.$$

By definition of RN we get $N \cap M \in \mathfrak{N}_x$.

[RN3]: Suppose that $N \in \mathfrak{N}_x$, and take $M = \text{Int}\,(N)$. Then for each $y \in M$, $y \in \text{Int}\,(N)$. Then $\mathbb{P}\{\omega: y \in \text{Int}\,(N) \subseteq \text{N}\} = 1$. So $N \in \mathfrak{N}_y$.

[RN4]: Suppose that $N \in \mathfrak{N}_x$. Thenthereexistsrandomopenset U withthe property that $\mathbb{P}\{\omega: x \in U(\omega) \subseteq N(\omega)\} = 1$. Then $\mathbb{P}\{\omega: x \in \mathrm{Int}(N(\omega)) \subseteq N(\omega)\} = 1$. If $\mathbb{P}\{\omega: N(\omega) \subseteq M(\omega)\} = 1$, then $\mathbb{P}\{\omega: \mathrm{Int}(N(\omega)) \subseteq \mathrm{Int}(M(\omega))\} = 1$. So $\mathbb{P}\{\omega: x \in \mathrm{Int}(M(\omega))\} = 1$ and hence $\mathbb{P}\{\omega: x \in \mathrm{Int}(M(\omega)) \subseteq M(\omega)\} = 1$. Therefore $M \in \mathfrak{N}_x$.

[RN5]: If G isa random openset, and$x \in G$. Since$G = \mathrm{Int}(G)$. Then$\mathbb{P}\{\omega: x \in \mathrm{Int}(G(\omega)) \subseteq G(\omega)\} = 1$. Hence $G \in \mathfrak{N}_x$. Conversely, if $G \in \mathfrak{N}_x$ forevery$x \in G$. Then thereexists random openset V_x suchthat $\mathbb{P}\{\omega: x \in V_x(\omega) \subseteq G(\omega)\} = 1$. (In fact $\{\omega: x \in V_x(\omega) \subseteq G(\omega)\} = \Omega$). Hence $\bigcup_{x \in G(\omega)} \mathrm{Int}(V_x(\omega)) = G(\omega)$. Therefore G is random open set by Theorem (2.4).

Definition 3.7 ($\mathbb{P}$ −limit point). Let $(\Omega, X, \mathfrak{R})$ be an TRS and A be a random set in $(\Omega, X, \mathfrak{R})$. Apoint $x \in X$ issaidtobe $\mathbb{P}$ −limit point of$A$if $\mathbb{P}\{\omega: [N(\omega) - \{x\}] \cap A(\omega) \neq \emptyset\} = 1$ for every RN N of x.
Thesetof all $\mathbb{P}$ −limitpoint of A iscalledthe $\mathbb{P}$ −derivedset and is denotedby $\mathbb{P} - D(A)$.

Example 3.8 Consider the TRS $(\Omega, \mathbb{R}, \mathfrak{R})$ given in Example 3.2 with $= (-1, \infty)$. Then -1 is $\mathbb{P}$ −limit point of .

Definition 3.9 ($\mathbb{P}$ −closed set) The (deterministic) subset$A$ of $X$is said tobe $\mathbb{P}$ −closedset if ithas all of its $\mathbb{P}$ −limit points points. That is A of$X$ is $\mathbb{P}$ −closedset if and only if $\mathbb{P} - D(A) \subseteq A$. Thecomplement of $\mathbb{P}$ −closedset is called $\mathbb{P}$ −openset.

Example 3.10 Consider Example 3.8. Then all intervals of the form $[a, \infty) \subseteq \mathbb{R}$ are $\mathbb{P}$ −closed.

Note 3.11. The set G is $\mathbb{P}$ −open set if and only if $\mathbb{P} - D(G) \cap G = \emptyset$.

Lemma 3.12 The finite union of all $\mathbb{P}$ −closedsets is $\mathbb{P}$ −closedset.

Proof. Assume that$\{A_i: i = 1,2, \dots, n\}$ bea collectionof $\mathbb{P}$ −closed sets. To showthat $A = \bigcup_{i=1}^{n} A_i$ isa $\mathbb{P}$ −closedset. Let$x \in X$ bea $\mathbb{P}$ −limitpoint of theset$A$. Then for every RN N of x wehave $\mathbb{P}\{\omega: [N(\omega) - \{x\}] \cap A(\omega) \neq \emptyset\} = 1$. Set $F = \{\omega: [N(\omega) - \{x\}] \cap A(\omega) \neq \emptyset\}$ with $\mathbb{P}(F) = 1$. Hence $[N(\omega) - \{x\}] \cap A(\omega) \neq \emptyset$, $\forall \omega \in F$. Then $[N(\omega) - \{x\}] \cap \bigcup_{i=1}^{n} A_i(\omega) \neq \emptyset$, $\forall \omega \in F$,or equivalently $\bigcup_{i=1}^{n} \{[N(\omega) - \{x\}] \cap A_i(\omega)\} \neq \emptyset$, $\forall \omega \in F$. Then there exists $i_0 \in \{1,2, \dots, n\}$ suchthat $[N(\omega) - \{x\}] \cap A_{i_0}(\omega) \neq \emptyset$, $\forall \omega \in F$. That is $\mathbb{P}\{\omega: [N(\omega) - \{x\}] \cap A_{i_0}(\omega) \neq \emptyset\} = 1$. Then x is a $\mathbb{P}$ −limit point of A_{i_0}. But A_{i_0} is a $\mathbb{P}$ −closed set, then $x \in A_{i_0}$. Consequently $x \in \bigcup_{i=1}^{n} A_i = A$. Since x is an arbitrary it follows that $\bigcup_{i=1}^{n} A_i = A$ contains all of its $\mathbb{P}$ −limit points. Hence $\bigcup_{i=1}^{n} A_i = A$ is $\mathbb{P}$ −closed set.

4. $\mathbb{P}$ −Hausdorff system.

In this final section the concept of $\mathbb{P}$ −Hausdorff system is introduced and studied. The " Hausdorff property" is one of the important properties in the study the topology and its applications. Therefore we focus our study to study this concept in terms of probability theory and random set.

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Definition 4.1 A topological random system $(\Omega, X, \mathfrak{R})$ is said to be $\mathbb{P}-$ Hausdorff ($\mathbb{P}-T_2$) if for every there exist two distinctpoints $x, y \in X$, tworandom opensets A and B in Xsuch that $x \in A$, $y \in B$ and$\mathbb{P}\{\omega: A(\omega) \cap B(\omega) \neq \emptyset\} = 0$ or equivalently$\mathbb{P}\{\omega: A(\omega) \cap B(\omega) = \emptyset\} = 1$.

Example 4.1 Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probabilityspace, where $\Omega := \{H, T\}$, $\mathcal{F} := 2^{\Omega}$ and $\mathbb{P}(\{H\}) = \mathbb{P}(\{T\}) = \frac{1}{2}$ and let $\mathbb{R}$ be the set of all real numbers endowed with the usualtopology. Define the collection $\mathfrak{R} := \{A_x : x \in \mathbb{R}\}$ of sub sets of $\mathbb{R}$, where

$$A_x : \Omega \to \mathcal{B}(\mathbb{R}) \text{ , } A_x(\omega) = \{x\}, \forall \omega \in \Omega.$$

Thus $\mathfrak{R} := \{A_x : x \in \mathbb{R}\}$be the collection of randomsets in $\mathbb{R}$. Hence the triple $(\Omega, \mathbb{R}, \mathfrak{R})$ is TRS. Since $\mathbb{P}\{\omega: A(\omega) \cap B(\omega) = \emptyset\} = \mathbb{P}(\Omega) = 1$. Then $(\Omega, \mathbb{R}, \mathfrak{R})$ is $\mathbb{P}-T_2$ space.

Theorem 4.3 The subspace of a $\mathbb{P}-T_2$ spaceis$\mathbb{P}-T_2$.

Proof: Let$(\Omega, X, \mathfrak{R})$ be atopological random system and (Y, τ_Y) be asubspace of(X, τ). Let$x, y \in Y$ with$x \neq y$. Then$x, y \in X$. Byhypothesis, thereexisttwo open randomsets $A$and Bin X suchthat $x \in A$,$y \in B$ and$\mathbb{P}\{\omega: A(\omega) \cap B(\omega) \neq \emptyset\} = 0$. Define $C(\omega) := A(\omega) \cap Y$ and $D(\omega) := B(\omega) \cap Y$ are two random sets in Y with $x \in C(\omega)$ and $y \in D(\omega)$. Let $F = \{\omega: A(\omega) \cap B(\omega) \neq \emptyset\}$, such that $\mathbb{P}(F) = 0$. Then $\{\omega: C(\omega) \cap D(\omega) \neq \emptyset\} \subseteq F$. By completeness of $(\Omega, \mathcal{F}, \mathbb{P})$ we have$\{\omega: C(\omega) \cap D(\omega) \neq \emptyset\} \in \mathcal{F}$. Thus$\mathbb{P}\{\omega: C(\omega) \cap D(\omega) \neq \emptyset\} \leq \mathbb{P}\{\omega: A(\omega) \cap B(\omega) \neq \emptyset\} = 0$. It follows from completenessof theprobability spacethat$\mathbb{P}\{\omega: C(\omega) \cap D(\omega) \neq \emptyset\} = 0$. Thus the subsystem $(\Omega, Y, \mathfrak{R})$ is $\mathbb{P}-T_2$.

Theorem 4.4 Every singletonset in a$\mathbb{P}-T_2$ space is$\mathbb{P}-$closedset.

Proof. Let$(\Omega, X, \mathfrak{R})$ be a $\mathbb{P}-T_2$ and let$x \in X$. Toshow that$\{x\}$is $\mathbb{P}-$closed set. Let $y \in X$, with $x \neq y$. To prove that y is not $\mathbb{P}-$limit point of $\{x\}$. i.e., thereexists RN $N$of $y$such that $\mathbb{P}\{\omega: [N(\omega) - \{y\}] \cap \{x\} = \emptyset\} = 1$. Since $(\Omega, X, \mathfrak{R})$ is $\mathbb{P}-T_2$, thereexist tworandom opensets$A$ and$B$ in X suchthat$x \in M$, $y \in N$ and$\mathbb{P}\{\omega: M(\omega) \cap N(\omega) = \emptyset\} = 1$. Set $F = \{\omega: M(\omega) \cap N(\omega) = \emptyset\}$, with $\mathbb{P}(F) = 1$. Then$M(\omega) \cap N(\omega) = \emptyset$, $\forall \omega \in F$. Then$x \notin N(\omega)$, $\forall \omega \in F$. That is$\mathbb{P}\{\omega: x \notin N(\omega)\} = 1$ or$\mathbb{P}\{\omega: \{x\} \cap N(\omega) = \emptyset\} = 1$. Hence$\mathbb{P}\{\omega: \{x\} \cap (N(\omega) - \{y\}) = \emptyset\} = 1$. Consequently $\{x\}$ is $\mathbb{P}-$closed set.

Corollary 4.5 A finite (deterministic) sub set of a $\mathbb{P}-T_2$ is $\mathbb{P}-$closed set.

Proof. Thisfollows fromTheorem 4.4andLemma3.12.

Theorem 4.6 The RS $(\Omega, X, \mathfrak{R})$ is $\mathbb{P}-T_2$ $\leftrightarrow$foreach pair$x, y \in X$, thereexists a$\mathbb{P}-$nhd N_y of$y$ suchthat$\mathbb{P}\{\omega: x \notin \overline{N_y}(\omega)\} = 1$.

Proof: Supposing$(\Omega, X, \mathfrak{R})is\mathbb{P}-T_2$and let$x, y \in X$with$x \neq y$. Then thereexist two randomopen sets$G$ and$H$ in$X$ suchthat$x \in G$, $y \in H$and$\mathbb{P}\{\omega: G(\omega) \cap H(\omega) = \emptyset\} = 1$. Then$\mathbb{P}\{\omega: y \in H(\omega) \subseteq X - G(\omega)\} = 1$. Then $X - G(\omega)$ is closed $\mathbb{P}-$nhd of y and $\mathbb{P}\{\omega: x \notin X - G(\omega)\} = 1$. Set$N_y = X - G(\omega)$, then $\overline{N_y} = N_y$, so N_y is $\mathbb{P}-$nhd and $\mathbb{P}\{\omega: x \notin \overline{N_y}(\omega)\} = 1$

Conversely, suppose that for each pair $x, y \in X$,there exists a $\mathbb{P}$ −nhd N_y of$y$ suchthat $\mathbb{P}\{\omega: x \notin \overline{N_y}(\omega)\} = 1$. Since $\overline{N_y} \supseteq N_y$, then by Theorem (3.6) RN4 $\overline{N_y}$ is $\mathbb{P}$ −nhd of y. Since $\overline{N_y}$ is closed random set, then $X - \overline{N_y}$ is open random set with $x \in X - \overline{N_y}$ and $y \notin X - \overline{N_y}$. Put $-\overline{N_y} = N_x$, we see that there is a $\mathbb{P}$ −nhd N_x of x and a $\mathbb{P}$ −nhd $\overline{N_y}$ of y such that $\mathbb{P}\{\omega: N_x(\omega) \cap \overline{N_y}(\omega) = \emptyset\} = 1$. Consequently $\mathbb{P}\{\omega: N_x(\omega) \cap N_y(\omega) = \emptyset\} = 1$. Therefore $(\Omega, X, \mathfrak{R})$ is $\mathbb{P} - T_2$.

Theorem 4.7 The RS $(\Omega, X, \mathfrak{R})$ is $\mathbb{P} - T_2$ if and only if for every collection $\{F_\lambda: \lambda \in \Lambda\}$ of closed $\mathbb{P}$ − nhd ofeach $x \in X$we have$\mathbb{P}\{\omega: \bigcap_{\lambda\in\Lambda} F_\lambda(\omega) = \{x\}\} = 1$.

Proof. Suppose that$(\Omega, X, \mathfrak{R})$ is a$\mathbb{P} - T_2$ RS. Let$x, y \in X$ with$x \neq y$. Then thereexist $G, H \in ROS$suchthat $x \in G, y \in H$and$\mathbb{P}\{\omega: G(\omega) \cap H(\omega) = \emptyset\} = 1$. Thus$\mathbb{P}\{\omega: G(\omega) \subseteq X - H(\omega)\} = 1$. Hence$X - H(\omega)$ is a closed $\mathbb{P}$− nhd of$x$and by completeness of $(\Omega, \mathcal{F}, \mathbb{P})$ we have $\mathbb{P}\{\omega: y \in X - H(\omega)\} = 0$. If $\{F_\lambda: \lambda \in \Lambda\}$ of closed $\mathbb{P}$ − nhd of each $x \in X$, then $\mathbb{P}\{\omega: y \in \bigcap_{\lambda\in\Lambda} F_\lambda(\omega)\} = 0$. Since y is an arbitrary, then $\mathbb{P}\{\omega: \bigcap_{\lambda\in\Lambda} F_\lambda(\omega) = \{x\}\} = 1$.
Conversely, suppose that $\mathbb{P}\{\omega: \bigcap_{\lambda\in\Lambda} F_\lambda(\omega) = \{x\}\} = 1$ for every collection $\{F_\lambda: \lambda \in \Lambda\}$ of closed $\mathbb{P}$ − nhd of each$x \in X$. Let$y \in X$with$x \neq y$. Since $\mathbb{P}\{\omega: y \in \bigcap_{\lambda\in\Lambda} F_\lambda(\omega)\} = 0$, then there exists closed $\mathbb{P}$ − nhd N of each x $\mathbb{P}\{\omega: y \notin N(\omega)\} = 1$. Thenthere exists$G \in ROS$ such that$\mathbb{P}\{\omega: x \in G(\omega) \subseteq N(\omega)\} = 1$.

Therefore $G, N^c \in ROS$ such that $x \in G$ and $y \in N^c$. Finally, we must show that$F = \{\omega: G((\omega) \cap N^c(\omega) = \emptyset\} \in \mathcal{F}$, and $\mathbb{P}(F) = 1$.
We have$F = \{\omega: G(\omega) \cap N^c(\omega) = \emptyset\} = \{\omega: G(\omega) \subseteq N(\omega)\}$

$\supseteq$

$\{\omega: x \in G(\omega) \subseteq N(\omega)\}$.
By completeness of $(\Omega, \mathcal{F}, \mathbb{P})$, $F \in \mathcal{F}$and$\mathbb{P}(F) = 1$. This means that $(\Omega, X, \mathfrak{R})$ is a $\mathbb{P} - T_2$ RS.

Corollary 4.8 The RS $(\Omega, X, \mathfrak{R})$ is $\mathbb{P} - T_2$ if and only if for every collection $\{N_\lambda: \lambda \in \Lambda\}$ of $\mathbb{P}$ − nhd of each$x \in X$ we have$\mathbb{P}\{\omega: \bigcap_{\lambda\in\Lambda} \overline{N_\lambda}(\omega) = \{x\}\} = 1$.

Proof Since the closure of random set is closed random set (by Theorem 2.4) then the result followed directly from Theorem 4.6.

Theorem 4.9 The RS $(\Omega, X, \mathfrak{R})$ is $\mathbb{P} - T_2$ if and only if for every finite (deterministic) subset $\{x_i: i = 1,2,..,n\}$ of X there exists RN N_i of x_i for every $i = 1,2,..,n$ such that for every$i, j = 1,2,..,n$ with $i \neq j$ we have$\mathbb{P}\{\omega: N_i(\omega) \cap N_j(\omega) = \emptyset\} = 1$.

Proof. Supposing that$(\Omega, X, \mathfrak{R})$ be a$\mathbb{P} - T_2$ and$\{x_i: i = 1,2,..,n\} \subseteq X$ with $x_i \neq x_j$, $\forall i, j = 1,2,..,n$ with$i \neq j$. By hypothesis there exist $N_{ij}, N_{ji} \in ROS$ such that $x_i \in N_{ij}$,$x_j \in N_{ji}$ and $\mathbb{P}\{\omega: N_{ij}(\omega) \cap N_{ji}(\omega) = \emptyset\} = 1$. Let $N_i(\omega) = \cap\{N_{ij}(\omega): j = 1,2, \dots, n, i \neq j\}$.

Then by Theorem2.4 $N_i(\omega) \in ROS$, for every $i = 1,2,..,n$. To show that $\mathbb{P}\{\omega: N_i(\omega) \cap N_j(\omega) = \emptyset\} = 1$, for every $i,j = 1,2,..,n$ with $i \neq j$. Set $F = \{\omega: N_i(\omega) \cap N_j(\omega) = \emptyset\}$, for every $i,j = 1,2,..,n$ with $i \neq j$. Let $i,j = 1,2,..,n$ with $i \neq j$. Then $\forall \omega \in F$ we have

$$N_i(\omega) \cap N_j(\omega) = (\cap_{i\neq j} N_{ij}(\omega)) \cap (\cap_{i\neq j} N_{ji}(\omega))$$
$$= \cap_{i\neq j}(N_{ij}(\omega) \cap N_{ji}(\omega)) = \emptyset.$$

Then $F = \{\omega: N_{ij}(\omega) \cap N_{ji}(\omega) = \emptyset\}$ and hence $\mathbb{P}(F) = 1$.

Conversely, suppose that for every finite (deterministic) subset $\{x_i: i = 1,2,..,n\}$ of X there exists RN N_i of x_i for every $i = 1,2,..,n$ such that for every $i,j = 1,2,..,n$ with $i \neq j$ we have $\mathbb{P}\{\omega: N_i(\omega) \cap N_j(\omega) = \emptyset\} = 1$. It follows in particular that for any two distinct points x, y there exist $M, N \in ROS$ such that $x \in N, y \in M$ and $\mathbb{P}\{\omega: M(\omega) \cap N(\omega) = \emptyset\} = 1$. Thus $(\Omega, X, \mathfrak{R})$ be a $\mathbb{P} - T_2$.

Theorem 4.10 Let $(\Omega, X, \mathfrak{R})$ be an RS. If each point of X admits a τ −closed $\mathbb{P}$ −nhd of x which is a $\mathbb{P} - T_2$ sub-system of $(\Omega, X, \mathfrak{R})$, then $(\Omega, X, \mathfrak{R})$ is $\mathbb{P} - T_2$.

Proof. Let $x \in X$ and let Y be a τ −closed $\mathbb{P}$ −nhd of x in X. Such that $(\Omega, Y, \mathfrak{R})$ is $\mathbb{P} - T_2$ sub-system of $(\Omega, X, \mathfrak{R})$. First we need to show that every τ_Y −closed $\mathbb{P}$ −nhd of x is a τ −closed $\mathbb{P}$ −nhd of x. N^* be a τ_Y −closed $\mathbb{P}$ −nhd of x. Then there is a τ −closed $\mathbb{P}$ −nhd N of x such that $N^* = N \cap Y$. Since N and Y are random τ −closed sets then by Theorem 2.4 $N^* = N \cap Y$ is random τ −closed set and so N^* is τ −closed $\mathbb{P}$ −nhd of x. Now, let $\{F_\lambda: \lambda \in \Lambda\}$ be a collection of τ_Y −closed $\mathbb{P}$ − nhd of x by hypothesis we have $\mathbb{P}\{\omega: \cap_{\lambda\in\Lambda} F_\lambda(\omega) = \{x\}\} = 1$. It follows that $\{F_\lambda: \lambda \in \Lambda\}$ be a collection of τ − −closed $\mathbb{P}$ − nhd of x. Consequently, $(\Omega, X, \mathfrak{R})$ is $\mathbb{P} - T_2$.

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حول النظم التبولوجية العشوائية الهاوزدورفية من النمط — $\mathbb{P}$

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المستخلص:

قدمنا في هذا البحث مفهوم النظام التبولوجي العشوائي الذي يعتبر كخليط بين فرعين اساسيين من فروع الرياضيات " التبولوجيا" و "نظرية الاحتمال". تم دراسة النظم التبولوجية العشوائية الهاوزدورفية من النمط — $\mathbb{P}$ وتم تقديم بعض خواص هذا النظام مع برهانها.

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Linear programming measures for solving the problem of a linear division of convex multiplayers.

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Abstraction

In this research, we are trying to solve Simplex methods which are used for successively improving solution and finding the optimal solution, by using different types of methods Linear, the concept of linear separation is widely used in the study of machine learning, through this study we will find the optimal method to solve by comparing the time consumed by both Quadric and Fisher methods.

Keyword

simplex method, linear programming problem, Quadratic simplex method, separating hyperplane

Mathematics Subject Classification: **39AXX**

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Introduction

The creation of computers and the associated accelerated development of mathematical theories, including mathematical cybernetics and discrete mathematics, as well as information technologies, made it possible to set and solve on the computer new tasks that until recently were exclusively within the competence of man. One of such fundamental problems is the solution of multidimensional problems of linear separation of convex polyhedra.

The concept of linear separation is widely used in the study of machine learning. It is also used for the preparation of support vector methods, by means of which problems of complex systems are solved. In this study, some methods for testing the linear separation between two classes of data will be presented[10].

In general form, this problem can be formulated as follows: two subsets (two classes) X and Y of R ^ d are considered. They are linearly separable if there exists a hyperplane P of R ^ d and if the elements of X and Y lie on opposite sides of each other.

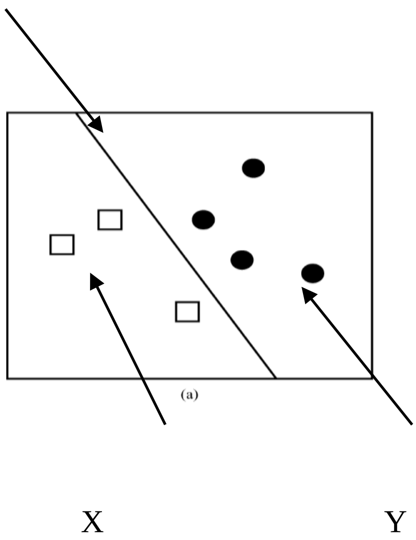
i.e.

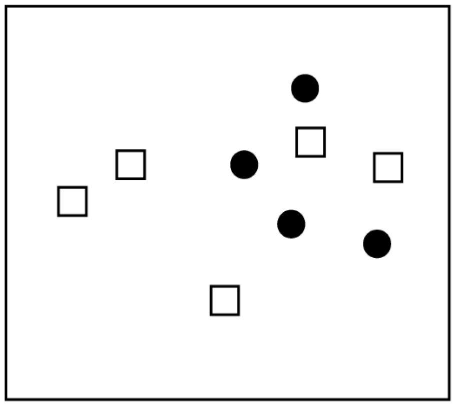
$$P\ (w,\gamma)\ =\ \{z \in\ R^{d}:<W,z>=\ \gamma\}.$$

$$< W,x > \leq \gamma \quad \forall\, x \in X.$$

$$< W,y > \leq \gamma \quad \forall\, y \in Y.$$

P (w,γ)





(b)

figure.1.

Squares and circles represent the two classes. (X and Y)

In Figure 1 (a) is represented by the linearly separable case, as in Figure 1 (b) is not separable case.

Purpose and objectives of the study

The purpose of this study is to develop a software package that numerically realizes and compare the solution of the problem of linear separation of convex polyhedral by various computational means:
1) By solving the linear programming problem by the simplex method;
2) By reducing to the problem of quadratic programming;
3) Through the application of the Fisher's method.
To achieve this goal, it was necessary to solve the following tasks:
-describe a general approach to solving the problem of linear separability of two classes on the basis of the listed methods;
- Design and implement a software package for solving the separation problem, using the developed methods and algorithms. Conduct computational experiments to analyze the effectiveness of the proposed approaches.

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Theoretical and practical value

The theoretical value of the study is that it gives a formal description of methods for solving the separation problem.
The practical value of the study lies in the fact that the proposed methods are implemented in the form of a software complex for the computer, which allows to effectively solve the problems of linear separability, in particular, used for dimension equal to 2, for the implementation of visual graphical interpretation.

In conclusion, summarizes the main results of the study, brief conclusions have presented the work.

1.1. The solution of the linear programming problem, Simplex - method.

When solving problems of linear programming, one of the most popular methods is the simplex method. The simplex method is a method of successively improving the solution and finding the optimal solution[1]. at the beginning if there is any admissible basic solution corresponding to one of the corner points of the solution polyhedron, and then this solution is purposefully improved by moving to a new basic solution at the neighboring corner point at which the value of the objective function does not decrease to the maximum , until an optimal solution is obtained. This method is universal, with the help of which it is possible to solve any linear programming problem.[2]
The simplex method is intended to solve linear programming problems in canonical form. Depending on the nature of the limits, linear programming problems can be solved with a natural and artificial basis.
If the limits are given by inequalities of the type "≤", then the problem is solved with a natural basis if the limits are given by inequalities of the type "≥" or the equalities "=", then the solution is conducted with an artificial basis.
There are three ways to solve the task :

1) The model is solvable.

2) The model is inadmissible.

3) The model is unlimited.

In this study, we use the simplex method to solve the problems of linear separability between two classes. Algorithms for solving the objective function are presented in Tables I and II. The first algorithm makes it possible to find the values of **q and p**, shows the obtaining of the optimal solution, or vice versa, shows that the problem does not have a solution

Table I. Algorithm of the objective function

- Data: an array A of size M × N is given, containing the constraints of linear expressions, where the rotation column and rows are represented as **p and q**.
- Result: the solution is, if the limits of the linear expression are solvable, and otherwise there is no solution.

```
Repeat
    q := 0;
    Repeat
        q := q + 1
    Until (q = M + 1) Or (a[0, q] < 0);
    p := 0;
    Repeat
        p := p + 1
    Until (p = N + 1) Or (a[p, q] > 0);
    For i := p + 1 To N Do
        If a[i, q] > 0
        Then
            If ((a[i, M + 1]/a[i, q]) < (a[p, M + 1]/a[p, q]))
            Then
                  p := i;
        If (q < M + 1) And (p < N + 1)
        Then
            pivot(p, q))
Until(q = M + 1) Or (p = N + 1)
```

Table II. Procedure of turning (p, q)

Data: the string p and the column q of the array M × N are given, containing the linear expression constraints that are used to perform the rotation

```
Begin
For j := 0 To N Do
    For k := M + 1 Downto 1 Do
        If (j <> p) And (k <> q)
        Then
              a[j, k] := a[j, k] − a[p, k] ∗ a[j, q]/a[p, q];
For j := 0 To N Do
    If j <> p
    Then
        a[j, q] := 0;
For k := 0 To M + 1 Do
    If k <> q
    Then
        a[p, k] := a[p, k]/a[p, q];
a[p, q] = 1;
End
```

These algorithms can be used to solve the jective function.
We illustrate the simplex method, showing Linear programming through some task Z[7].
Task Z: Let X = {(1,1)} and $Y = \{(0,0),(1,0),(0,1)\}$ be the initial data for two classes that define the problem Z. We check $X \,||\, Y$?

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Adj: is the operation of adding a value, in this case, we add *1* if the point belongs to class *1*, otherwise *-1* and the coordinates of points from *2* classes change the sign.$S = Adj(X,1) \cup -Adj(Y,1)\{1,1,1),(0,0,-1),(0,-1,-1),(-1,0,-1)\}$. So, to find out X || Y, we need to find a set of values for weights w_1 $+w_1$+t, and threshold *t*, so that it minimizes one of the indicated inequalities.

On Condition
$$\begin{cases} w_1 + w_2 + t > 0 \\ -t > 0 \\ -w_2 - t > 0 \\ -w_1 - t > 0 \end{cases} \quad (1.1)$$

Since the Simplex method limits the values of variables that are ≥ 0, and the weight value can be either positive or negative, we transform each of our original variables as a difference of two variables. That is, we reduce it to the canonical form:

$$\begin{cases} w_1 = w_{11\_aux} - w_{12\_aux} \\ w_2 = w_{21\_aux} - w_{22\_aux} \\ t = t_{1\_aux} - t_{2\_aux} \end{cases} \quad (1.2)$$

Since the simplex method does not take a strict inequality, we change our sign > 0 by ≥ 1.

Using the above transformations, our new set of constraints becomes:

$$\begin{cases} (w_{11_{aux}} - w_{12_{aux}}) + (w_{21_{aux}} - w_{22_{aux}}) + (t_{1_{aux}} - t_{2_{aux}}) \geq 1 \\ (-t_{1_{aux}} - t_{2_{aux}}) \geq 1 \\ (-w_{11_{aux}} + w_{12_{aux}}) - (t_{1_{aux}} + t_{2\_aux}) \geq 1 \\ (-w_{21_{aux}} - w_{22_{aux}}) - (t_{1_{aux}} + t_{2\_aux}) \geq 1 \end{cases}$$

Applying the simplex method next, we obtain an admissible solution which gives the following result:

$$w_{11_{aux}} = 2 \quad w_{12_{aux}} = 0$$
$$w_{21_{aux}} = 2 \quad w_{22_{aux}} = 0$$
$$t_{1_{aux}} = 0 \quad t_{2\_aux} = 3$$

[8] Thus, we can conclude that task Z is solved. Now we can define the values for the variables: $w_1 = 2, w_2 = 2$ t= –3. These variables form a hyperplane that separates the two classes.

The disadvantage of the Simplex method is that the best solution is not guaranteed in advance, it is impossible to say how many steps it takes to achieve the optimal solution. The complexity of this method depends on the rotation rule used, which was indicated in Table II.

1.2. The solution of the quadratic programming problem

The process of programming this method consists of solving Quadratic programming. It occurs in the use of the support vector method (SVM). The support vector method is a set of algorithms used for classification problems and regression analysis. Belongs to the family of linear classifiers. A special property of the support vector method is a continuous decrease in the empirical classification error and an increase in the gap, so the method is also known as the classifier method with the maximum gap. The main idea of the method is the translation of the initial vectors into a space of higher dimension and the search for a separating hyperplane with the maximum gap in this space. Two parallel hyperplanes are constructed on both sides of the hyperplane that separates our classes. The separating hyperplane is a hyperplane that maximizes the distance to two parallel hyperplanes. The algorithm works under the assumption that the greater the difference or the distance between these parallel hyperplanes, the smaller will be the average classifier error.

The hyperplane separating the two classes is defined as follows:

$$\begin{cases} \overrightarrow{w^T}\overrightarrow{x_i} + \mathrm{b} \geq 1 \quad c_i = 1 \\ \overrightarrow{w^T}\overrightarrow{x_i} + \mathrm{b} \leq 1 \quad c_i = -1 \end{cases} \quad (1.4)$$

Initial data from us $\vec{x}_i \in R^n, i = 1, \dots, l$ and $c_i, c \in \{-1,1\}$,

We find the inverse matrix by the Gauss method, for the convenience of the definition we denote $S_w = A$ *and* $S_w^{-1} = X$, then we use the equation A*X=E, where: X is the inverse matrix of the matrix A, and E is the identity matrix,

We reduce the matrix A to triangular matrix using the Gauss algorithm"Direct motion".

In parallel, we perform similar operations with the unit matrix E.

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We finally represent the obtained equation in its expanded form (fig 1.2)

The method of support vectors should find a solution to the minimization problem: $\frac{1}{2}\overrightarrow{\|w\|^2}$

On Condition $c_i(\overrightarrow{w^T}\overrightarrow{x_i} + \text{b}) \geq 1$ Where C> 0 represents the penalty parameter.

We solve the following minimization problem: $\frac{1}{2}\overrightarrow{\|w\|^2}$ (1.5)

Formal description of the task:

We believe that the points have the form:

$$\{(x_1, c_1), (x_2, c_2), \dots, (x_n, c_n)\}$$

Where c_i takes the values 0 or 1, depending on which class the point x_i belongs to. Each x_i is a p-dimensional real vector, usually normalized by the values [0,1]. If the points are not normalized, the point with large deviations from the average coordinates of the points will affect the classifier too much. We can treat this as a learning collection, in which the class to which it belongs is already assigned to each element. We want the algorithm of the support vector method to classify them in the same way. To do this, we construct a separating hyperplane that looks like this:

w*x-b=0; (1.6)

Where w, b - are unknown, they need to be found.

The vector w is the perpendicular to the separating plane (hyperplane);

W=$(w_1, w_2, \dots, w_n)$;

$\frac{b}{\|w\|}$- the distance from the hyperplane to the origin;

If the parameter b is zero, the hyperplane passes

through the origin, which limits the solution.

$\|w\| = \sqrt{\sum_{i=1}^{n} w_i^2}$ -Is the length of the vector (1.7)

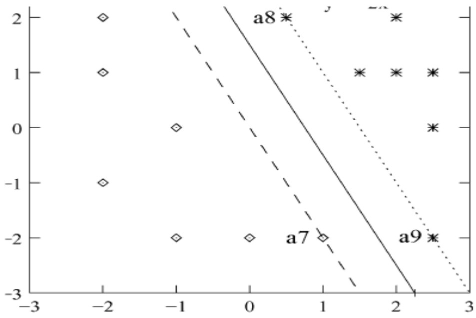
Since we are interested in the optimal separation, we are interested in support vectors and hyperplanes parallel to the optimal and closest to the supporting vectors of two classes. It can be shown that these parallel hyperplanes can be described by the following equations (up to normalization).

w*x-b=1, (1.8)

w*x-b=−1 . (1.9)

If the training sample is linearly separable, then we can choose hyperplanes in such a way that no points of the training sample lie between them and then maximize the distance

between the hyperplanes. The width of the strip between them is easy to find from considerations of geometry, it is $\frac{2}{||\text{w}||}$, so our task is to minimize $||\mathbf{w}||$.



$$\frac{2}{||\mathbf{w}||}$$

To exclude all points from the strip, we must verify for all i that

$$\begin{cases} \text{w} * \text{x}_\text{i} - \text{b} \leq -1, c_i = -1 \\ \text{w} * \text{x}_\text{i} - \text{b} \geq 1, c_i = 1 \end{cases} \quad (1.10)$$

This can also be written in the form:

$c_i(w * x_i - b) \geq 1; \quad 1 \leq i \leq n$ (1.11)

The problem of constructing an optimal separating hyperplane reduces to minimizing ‖ w ‖ , Under the condition (1.11). This is the problem of quadratic optimization, which has the form:

$$\begin{cases} \|\boldsymbol{w^2}\| \to \boldsymbol{min} \\ c_i(w * x_i - b) \geq 1; \quad 1 \leq i \leq n \end{cases} \quad (1.12)$$

By the Kuhn-Tucker theorem (necessary conditions for solving the problem of non-linear programming), this problem is equivalent to the dual problem of finding the saddle point of the Lagrange function.

$$\begin{cases} L(w, b, h) = \frac{1}{2}\|\boldsymbol{w^2}\| \\ -\sum_{i=1}^{n} \boldsymbol{h_i}(\boldsymbol{c_i}((\boldsymbol{w}, \boldsymbol{x_i}) - \boldsymbol{b}) - \boldsymbol{1} \to \boldsymbol{min_{w,b}max_{\lambda}} \\ \boldsymbol{\lambda_i \geq 0, 1 \leq i \leq n} \end{cases} \quad (1.13)$$

$\lambda = (\lambda_{1,\dots,}\lambda_n)$ a vector of dual variables.

We reduce this problem to an equivalent quadratic programming problem, which contains only dual variables:

$$\begin{cases} -L(\boldsymbol{\lambda}) = -\sum_{i=1}^{n} \boldsymbol{\lambda_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \boldsymbol{\lambda_i \lambda_j c_i c_{j_i}}(\boldsymbol{x_i x_j}) \to \boldsymbol{min\lambda_i} \\ \boldsymbol{\lambda_i \geq 0, 1 \leq i \leq n} \\ \sum_{i=1}^{n} \boldsymbol{\lambda_i c_i = 0} \end{cases}$$

(1.14)

All variables are λ
We single out the linear and quadratic parts as follows:

$$-L(\lambda) = -\underbrace{\sum_{i=1}^{n}\lambda_i}_{CX} + \underbrace{\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\lambda_j c_i c_j (x_i x_j)}_{X^T DX} \rightarrow \min\lambda_i \quad (1.15)$$

Next, we make a system check (1.10). Namely, a quadratic function having a nonnegative definite quadratic form is minimized, hence, the objective function is convex. An admissible region is a polyhedron, which means that we are faced with the problem of quadratic programming.

Hence we can apply the *Kuhn-Tucker* optimality theorem.

We have a task: $\mathbf{CX} + \frac{1}{2}\boldsymbol{X^T}\mathbf{DX} \rightarrow \boldsymbol{min}$(1.16)
Under the constraints: AX$\leq b, X \geq 0$,

Next, we apply the *Kuhn-Tucker* theorem. The convexity conditions of the objective function and the admissible domain are allowed to use this theorem.
We record the *Kuhn-Tucker* system:

$$\begin{cases} \text{Ax} + Y = b \\ -Dx - A^{\{T\}U} + V = C \\ V^{\{T\}x} = 0, \ \ U^{\{T\}Y} = 0 \\ \text{X, Y, U, V} \geq 0 \end{cases} \quad (1.17)$$

Hence we have a system of equations.
$V^{\{T\}x}=0$, $U^{\{T\}Y}=0-$ is called the complementary non-rigidity condition.
That is, if we introduce vectors $y = (y_1...,y_n)$ и $\boldsymbol{v} = (v_1...,v_m)$, Then relation

$$(\boldsymbol{c+2xD+uA})\boldsymbol{x}^T\boldsymbol{=0}$$

$$\boldsymbol{xAT—b} \text{ £}\boldsymbol{0} \quad (1.18)$$

$$(\boldsymbol{xAT—b})\boldsymbol{u}^T \boldsymbol{=0}$$

Will look like:

$$\boldsymbol{c} + 2\,\boldsymbol{x}\,\boldsymbol{D} + \boldsymbol{u}\,\boldsymbol{A} — \boldsymbol{в} = \boldsymbol{0} \quad (1.19)$$

$$\boldsymbol{x}\,\boldsymbol{AO} — \boldsymbol{b} + \boldsymbol{v} = \boldsymbol{0} \quad (1.20)$$

$$\boldsymbol{в}\,\boldsymbol{xT} = \boldsymbol{0},\ \boldsymbol{v}\,\boldsymbol{uT} = \boldsymbol{0} \quad (1.21)$$

The system (1.19) - (1.20) consists of $n + m$ linear equations relatively $2(m+n)$ Unknown $xj,\ yj\ (j = 1...,n), ui,\ vi\ (i = 1...,m)$.
In addition, as follows from conditions (1.18), there should be:

If $x_j > 0$, then $y_j = 0$ (1.22)

If $y_j > 0$, then $x_j = 0$ (1.23)

If $u_i > 0$, then $v_i = 0$ (1.24)

If $v_i > 0$, then $u_i = 0$. (1.25)

Consequently, the desired solution of the system (1.19) - (1.20) can be an arbitrary inseparable basic solution of it, but such that the variables *xj* and *yj* (and also *ui* and *vi*) with the same indices cannot be basic at the same time. To find such a solution, one can apply any of the known methods of ***LP***, in particular, the artificial base method.
To this goal, we write the system (1.19) - (1.20) in the form
2 x D + u A — в = — c (1.26)
x AO + v = b. (1.27)

Without limiting universality, we will assume that the right-hand parts of this system are inalienable. According to the artificial basis method, in every equation of the system (1.26)—(1.27), which does not contain a basic variable, we introduce an artificial variable. Since the variables vi (i=1...,m) Can be considered basic, then the artificial variables z=(z1...,zn) We introduce only Eq. (1.26) and consider the auxiliary CLRF.
z iT min (1.28)

2 x D + u A — в + z = — c (1.29)
x AT+ v = b (1.30)

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Where i = (1,1...,1) — n Dimensional unit vector.

Next, we solve this system, applying the rule, the transition from one basis to another, which is used in the simplex method.

We have, that $\boldsymbol{V_i x_i}$ = 0 = $\boldsymbol{U_j Y_j}$ For all i, j. The fulfillment of this condition means that if the variable $\boldsymbol{V_i}$ in the basic solution takes a positive value, then the variable $\boldsymbol{U_j}$can not be a basic variable and take a positive value. Similarly, the variables $\boldsymbol{x_i}$and $\boldsymbol{Y_j}$ cannot simultaneously take positive values. That is, this rule is also applied in the simplex method.

Quadratic simplex method

If the found admissible basic solution of this problem satisfies the conditions of complementary non-rigidity, then it determines the optimal solution of the original. Otherwise, we must pass to a new admissible basic solution. In this case, a new variable with a zero estimate is included in the basic variables. The simplex method with the conditions (1.22) - (1.25) for the solution of the auxiliary CLPP (1.28) - (1.30), constructed on the basis of the problem of convex quadratic programming (1.16), is called the quadratic simplex method (the algorithm of the ordinary simplex method)[9].

If in the optimal solution of the auxiliary CPRF (1.28) - (1.30) all the artificial variables zj (j=1...,n) take zero values, then discarding them, we obtain the DBR of the system (1.26) - (1.27). That part of it that corresponds to the variables of the initial problem of convex quadratic programming (1.16) is its optimal solution.

If the value of at least one of the artificial variables is different from zero in the optimal solution of the auxiliary CLRP (1.28) - (1.30), then the system (1.26) - (1.27) has no solutions, and consequently the set of saddle points of the Lagrange function of the initial problem of convex Quadratic programming is empty.

Let's say we solved the problem and found $\boldsymbol{\lambda} = (\boldsymbol{\lambda_{1,...,}} \boldsymbol{\lambda_n})$ and now we can find a hyperplane that separates two classes:

w=$\sum_{i=1}^{n} \boldsymbol{\lambda_i c_i x_i}$, (1.31)

b=w$\boldsymbol{x_i}$-$\boldsymbol{c_i}$, $\boldsymbol{\lambda_i > 0}$ (1.32)

The summation does not go over the entire sample, but only over reference ones, for which $\boldsymbol{\lambda_i \neq 0}$.

The advantage of support vector methods is that the problem of convex quadratic programming has a unique solution. And also the support vector method is the best method of linear classification.

1.3. The solution of the problem of linear separation by the Fisher method

Fisher's linear discriminant in the original value is a method that determines the distance between the distributions of two different classes of objects or events. It can be used in problems of machine learning with a statistical (Bayesian) approach to solving classification problems[3].

Suppose that the training sample satisfies the following hypotheses in addition to the basic hypotheses of the Bayesian classifier:

Classes are distributed according to the normal law

Covariance matrices of classes are equal

Such a case corresponds to the best division of classes by the Fisher discriminator (in the original value). Then the statistical approach leads to a linear discriminant, and it is this classification algorithm that is now often understood as the linear discriminant of Fisher[4].

Under certain general assumptions, the Bayesian classifier reduces to the formula:

a(x)=arg $\max_{y \in Y} \lambda_y P_y p_y(x)$ (1.33)

Where Y is the set of answers (classes), x belongs to the set of objects X, P_y is the a priori probability of class y, p_y(x)is the likelihood function of class y, $\lambda_y$is the weighting factor (error price on object of class Y).

In the extension of all the above hypotheses, in addition to the hypothesis of the equality of covariance matrices, this formula has the form:

a(x)=arg $\max_{y \in Y}(ln(\lambda_y P_y) - \frac{1}{2}(x-\mu_y)^T \sum_y^{-1}(x - \mu_y) - \frac{1}{2} ln (|\sum_y^{-1}|) - \frac{n}{2}\ln(2\pi)),$ (1.34)

Where $\mu_y = \frac{1}{l_y}\sum_{\substack{i=1 \\ y_i=y}}^{l} x_i \sum_y = \frac{1}{l_y}\sum_{\substack{i=1 \\ y_i=y}}^{l} (x_i - \mu_y)(x_i - \mu_y)^T$ Approximations of the expectation vector and the covariance matrix of class Y, obtained as maximum likelihood estimates, l is the length of the training sample, $l_y$is the number of objects of class Y in the training sample x∈ R^n n.

This classification algorithm is

A quadratic discriminant. It has a number of drawbacks, one of the most important of which is the poor conditionality or degeneracy of the covariance matrix $\sum y$ with a small number of training elements of class Y, which can result in a strongly distorted result when the given matrix $\sum_y^{-1}$ in the past, And the entire classification algorithm is unstable, it will work poorly

(it is also possible that the inverse matrix $\sum_y^{-1}$ does not exist at all).

The linear discriminant of Fisher solves this problem

The main idea of the algorithm

When accepting the hypothesis of equality between covariance matrices, the classification algorithm takes the form:

$$a(x)=\arg\max_{y\in Y}(\ln(\lambda_y P_y) - \frac{1}{2}\mu_y{}^T \textstyle\sum_y^{-1}\mu_y + x^T \sum_y^{-1}\mu_y), \text{ OR } a(x)=\arg\max_{y\in Y}\left(\beta_y + x^T\alpha_y\right) \quad (1.35)$$

Simplicity of classification by the linear *discriminant Fisher* -

One of the advantages of the algorithm: in the case of two classes in the two-dimensional characteristic space, the separating surface will be a straight line. If there are more than two classes, then the dividing surface will be piecewise-linear. But the main advantage of the algorithm in comparison with the quadratic discriminant is the decrease in the effect of the poor conditionality of the covariance matrix with insufficient data.

For small l_y approximations

$$\Sigma_y = \frac{1}{l_y}\sum_{\substack{i=1\\ y_i=y}}^{l} (x_i - \mu_y)\,(x_i - \mu_y)^T \quad (1.36)$$

Will give a bad result, therefore, even in those problems where it is known that classes have different forms, sometimes it is advantageous to use the heuristic of the Fisher discriminator and to consider the covariance matrices of all classes to be equal. This will allow us to calculate some "average" covariance matrix using the entire sample[6]:

$$\sum = \frac{1}{l_y}\sum_{i=1}^{l}(x_i - \mu_y)\,(x_i - \mu_y)^T \quad (1.37)$$

The use of which in most cases will make the classification algorithm more stable.

Formulation of the problem:

Let X_1 and X_2 represent a certain problem of classifying points into two classes. And let $X_1 = \{x_1^1, x_2^1, \dots, x_{l_1}^1\}$ Set of points belonging to class X

and $X_2 = \{x_1^2, x_2^2, \dots, x_{l_2}^2\}$ $x_i^j \in R^d$, Belong to the class

Y.

By Fisher's method, we solve the problem posed and determine whether the points are separable or not. The linear Fisher discriminator is given by the vector w, which maximizes

$J(\overrightarrow{w}) = \overrightarrow{w^T}\, S_w \vec{w} \,/\, \overrightarrow{w^T}$ SW $\overrightarrow{w'}$
Where S_w Selective covariance matrix[8]

The **covariance matrix** in probability theory is a matrix composed of the pairwise covariances of the elements of one or two random vectors. The covariance matrix of a random vector is a square symmetric non-negative definite matrix on the diagonal of which the variances of the vector components are located, and the off-diagonal elements are the covariance between the components. The covariance matrix of a random vector is a multidimensional analog of the variance of a random variable for random vectors. The covariance matrix of two random vectors is a multidimensional analog of the covariance between two random variables.
It is given by the following formulas:

$S_w = (m_1 - m_2)(m_1 - m_2)^T$ (1.38)
$S_W = \sum_{i\epsilon\{1,2\}}\sum_{j=1}^{l_i}(x_j^i - m_i)\,(x_j^i - m_i)^T$ (1.39)
$m_i = l - \frac{1}{i}\sum_{j=1}^{l_i} x_j^i$;
$m_i = \sum_{j=0}^{l_i} x_j^i \,\frac{1}{l_i}$
l_i - Number of elements in a class l .

The parameter of the linear classifier is given by the vector $\overrightarrow{W}$:

q(x) = 0, for $\overrightarrow{W}' \times \vec{x}$ +b ≥ 0 (1.40)
q(x) = 1, for $\overrightarrow{W}' \times \vec{x}$ +b < 0 (1.41)

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Example 1. Let the points belong to the classes X and Y.

X = {(−1; 3); (−1; −2); (−3; 2)}
Y = {(−1; 2); (2; 1)}
$m_1 -$ *Mean point value* x_i ;
$m_2 -$ *Mean point value* y_i ;
For each class, calculate the mean of the points:

$m_1 = (-1{,}6666;\ -1)$;
$m_2 = (0{,}5;\ 1{,}5)$;
S_1 *Matrix of selective covariance of points*
S_2 *Matrix of selective covariance of points* y_i
$S_1[i,j] = \mathrm{Cov}(x_i, x_j)$; (1.42)
$S_2[i,j] = \mathrm{Cov}(y_i, y_j)$; (1.43)
S_W – Sum of two matrices
$S_W = S_1 + S_2 \Rightarrow S_W[i,j] = \mathrm{Cov}(x_i, x_j) + \mathrm{Cov}(y_i, y_j)$; (1.44)

$$S_W = \begin{pmatrix} 5{,}8333; & -4{,}5 \\ -4{,}5; & 7{,}5 \end{pmatrix}$$

S_W^{-1} – inverse matrix
The inverse matrix is a matrix S_W^{-1} multiplied by which the original matrix S_W gives the unit matrix E: $S_W * S_W^{-1} = S_W^{-1} * S_W = E$ (1.45)
We find the inverse matrix by the Gauss method, for the convenience of the definition we denote $S_W = A$ and $S_W^{-1} = X$, we use the equation A * X = E, where: X is the inverse matrix of the matrix A, and E is the identity matrix. We reduce the matrix A to a triangular matrix using the Gauss algorithm "direct motion". In parallel, we perform similar operations with the unit matrix E. We represent the obtained equation in its expanded form (see Figure 1.2)[9].

$$\begin{pmatrix} a_{11} + a_{12} + a_{13} + \dots & +a_{1n} \\ 0 + a_{22}^{(1)} + a_{23}^{(1)} + \dots & +a_{2n}^{(1)} \\ 0 + 0 + a_{33}^{(2)} + \dots & +a_{3n}^{(2)} \\ \vdots \quad \vdots & \\ 0 + 0 + 0 + \dots & +a_{nn}^{(n-1)} \end{pmatrix} \cdot \begin{pmatrix} x_{11} + x_{12} + x_{13} + \dots + x_{1n} \\ x_{21} + x_{22} + x_{23} + \dots + x_{2n} \\ x_{31} + x_{32} + x_{33} + \dots + x_{3n} \\ \vdots \quad \vdots \\ x_{n1} + x_{n2} + x_{n3} + \dots + x_{nn} \end{pmatrix} = \begin{pmatrix} e_{11} + 0 + 0 + \dots & +0 \\ e_{21}^{(1)} + e_{22}^{(1)} + 0 + \dots & +0 \\ e_{31}^{(2)} + e_{32}^{(2)} + e_{33}^{(2)} + \dots & +0 \\ \vdots \quad \vdots & \\ e_{n1}^{(n-1)} + e_{n2}^{(n-1)} + e_{n3}^{(n-1)} + \dots & +e_{nn}^{(n-1)} \end{pmatrix}$$

Figure.1.2 finding the inverse matrix by the Gauss method

Starting from the equation (Figure. 1.2), we express the values of the inverse matrix X. Multiplying the last row of the matrix A by the first column of the matrix X, we obtain the equality

$0 * X11 + 0 * X21 + 0 * X31 + 0 * X41 + \dots + Ann * Xn1 = En1$, Here we express $Xn1 = \frac{En1}{Ann}$. Similarly, we obtain from the following equation $0 * X11 + 0 * X21 + 0 * X31 + \dots + A(n-1)(n-1) * X(n-1)(1) + A(n-1)(n) * X(n)(1) = E(n-1)(1)$,
Express the $X(n-1)(1) = E(n-1)(1)/A(n-1)(n-1) - Xn1 * A(n-1)(n)/A(n-1)(n-1)$
and so on.
This operation is called the "reverse course of finding the elements of the inverse matrix X". Using formula (1.42), it is not difficult to calculate all the elements of the matrix X.
Matrix elements X.

$X_{ji} = \frac{1}{A_{jj}}(E_{ji} - \sum_{k=n}^{j} A_{jk} X_{ki})$ (1.46)

i=1…n,
j=n…1

$$S_W^{-1} = \begin{pmatrix} 0{,}319; & 0{,}19 \\ 0{,}19; & 0{,}248 \end{pmatrix}$$

W – The normal of the hyperplane;

$W = S_W^{-1}(m_1 - m_2)$; (1.47)

$W = (-1{,}17;\ -1{,}035)$;

Finally, we calculate the hyperplane shift:

b = $(w * m_1 - w * m_2)$ / 2 (1.48)

$< w, x > -b \leq 0$;

$< w, x > \leq b$;

Known Known b = 0,42337 ;

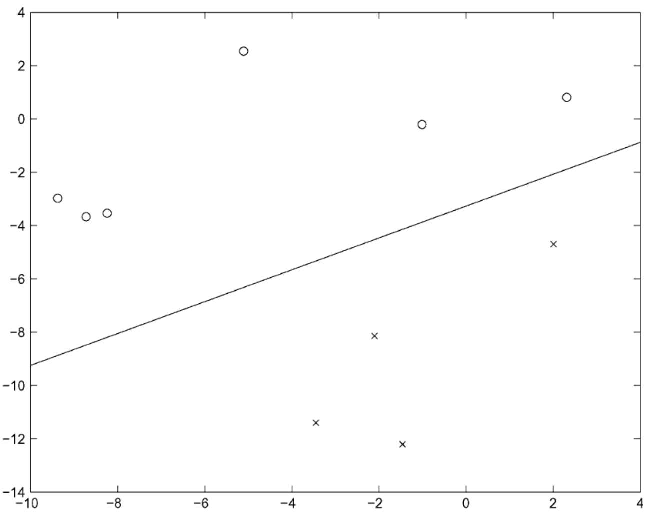


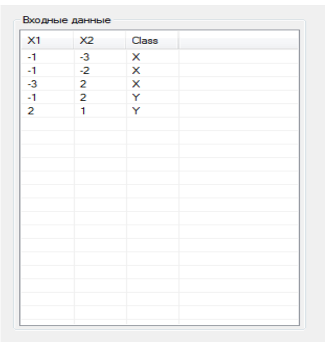
Figure 1.3. Hyperplane for linear Fisher discriminator.

The heuristic of the *linear Fisher discriminator* is in some way a simplification of the quadratic discriminant. It is used to obtain a more stable classification algorithm. It is most appropriate to use *the linear discriminant of Fisher* when Data for training is not enough. Owing to the main hypothesis on which the algorithm is based, it is most successfully solved by simple classification problems in which the forms are "similar" to each other in forms. The classification process by the linear Fisher discriminator can be described by the following scheme:

- Estimation of mathematical expectations
- Calculation of the general covariance matrix

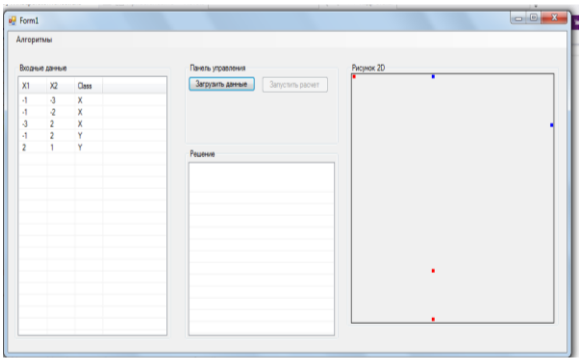
  and its inversion

Our sample of data

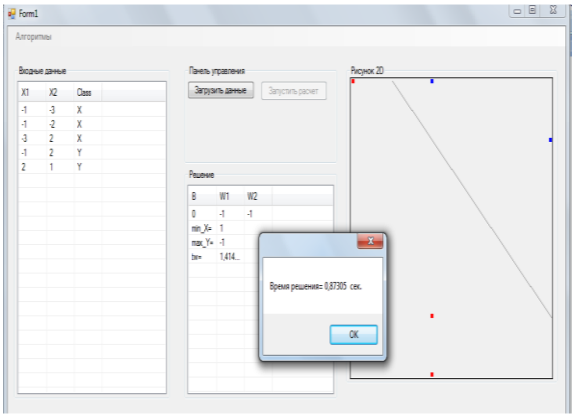

Входные данные

X1	X2	Class
-1	-3	X
-1	-2	X
-3	2	X
-1	2	Y
2	1	Y

(Fig 1) Feeding this data to the program



We start on calculation and we see the following result:



In this case, the points are linearly separable. And they are separated from each other by a separating hyperplane. Where *b* is the hyperplane shift, w is the normal of the hyperplane, *min_X* is the minimal scalar product of the vectors z_i by the normal, is calculated by the formula:

$$\min_{i=\overline{1,l}} < \boldsymbol{w}, \boldsymbol{z_i} >$$

max_Y is the maximum scalar product of the vectors z_i by the normal, is calculated by the formula:

$$\max_{j=\overline{1,m}} < \boldsymbol{w}, \boldsymbol{p_j} >$$

$< \boldsymbol{w}, \boldsymbol{x} > = \boldsymbol{b}$ Hyperplane shift.

The thickness of the hyperplane t_w in separable cases is calculated as follows:

$$\widehat{W} = \frac{W}{\|w\|}.t\left(\frac{W}{\|w\|}\right) = \left|\min_{i=\overline{1,l}} < z_i, \frac{W}{\|w\|} > - \max_{j=\overline{1,m}} < p_j, \frac{W}{\|w\|} >\right| t(\widehat{W}) = \left|\min_{i=\overline{1,l}} < z_i, \widehat{W} > - \max_{j=\overline{1,m}} < p_j, \widehat{W} >\right|$$

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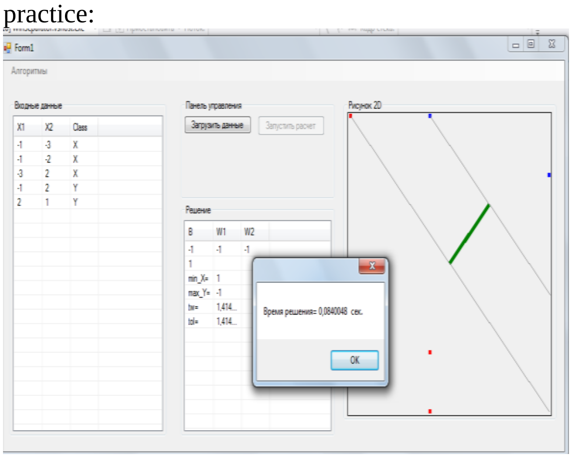
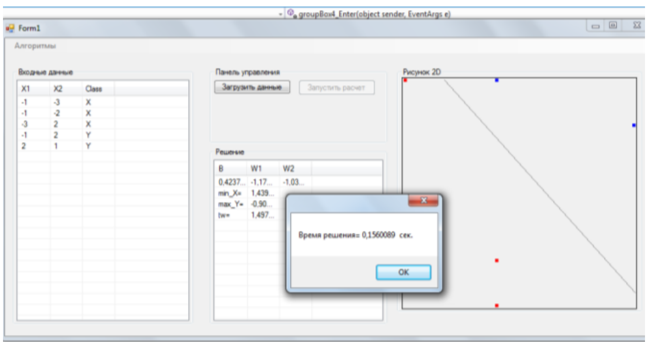
The quadratic method and see the following result in practice:



Time consumed 0.0840048 Sec

The Fisher method in practice:

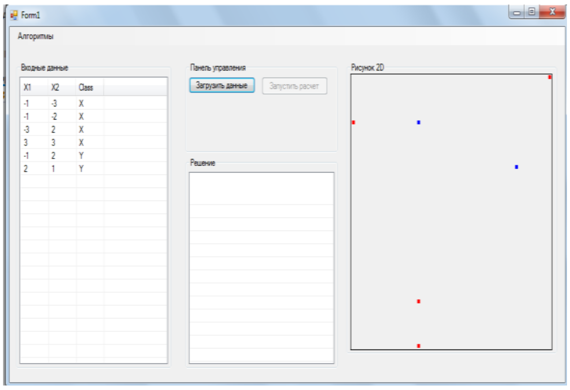


Time consumption 0.01560089 Sec

Results

METHOD	TIME CONSUMPTION
EXAMPLE 1	
Quadric	0.0840048
Fisher	0.01560089
EXAMPLE 2	
Quadric	0.0890051
Fisher	0.1480084
EXAMPLE 3	
Quadric	0.0320019
Fisher	0.0890051
EXAMPLE 4	
Quadric	0.035002
Fisher	0.0920052

Another side view where there is no solution



The both methods did not find solution.

Conclusion

In the study, the problems related to the methods of solving the linear separation problem were considered. A software package was developed that numerically realizes the solution of the problem of linear separation of convex polyhedral by various computational means:

1) By solving the linear programming problem by the simplex method;

2) By reducing to the problem of quadratic programming;

3) Through the application of the Fisher method.

A general approach to solving the problem of linear separability of two data classes based on the listed methods was described. Computational experiments were conducted to analyze the effectiveness of the proposed approaches. A comparison of the effectiveness of the approaches considered was carried out for problems of the different dimensionality of space and the number of points defining separable sets with respect to the following indicators:

1) The time for solving the problem,

2) The thickness of the separation layer (formed between two hyperplanes supporting to shared sets) of two classes of points.

Comparison of methods

The task was to compare methods and determine which one is effective. When comparing, it is necessary to take into account the complexity of the task, the time of performance of work and the value of the measure of the thickness of the compartment.

1. When input data of dimension $n = 2$, from the point of view of the thickness of separation by a hyperplane, the Simplex method and the quadratic programming method are effective. In most cases, they have the same thickness of separation t_w. The results obtained (for dimension $n = 2$) can also be compared by the time of the work. After doing a lot of experiments, we saw that the *quadratic programming method* is optimal for the time of execution of the work.
2. If the input data has the dimension $n = 3, \ldots n = 10$, etc., then the best method is the method based on *quadratic programming*. The effectiveness of the method of quadratic programming does not depend on the dimension. When comparing the thickness of the compartment, in these tests quadratic programming provided the greatest thickness.

And by the time of the work, the optimal is the simplex method.

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المستخلص :

في هذا البحث، نحاول حل العادلات البسيطة التي تستخدم في إيجاد الحل الأمثل ، وذلك باستخدام أنواع مختلفة من المعادلات الخطية، ان استخدام مفهوم تعلم الاله (Machine Learning) الان يعتبر من اهم المواضع على نطاق العالم حيث استخداماتة واسعة تصل حتى لقيادة السيارة ولهذا الغرض سنستخدم مفهوم الفصل الخطي يستخدم على نطاق واسع في دراسة تعلم الآلة، من خلال هذه البحث وسوف نجد الطريقة المثلى لحل عن طريق مقارنة الوقت المستهلكة من قبل كل من طرق كوادريك و فيشر.

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Non-polynomial spline finite difference method for solving second order boundary value problem

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Abstract

Second -order two-point boundary value problems (BVPs) were solved based on a of non-polynomial spline general functions with finite difference method.In this paper we discusses a method depended on using special finite-difference approximations for derivatives and formatting a formula that can be deal with endpoints that exceed the usual finite-difference formula for derivatives. Convergence analysis of the method is discussed. The numerical description of the method is shown by four examples. So the results obtained by our method are very encouraging over other existing methods.

Keywords: Non-polynomial spline method, Finite difference, Boundary value problem, Truncation error, Exact solution.

Mathematics Classification 65L1265M0641A15.

1. Introduction

In this paper our concern is approximating the solution of the second order linear BVP by using non-polynomial spline function[5-9,11-13],we discussed the numerical solution for two cases of the problem using non-polynomial spline function.
consider the linear second order BVP of the form [7,10]

$$y''(x)+r(x)y'+s(x)y=g(x) \quad ,x\in[a,b] \qquad (1)$$

subject to the boundary conditions (B.Cs)

$$(A) \quad y(a)=\alpha_1 \quad ,y(b)=\beta_1$$

$$(B) \quad y'(a)=\alpha_2 \quad ,y'(b)=\beta_2 .$$

Where $r(x)$, $s(x)$ and $g(x)$ are continuous function on $[a,b],\alpha_1,\beta_1,\alpha_2$ and β_2 are real constant .The main objective of our research is to introduce a new spline method to approximate the second order BVP as in (1).

This paper is organized as follows. In section2, derive of the method with (B.Cs)-(A). Analysis of the method in section 3.In section4, derive of the method with (B.Cs)-(B). In section 5, non-polynomial spline solution.Section 6 convergence analysis .Section 7 some numerical examples to show the performance of the proposed method and for comparison purposes with another numerically methods. Finally, the conclusion is given in section 8.
The main objective of our research is to introduce a new non-polynomial spline method to approximate the second order boundary value problem as in (1).

2 . Derive of the method with B.Cs -(A)

We introduced a finite set of grid point x_i by dividing the interval $[a,b]$ into n equal parts where $x_i = a + ib$,i=0,1,2,...,n, $x_0 = a$ $x_n = b$ and $h = \dfrac{b-a}{n+1}$.

Let $y(x)$ be the exact solution of problem (1) and y_i be the approximation value to $y(x_i)$ value obtained by the spline function $\varphi_i(x)$ passing through the point (x_i, y_i) and (x_{i+1}, y_{i+1}).

Each non-polynomial spline segment has the form [4]

$$\varphi_i(x) = a_i + b_i(x - x_i) + c_i \sinh(k(x - x_i)) + d_i \cosh(k(x - x_i)) \quad (2)$$
$$, i = 0,1,2,\dots,n-1$$

Where a_i, b_i, c_i, d_i are constants and k is free parameter to be determined later. Our non-polynomial spline is now defined by the relations

(i)

$$y_i(x) = \varphi_i(x) \quad , x \in [x_i, x_{i+1}] \quad i = 1,2,\dots,n$$

.

(ii)

$$\varphi_i(x) \in C^{\infty}[x_i, x_{i+1}].$$

(3)

First, we developd expressions for the four coefficients a_i, b_i, c_i, d_i of (2) in terms of $y_i, y_{i+1}, \psi_i, \psi_{i+1}$, where

$$(i)\varphi_i(x_i) = y_i \qquad (ii)\varphi_i(x_{i+1}) = y_{i+1}$$
$$(i)\varphi_i''(x_i) = \psi_i \qquad (ii)\varphi_i''(x_{i+1}) = \psi_{i+1}$$

(4)

From Eq. (2) and (4), we obtained via a straight forward calculation the following expression:

$$a_i = y_i - \frac{h^2}{\theta^2}\psi_i \; , \; b_i = \frac{1}{h}(y_{i+1} - y_i) + \frac{h}{\theta^2}(\psi_i - \psi_{i+1}) \; ,$$
$$d_i = \frac{h^2}{\theta^2}\psi_i \quad and \quad c_i = \frac{h^2}{\theta^2 \sinh\theta}(\psi_{i+1} - \psi_i \cosh\theta)$$
$$where \quad i = 0,1,2,\dots,n-1 \quad and \quad \theta = kh. \quad (5)$$

Now using the continuity of the first derivatives at the point (x_i, y_i) .that is $\varphi'_{i-1}(x_i) = \varphi'_i(x_i)$ yield the following relations:

$$y_{i-1} - 2y_i + y_{i+1} = h^2(\xi\psi_{i-1} + 2\eta\psi_i + \xi\psi_{i+1}) \quad (6)$$

where

$$\psi_i = -r(x_i)y' - s(x_i)y_i + g_i(x_i),$$
$$\xi = \left(\frac{h}{\theta^2} - \frac{h}{\theta\sinh\theta}\right) \; and \; \eta = \left(\frac{-2}{\theta^2} + \frac{2\cosh\theta}{\theta\sinh\theta}\right) \; .$$

The truncation errors, t_i , $i = 1,2,\dots,n$ associated with the scheme (6) can be obtained as follows: first we re-write the scheme (6) in the form

$$y_{i-1} - 2y_i + y_{i+1} = h^2[\xi y''_{i-1} + 2\eta y''_i + \xi y''_{i+1}] + t_i$$
$$for\; i = 1,2,3,\dots,n \quad (7)$$

The terms y_{i-1}, y_i, y_{i+1}. in Eq.(6)are expanded around the point x_i using the Taylor series and the expansion for t_i i=1,2,3,….n, can be obtained.

$$t_i = \left\{ \begin{array}{l} (1-(2\xi+2\eta)h^2 y_i'' + \frac{h^4}{12}(1-12\xi)y_i^{(4)} + \\ \frac{24}{360}(1-30\xi)h^6 y_i^{(6)} + O(h^8) \\ i = 1,2,3,\dots,n. \end{array} \right\} \quad (8)$$

The scheme (6) gives rise to family of method of different order as follows:
1-second order methods

$$As \quad k \to 0 \quad , \xi = \frac{1}{6} \; and \; \eta = \frac{1}{3}$$ .Then

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local truncation error is

$$t_i = -\frac{1}{12} h^4 y_i^{(4)} + O(h^6) \qquad i = 1, 2, \dots, n \qquad (9)$$

2-Forth order methods

For $\xi = \frac{1}{12}$ *and* $\eta = \frac{5}{12}$ .Then the truncation errors in Eq.(6) is

$$t_i = -\frac{h^6}{240} h^6 y_i^{(6)} + O(h^8) \qquad i = 1, 2, \dots n$$

(10)

3 .Analysis of the method with B.Cs.(A)

To illustrate the application of the spline method developed in the previous section we considered the linear boundary value problem that is given in Eq. (1)at grid point (x_i, y_i) . we choose an integer $n > 0$ and divide the interval $[a,b]$ in n equal subintervals .Where the step size $h = \frac{b-a}{n+1}$,also approximate forth derivative in Eq.(1) by using non-polynomial spline by substituting $\varphi_i = y_i''$ in Eq.(1) we get the following Eqs.

$$\psi_{i-1} = -r(x_{i-1}) y'_{i-1} - s(x_{i-1}) y_{i-1} + g(x_{i-1}) \qquad (11)$$

$$\psi_i = -r(x_i) y'_i - s(x_i) y_i + g(x_i) \qquad (12)$$

$$\psi_{i+1} = -r(x_{i+1}) y'_{i+1} - s(x_{i+1}) y_{i+1} + g(x_{i+1}) \qquad (13)$$

The first derivative approximate by using finite difference as follow[4]:

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h}, \quad y'_{i-1} = \frac{-3y_{i-1} + 4y_i - y_{i+1}}{2h}$$

$$y'_{i+1} = \frac{y_{i-1} - 4y_i + 3y_{i+1}}{2h} \qquad (14)$$

So Eqs.(11)-(13) and using (14) are become in the form:

$$\psi_{i-1} = \frac{-r(x_{i-1})(-3y_{i-1} + 4y_i - y_{i+1})}{2h} - s(x_{i-1}) y_{i-1} + g(x_{i-1}).$$

$$\psi_i = \frac{-r(x_i)(y_{i+1} - y_{i-1})}{2h} - s(x_i) y_i + g(x_i).$$

$$\psi_{i+1} = \frac{-r(x_{i+1})(y_{i-1} - 4y_i + 3y_{i+1})}{2h} - s(x_{i+1}) y_{i+1} + g(x_{i+1}). \qquad (15)$$

Now the substituting of the Eq. (15) in (5) we get the following n linear algebraic Eqs. with n unknown

$$\left[-1 + \xi h\left(\frac{3r(x_{i-1})}{2h} - \frac{r(x_{i+1})}{2} - h^2 s(x_{i-1})\right) + \eta h r(x_i)\right] y_{i-1}$$

$$+\left[2 - 2\xi h(r(x_{i-1}) - r(x_{i+1})) - 2\eta h^2 s(x_i)\right] y_i$$

$$\left[-1 + \xi h\left(\frac{r(x_{i-1})}{h} - \frac{3r(x_{i+1})}{2}\right) - h^2 s(x_{i+1})) - \eta h r(x_i)\right] y_{i+1}$$

$$= -h^2 \xi (r(x_{i-1}) + r(x_{i+1})) - 2h^2 \eta r(x_i) \qquad for\ i = 1, 2, \dots, n$$

(16)

4. Derive the method with B.Cs-(B)

Proceeding like in section(2),but we needed to **derive special formula when** $i = 1, n$ **using the boundary conditions** , finite difference (14) and the step size $h = \frac{b-a}{n+1}$ as following:

$$\left[-\frac{2}{3} + \xi h^2\left(\frac{-4r(x_2)}{3h} + \frac{4s(x_0)}{3}\right) + 2\eta h^2\left(\frac{2r(x_1)}{3} + s(x_1)\right)\right] y_1$$

$$+\left[\frac{2}{3} + \xi h^2\left(\frac{4r(x_2)}{3h} - \frac{s(x_0)}{3} + s(x_2)\right) + 4\eta h^2 \frac{r(x_1)}{3}\right] y_2$$

$$= \left[\frac{2h}{3} + \xi h^2\left(\frac{r(x_2)}{3} - r(x_0) + \frac{2hs(x_0)}{3}\right) - \eta h^2 \frac{r(x_1)}{3}\right] \alpha_3$$

$$+h^2 \xi (r(x_0) + r(x_2)) + 2h^2 \eta r(x_1) \qquad for\ i = 1$$

(17)

$$\left[\frac{2}{3}+\xi h^2\left(\frac{-4r(x_{n-1})}{3h}-\frac{s(x_0)}{3}+s(x_{n-1})\right)-2\eta h^2\frac{r(x_n)}{3}\right]y_{n-1}$$
$$+\left[\frac{-2}{3}+\xi h^2\left(\frac{4r(x_{n-1})}{3h}-\frac{4s(x_{n+1})}{3}\right)+2\eta h^2\left(\frac{2r(x_n)}{3h}+s(x_n)\right)\right]y_n$$
$$=\left[\frac{-2h}{3}+\xi h^2\left(\frac{r(x_{n-1})}{3}-r(x_{n+1})-\frac{2hs(x_{n+1})}{3}\right)-\eta h^2\frac{r(x_n)}{3}\right]\alpha_4$$
$$+h^2\xi(r(x_{n-1})+r(x_{n+1}))+2h^2\eta r(x_n) \qquad for\ i=n.$$
(18)

5. Non-polynomial spline solutions

The scheme (16) gives rise to a linear system of order $(n\times n)$ and may be written in the matrix form as

$$Ay+h^2DG=Q$$
(19)

Where $A=C+hBr-h^2Bs$,

$$,C=\begin{bmatrix}2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & & \ddots & \\ & & & -1 & 2\end{bmatrix},$$

and $Br=Z_{ij}$, $Bs=U_{ij}$ are define as

$$M_{ij}=\begin{cases}-2\xi(r(x_0)-r(x_2)) & i=1,\ j=1,\\ \xi\left(\frac{3r(x_{i-1})}{2}-\frac{r(x_{i+1})}{2}\right)-h\eta r(x_i) & i>j,\\ -2\xi(r(x_{i-1})-r(x_{i+1})) & i=j,\\ \xi\left(\frac{r(x_{i-1})}{2}-\frac{r(x_{i+1})}{2}\right)-h\eta r(x_i) & i<j,\\ -2\xi(r(x_{n-2})-r(x_n)) & i=j=n-1,\end{cases}$$

$$N_{ij}=\begin{cases}2\eta s(x_i), & i=j=1,2,\dots,n-1\\ \xi s(x_{i-1}), & i>j,\\ \xi s(x_{i+1}), & i<j\end{cases}$$

$$G=(g(x_1),g(x_2),\dots,g(x_n))^T,$$
$$Y=(y_1y_2,\dots,y_n)^T$$

$$D=\begin{bmatrix}2\eta & \xi & & & & \\ \xi & 2\eta & \xi & & & \\ & \xi & 2\eta & \xi & & \\ & & & \ddots & & \\ & & & & \xi & 2\eta\end{bmatrix}$$

and $Q=(q_1,q_2,\dots,q_n)^T$

where

$$q_1=-h^2\xi r(x_0)-(-1+h\xi(\frac{3r(x_0)}{2}+\frac{r(x_0)}{2})-hs(x_0))+h\eta r(x_1))y_0$$

$$q_i=0\quad ,i=2,3,\dots,n-2.$$

$$q_{n-1}=-h^2\xi r(x_n)-(-1+h\xi(\frac{r(x_{n-2})}{2}-\frac{3r(x_n)}{2})-hs(x_{n-2}))+h\eta r(x_{n-1}))y_n$$

we assume that

$$\bar{Y}=(y(x_1),y(x_2),\dots,y(x_{n-1}))^T.$$

Be the exact solution of the given boundary value problem (1) at nodal point x_i For $i=1,2,\dots,n$ .then we have

$$A\bar{Y}+h^2DG=T(h)+Q, \qquad (20)$$
$$A(\bar{Y}-Y)=AE=T(h)$$
(21)

6. Convergence analysis

In this section we discuss the convergence property of (6),we now turn back to the error eq. in (18) and rewrite it in the form:

$$E=A^{-1}T=[C+hBr-h^2Bs]^{-1}T=[I+C^{-1}(hBr-h^2Bs)]^{-1}C^{-1}T$$
$$\|E\|_\infty\le\|[I+C^{-1}(hBr-h^2Bs)\|_\infty\|C^{-1}\|_\infty\|T\|_\infty$$
(22)

In order to derive the bound on $\|E\|_\infty$,the following two lemmas are needed.

Lemma(1) :The matrix ($C + hBr - h^2Bs$) is nonsingular if

$$\|r\|_\infty < \frac{8h\varepsilon}{(a-b)^2(8\xi+2\eta)} \ and \ \|s\|_\infty < \frac{8(1-\varepsilon)}{(a-b)^2}$$

$$where \ 0<\varepsilon<1.$$

Proof:

Since ,

$$A = C + hBr - h^2Bs = [I + C^{-1}(hBr - h^2Bs)]C$$

and the matrix N is nonsingular ,so to prove A nonsingular it's sufficient to show

$[I + C^{-1}(hBr - h^2Bs)]$nonsingular.

Since

$$\|C^{-1}(hBr - h^2Bs)\|_\infty \le \|C^{-1}\|_\infty \|hBr - h^2Bs\|_\infty \le \|C^{-1}\|_\infty (\|hBr\|_\infty + \|h^2Bs\|_\infty) \quad (23)$$

Moreover,

$$\|C^{-1}\|_\infty \le \frac{(a-b)^2}{8h^2} \quad [4, 10]$$

$$\|hBr\|_\infty \le h(8\xi+2\eta)\|r\|_\infty, \quad and \quad \|h^2Bs\|_\infty \le h^2\|s\|_\infty$$

Where

$$\|r\|_\infty = \max_{a\le x_i \le b}|r(x_i)|, \quad and \quad \|s\|_\infty = \max_{a\le x_i \le b}|s(x_i)|$$

substituting $\|hBr\|_\infty, \|C\|_\infty, \ and \ \|hBs\|_\infty$ in Eq. (23) we get,

$$\|C^{-1}(hBr - h^2Bs)\|_\infty \le \frac{(b-a)^2}{8h}(8\xi+2\eta)\|r\|_\infty + \frac{(b-a)^2}{8}\|s\|_\infty$$

since

$$\|r\|_\infty < \frac{8h\varepsilon}{(b-a)^2(8\xi+2\eta)} \quad and \quad \|s\|_\infty < \frac{8(1-\varepsilon)}{(b-a)^2} \quad (24)$$

Eq. (23) lead to $\|C^{-1}(hBr - h^2Bs)\|_\infty \le 1.$

from lemma (1), show that the matrix A is nonsingular.

Since $\|C^{-1}(hBr - h^2Bs)\|_\infty \le 1.$so using lemma (1) and Eq. (22) we obtine

$$\|E\|_\infty \le \frac{\|C^{-1}\|_\infty \|T\|_\infty}{1 - \|C^{-1}\|_\infty \|(hBr - h^2Bs)\|}$$

From Eq. (6) we have:

$$\|T_i\|_\infty = \frac{h^4}{12}F_4, F_4 = \max_{a\le x\le b}|y^{(4)}(x_i)|.$$

then

$$\|E\|_\infty \le \frac{\|C^{-1}\|_\infty \|T\|_\infty}{1 - \|C^{-1}\|_\infty \|(hBr - h^2Bs)\|} \cong O(h^2) \quad (25)$$

Also from eq. (9) we have:

$$\|T_i\|_\infty = \frac{h^6}{240}F_6, F_6 = \max_{a\le x\le b}|y^{(6)}(x_i)|.$$

then

$$\|E\|_\infty \le \frac{\|C^{-1}\|_\infty \|T\|_\infty}{1 - \|C^{-1}\|_\infty \|(hBr - h^2Bs)\|} \cong O(h^4) \quad (26)$$

Theorem (1) [3]:

Let y(x) is the exact solution of the continuous boundary value problem (1) with the B.Cs-(A) and (B) and let $y(x_i), i = 1,2,\dots,n-1$,satisfies the discrete boundary value problem (18) in further ,if $e_i = y(x_i) - y_i$ then

1- $\|E\|_\infty \cong O(h^2)$ for second order convergent method.

2- $\|E\|_\infty \cong O(h^4)$ for forth order convergent method.

Whose $\|E\|_\infty$ given by(25)and(26),respectively neglecting all errors due to round off.

7. Numerical examples

We now consider four numerical examples to illustrate the comparative performance of non-polynomial spline finite difference method in scheme (6) .All calculation are implemented by maple 18. In example (1) andexample (2),we the scheme (6) is being applied to solve this problem for n=16 and compared with exact solution in tables (1) and(2)respectively ,moreover for n=8,16,32,64 and 128**.**We are compute solutions at grid point, the observed maximum absolute errors $L_\infty = |y_i - y(x_i)|$ where y_i is the numerical solution and $y(x_i)$ is exact solution all are tabulated in tables (3) , (4) respectively, In table (5) our results with the results given in [1]are compared .

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We deduce that our result are more accurate, The figures (1),(2) and(3) illustrate the comparison of the real solution with numerical solution.

In example (3) and example (4) ,We applied the scheme (6)with Eq.(17)and Eq. (18) to solve this problem for different values n and compared with exact solution in tables (6) and (7) ,moreover n=8,16,32,64 and 128,the observed maximumabsolute errors are tabulated in tables (8) and (9) ,the figures (3) and (4) illustrate the comparison of the real solution with numerical solution.

Example 1 [1]:Consider the boundary value problem

$$y''-(x+1)y'-2y=(1-x^2)e^{-x} \qquad 0\le x\le 1$$

$$y(0)=-1, y(1)=0.$$

With exact solution

$$y(x)=(x-1)e^{-x}$$

Example 2 [1]:Consider the boundary value problem

$$y''-y'=-e^{x-1}-1 \qquad 0\le x\le 1$$

$$y(0)=0, y(1)=0.$$

With exact solution $y(x)=x(1-e^{x-1})$.

Example 3 [11]:Consider the boundary value problem

$$y''+y=-1 \qquad 0\le x\le 1$$

$$y'(0)=\frac{1-\cos(1)}{\sin(1)}=-y'(1).$$

With exact solution

$$y(x)=\cos x\,\frac{1-\cos(1)}{\sin(1)}\sin x-1.$$

Example 4 [9]:Consider the boundary value problem

$$-y''=(2-4x^2)y \qquad 0\le x\le 1$$

$$y'(0)=0, y'(1)=\frac{-2}{e^1}.$$

With exact solution $y(x)=e^{-x^2}$.

Table 1: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (1).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	-0.880669135	-0.880668474	-0.880699746
0.1250	-0.772131021	-0.772128861	-0.77218479
0.1875	-0.673515692	-0.673511558	-0.673586159
0.2500	-0.584018988	-0.584012695	-0.584100587
0.3125	-0.50289777	-0.502889373	-0.502985745
0.3750	-0.429465486	-0.429455222	-0.429555799
0.4375	-0.363088043	-0.363076287	-0.363177296
0.5000	-0.303179969	-0.303167199	-0.30326533
0.5625	-0.249200851	-0.249187614	-0.249279986
0.6250	-0.200652029	-0.200638912	-0.200723036
0.6875	-0.157073513	-0.157061126	-0.157134868
0.7500	-0.118041136	-0.118030087	-0.118091638
0.8125	-0.083163896	-0.083154783	-0.083202621
0.8750	-0.052081495	-0.052074892	-0.052107752
0.9375	-0.024462057	-0.024458503	-0.024475352

Table 2: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (2).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	0.038043273	0.038039019	0.038024648
0.1250	0.072928609	0.072920342	0.072892248
0.1875	0.104350364	0.104338372	0.104297379
0.2500	0.131976602	0.131961227	0.131908362
0.3125	0.155449676	0.155428619	0.155365132
0.3750	0.174370442	0.174349572	0.174276964
0.4375	0.188322801	0.188299959	0.188220014
0.5000	0.196844046	0.196819852	0.19673467
0.5625	0.199435511	0.199410674	0.199322704
0.6250	19555679100	0.195532119	0.195444201
0.6875	0.184622441	0.184598846	0.184514255
0.7500	0.165998192	0.165976919	0.165899413
0.8125	0.138998192	0.13897997	0.138913842
0.8750	0.102878735	0.102865077	0.10281521
0.9375	0.056835959	0.056828319	0.056800254

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Table 3: The maximum absolute errors ofnon-polynomial spline finite difference method for example (1).

n	max norm(L)	
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	3.63E-04	4.04E-04
16	9.03129E-05	1.01E-04
32	2.25E-05	2.53E-05
64	5.57478E-06	6.26843E-06
128	9.87E-07	1.2426E-06

Table 4:The maximum absolute errors of non-polynomial spline finite difference method for example (2).

n	max norm(L)	
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	0.00045128	0.000352401
16	1.12E-04	8.79708E-05
32	2.83E-05	2.20701E-05
64	7.06974E-06	5.51684E-06
128	1.76738E-06	1.38E-06

Table 5:Comparison the maximum absolute errors ofnon-polynomial spline finite difference method for example (1) and example (2) for (n=32) with the maximum absolute errors of B-spline method[1].

	maximum error		
	non-polynomial spline finite difference method		
	ξ=1/6 , η=1/3	ξ=1/12 , η=5/12	B-spline method [1]
example 1	2.25E-05	2.53E-05	5.70E-04
example 2	2.83E-05	2.27E-05	6.88E-04

Table 6: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (3).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	0.031299775	0.030943688	0.032169192
0.1250	0.05940484	0.059048874	0.060307784
0.1875	0.083374298	0.083018496	0.084305898
0.2500	0.103114577	0.102758954	0.104069821
0.3125	0.118548619	0.118193162	0.119522374
0.3750	0.129616173	0.129260849	0.130603217
0.4375	0.136274035	0.159187958	0.137269078
0.5000	0.138496213	0.138141004	0.139493927
0.5625	0.136274035	0.135918796	0.137269078
0.6250	0.129616173	0.129260849	0.130603217
0.6875	0.118548619	0.118193162	0.119522374
0.7500	0.103114577	0.102758954	0.104069821
0.8125	0.083372975	0.083018496	0.084305898
0.8750	0.05940484	0.059048874	0.060307785
0.9375	0.031299775	0.030943688	0.032169192

Table 7: Comparison numerical solution of non-polynomial spline finite difference method for (n=16) with exact solution of example (4).

x	numerical solution		exact solution
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)	
0.0625	0.994624959	0.993898323	0.99610137
0.1250	0.983104909	0.982364394	0.984496437
0.1875	0.964172404	0.963409595	0.965454552
0.2500	0.938262193	0.937469997	0.939413063
0.3125	0.905959649	0.905132689	0.906960618
0.3750	0.867978854	0.867113733	0.868815056
0.4375	0.825136448	0.824231864	0.82579704
0.5000	0.778322587	0.777379302	0.778800783
0.5625	0.728470452	0.72749111	0.72876333
0.6250	0.676525748	0.675514562	0.676633846
0.6875	0.623417583	0.622379908	0.623344309
0.7500	0.570031932	0.568973763	0.569782825
0.8125	0.517188709	0.51611612	0.516770583
0.8750	0.465623173	0.464541742	0.465043188
0.9375	0.415972143	0.414886376	0.415236829

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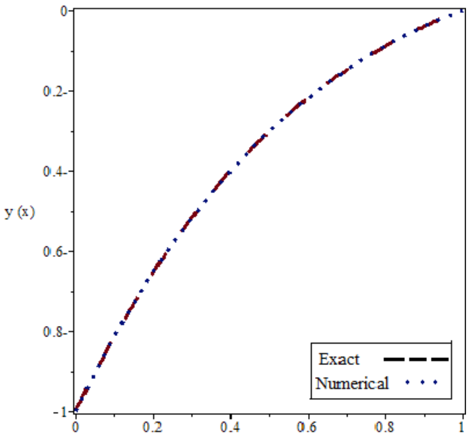
Table 8: The maximum absolute errors of non-polynomial spline finite difference method for example (3).

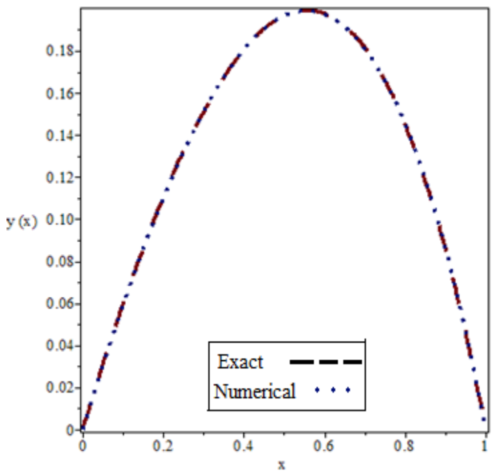
N	Maximum error	
	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	3.45E-03	4.87E-03
16	9.97E-04	1.35E-03
32	2.66E-04	3.55E-04
64	6.82946E-05	9.08673E-05
128	1.86E-05	2.27E-05

Table 9: The maximum absolute errors of non-polynomial spline finite difference method for example (4).

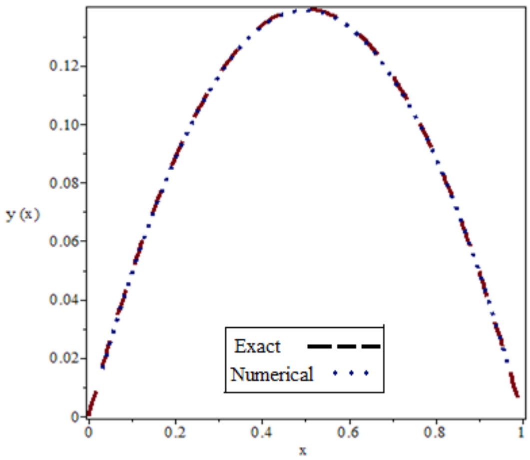
n	Second order method(ξ=1/6 , η=1/3)	Forth order method (ξ=1/12 , η=5/12)
8	7.15E-03	7.15E-03
16	1.48E-03	2.20E-03
32	4.20E-04	6.03E-04
64	1.11E-04	3.93E-05
128	2.68E-05	3.93377E-05



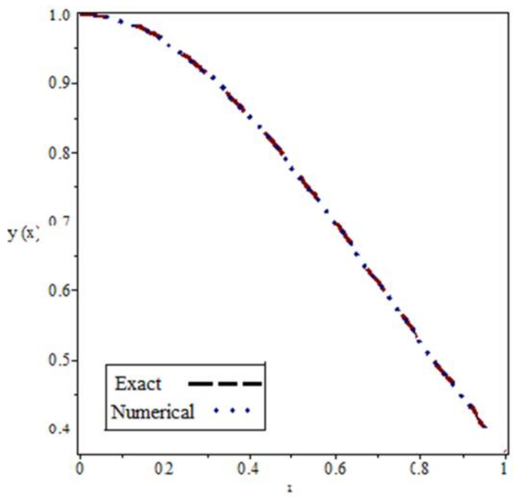
Figure(1): Comparison the exact and numerical solution (h=1/32) for



Figure(2): Gomparison the exact and numerical solution (h=1/32) for example(2).



Figure(3): Comparison the exact and numerical solution (h=1/32) for example (3).



Figure(4): Comparison the exact and numerical solution (h=1/32) for example (4) .

8. Conclusion

In this paper, the non-polynomial spline finite difference method was used to solve the general linear of the second boundary value problem, and show that this method is better in terms of accuracy and application, these have been verified by maximum absolute errors.

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طريقة الغير متعددات الحدود السبلاين مع االفروقات المحددة لحل مسائل قيم حدودية من الرتبة الثانية

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المستخلص :
تم حل مسائل قيم حدودية (BVPs) من الرتبة الثانية بطريقة غير متعددة الحدود السبلاين مع طريقة الفروقات المحددة. في هذا البحث نناقش طريقة تعتمد على استخدام التقريب الخاص للفروقات المحددة للمشتقات وتنسيق صيغة يمكن أن تتعامل مع نقاط النهاية التي تتجاوز صيغة الفروقات المحددة للمشتقات.تتم مناقشة تحليل التقارب لهذه الصيغة. يظهر الوصف العددي للطريقة بأربعة أمثلة. وبالتالي فإن النتائج التي تم الحصول عليها من خلال أسلوبنا مشجعة للغاية بالمقارنة مع الطرق الأخرى .

الكلمات المفتاحية: طريقة الغير متعددة الحدود السبلاين ،الفروقات المحددة، مسائل القيم الحدودية، خطأ القطع، الحل الحقيقي.

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On the third natural representation module *M* (n-3,3) of the permutation groups

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Abstract:

The main purpose of this work is to propose the third natural representation M (n-3,3)of the symmetric groups over a field **F** and prove that M (n-3,3)is split iff p does not divide $\frac{n(n-1)(n-2)}{6}$.

Keywords: symmetric group, group algebra FS_n , FS_n −module, Spechet module, exact sequence.

Mathematics Classification :20C30.

1. Introduction

In 1935, W .Specht introduced tableau correspondence polynomials ,known Specht polynomials, that proved how a given polynomial can be written as a linear combination of other polynomials.(see[Kerber:2004]).This was the results of Specht study on representation theory of symmetric groups, after he faced the problem when the symmetric group acts, in natural way, as a tableaux. However, the result of permutation a standard tableau can be a nonstandard tableau and this nonstandard tableau can be written as a linear combination of Specht polynomials. On the other hand, the representation with partition $\mu = (n-1,1)$ for a positive integer n, was first studied by H.K.Farahat in 1962 [Farahat:1962]. This type of representation is called the natural representation. Seven years later, M.H.Peel introduced in [Peel:1969] and [Peel:1971] the second representation of the symmetric groups and renamed Farahat natural representation by the first natural representation .In Peel's representation, the partition was then $\mu = (n-2,2)$ for a positive integer n. He also represented the r^{th}-Hook representation where the partition $\mu = (n-r, 1^r)$, for any $r \geq 1$.

For the author's knowledge, no one has studied the 3rd-natural representation so far . Therefore, this work represent of the symmetric groups over a field **F** and $x_1, x_2, \dots, x_n$ defined to be linearly independent commuting variables over **F**.

2. Preminaries

Definition 1: Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite set, then the symmetric group on X is the group whose elements "permutations" can be viewed as a bijective function from $\mathbf{F}[x_1, x_2, \dots, x_n]$ onto $\mathbf{F}[x_1, x_2, \dots, x_n]$.The symmetric group on X is denoted by $\mathbf{S_X}$ or $\mathbf{S_n}$. Then $\mathbf{FS_n}$ is called the group algebra of the symmetric group $\mathbf{S_n}$ with respect to addition of functions, composition of functions and product of functions by scalars [Joyce:2008].

Definition 2: Let n be a natural number then the sequence $\mu = (\mu_1, \mu_2, \dots, \mu_l)$ is called a partition of n if $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_l > 0$ and $\mu_1 + \mu_2 + \cdots + \mu_l = n$, the set $D_\mu = \{(i,j) | i = 1,2, \dots, l; 1 \leq j \leq \mu_i\}$ is called $\mu-$diagram and any bijective function $t : D_\mu \to \{x_1, x_2, \dots, x_n\}$ is called a μ–tableau. A μ–tableau may be thought as an array consisting of l rows and μ_1 columns of distinct variables $t((i,j))$ where the variables appear in the first μ_i positions of the i^{th} row and each variable $t((i,j))$ appears in the i^{th} row and the j^{th} column ((i,j)-position) of the array.$t((i,j))$ will be denoted by $t(i,j)$ for each $(i,j) \in D_\mu$. The set of all μ-tableaux will be denoted by T_μ. i.e $T_\mu = \{t | t \text{ is a } \mu - tableau\}$. Then the function $h: T_\mu \to F[x_1, x_2, \dots, x_n]$ which is defined by $h(t) = \prod_{i=1}^{l} \prod_{j=1}^{\mu_i} (t(i,j))^{i-1}$,$\forall\, t \in T_\mu$ is called the row position monomial function of T_μ, and for each μ–tableau t, $h(t)$ is called the row position monomial of t .So $M(\mu)$ is the cyclic FS_n –module generated by $h(t)$ over FS_n.[Ellers:2007]

3.The Third Natural Representation of S_n

In the beginning, we determine some denotations which we need them in this paper.

1. Let $\sigma_1(n) = \sum_{i=1}^{n} x_i$.

2. Let $\sigma_2(n) = \sum_{1 \leq i \langle j \leq n} x_i x_j$.

3. Let $\sigma_3(n) = \sum_{1 \leq i < j < k \leq n} x_i x_j x_k$.

4. Let $C_l(n) = x_l\,(\sigma_2(n) - \sum_{\substack{j=1 \\ j \neq l}}^{n} x_l\, x_j); l = 1,2, \dots, n$. Then $\sum_{i=1}^{n} c_{i_i}(n) = \sigma_3(n)$ and $dim_F(F\sigma_1(n))$ =$dim_F(F\sigma_2(n))$= $dim_F(F\sigma_3(n))$ = 1. $F\sigma_1(n), F\sigma_2(n)$ and $F\sigma_3(n)$ are all FS_n-modules, since $\tau\sigma_k(n) = \sigma_k(n)\ \forall\, k = 1,2,3$.

5. Let $u_{ij}(n) = C_i(n) - C_j(n)$; $i, j = 1,2, \dots, n$.

We denote $\bar{V}$ to be the FS_n-modules generated by $C_1(n)$ over FS_n and $\bar{V}_0$ to be the FS_n-submodule of $\bar{V}$ generated by $u_{12}(n)$ over S_n .

Definition3.1: The FS_n-module $M(n - r, r)$ defined by

$$M(n - r, r) = FS_n x_1 x_2 \dots x_r \quad ; \quad n \geq r$$

is called r^th^- natural representation module of S_n over F.

Lemma3.2: The set $B(n - 3,3) = \{x_i x_j x_l : 1 \leq i < j < l \leq n\}$is a F-basis of $M(n - 3,3)$ and $dim_F M(n - 3,3) = \binom{n}{3}; n \geq 3$.

Proof: Clear

Theorem3.3: The set $B_0(n - 3,3) = \{x_i x_j x_l - x_1 x_2 x_3 : 1 \leq i < j < l \leq n, (i,j,l) \neq (1,2,3)\}$ is a F-basis of $M_0(n - 3,3)$and $dim_F M_0(n - 3,3) = \binom{n}{3} - 1\,; n \geq 3$.

Proof: Since $M_0(n - 3,3) = \{ \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_i x_j x_l$: $\sum_{1 \leq i < j < l \leq n} k_{ijl} = 0$ and $k_{ijl} \in F\}$, we get that $B_0(n - 3,3) \subset M_0(n - 3,3)$.To prove $B_0(n - 3,3)$ generates $M_0(n - 3,3)$ over F .

Let $x \in M_0(n - 3,3)$.$\Rightarrow x = \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_i x_j x_l$; $\sum_{1 \leq i < j < l \leq n} k_{ijl} = 0$

$$\Rightarrow x = \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_i x_j x_l - 0.\, x_1 x_2 x_3$$

$$\Rightarrow x = \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_i x_j x_l - (\sum_{1 \leq i < j < l \leq n} k_{ijl})x_1 x_2 x_3$$

$$\Rightarrow x = \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_i x_j x_l - \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_1 x_2 x_3$$

$$\Rightarrow x = \sum_{1 \leq i < j < l \leq n} k_{ijl}\, (x_i x_j x_l - x_1 x_2 x_3)$$

with the term 123 excluded from the summation since $k_{ijl}(x_1 x_2 x_3 - x_1 x_2 x_3) = 0$.Hence $B_0(n - 3,3)$ generates $M_0(n - 3,3)$ over F .Moreover $B_0(n - 3,3)$ is linearly independent since if

$$\sum_{1 \leq i < j < l \leq n} k_{ijl}\, (x_i x_j x_l - x_1 x_2 x_3) = 0 \quad \Rightarrow$$

$$\sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_i x_j x_l - \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_1 x_2 x_3 = 0$$

$\Rightarrow \sum_{1 \leq i < j < l \leq n} k_{ijl}\, x_i x_j x_l = 0$,where $k_{123} = -\sum_{1 \leq i < j < l \leq n} k_{ijl}$ with $\sum_{1 \leq i < j < l \leq n} k_{ijl} = 0$ and $(i,j,l) \neq (1,2,3)$. By lemma (3.2) we have $B(n - 3,3)$ is linearly independent. Thus we get $k_{ijl} = 0\ \forall i,j,l\,; 1 \leq i < j < l \leq n$. Hence $B_0(n - 3,3)$ is a F-basis of $M_0(n - 3,3)$and $dim_F M_0(n - 3,3) = \binom{n}{3} - 1\,; n \geq 3$.■

Theorem3.4: The set $B = \{C_i(n) | i = 1,2, \ldots, n\}$ is a F-basis for $\bar{V}(n) = FS_n C_1(n)$.

Proof: Let $\tau_i = (x_1 x_i) \in S_n; 1 < i \leq n$. Then $\tau_i(C_1(n)) = C_i(n); i = 1,2, \ldots, n$.

Thus $C_i(n) \in \bar{V}(n); i = 1,2, \ldots, n$ .Hence $B \subset \bar{V}(n)$.

Now if $w \in \bar{V}(n). \Longrightarrow w = \sum_{i=1}^{(n-1)!} \sum_{j=1}^{n} k_{ij} \tau_{ij} C_1(n)$ where $\tau_{ij} \in S_n, k_{ij} \in F$ and $\tau_{ij}(x_1) = x_j$,which implies that $\tau_{ij}(C_1(n)) = C_j(n)\,; j = 1,2, \ldots, n$.

$\Longrightarrow w = \sum_{i=1}^{(n-1)!} \sum_{j=1}^{n} k_{ij} \tau_{ij} C_1(n)$

$= \sum_{j=1}^{n} (\sum_{i=1}^{(n-1)!} k_{ij}) C_j(n) = \sum_{j=1}^{n} d_j C_j(n)$

,where $d_j = \sum_{i=1}^{(n-1)!} k_{ij}$ Hence B generates $\bar{V}(n) = FS_n C_1(n)$ over F.

If $\sum_{i=1}^{n} k_i C_i(n) = 0. \Rightarrow k_1 C_1(n) + k_2 C_2(n) + \cdots + k_n C_n(n) = 0$.

$\Longrightarrow k_1 + k_2 + \cdots + k_n = 0$ since $C_l(n) = \sum_{1 \leq i < j < l \leq n} x_i x_j x_l$.Thus B is a linearly independent. Therefore B is a basis of $\bar{V}(n)$ and $dim_F \bar{V}(n) = n$.

■

Theorem3.5: $\bar{V}(n) = FS_n C_1(n)$ and $M(n-1,1)$ are isomorphic over FS_n.

Proof: Let $\varphi : M(n-1,1) \to \bar{V}(n)$ be defined as follows:

$\varphi(x_i) = C_i(n); \; i = 1,2, \ldots, n$. Then for each $\tau = (x_i x_j) \in S_n$ such that $\tau(x_i) = x_j$ we get that $\varphi(\tau x_i) = \varphi(x_j) = C_j(n) = \tau C_i(n) = \tau\varphi(x_i)$ .Hence φ is a FS_n-homomorphism .Also $y = \sum_{i=1}^{n} k_i C_i(n)$ for any $y \in \bar{V}$.Thus for all $y \in \bar{V}, \; \exists \; w = \sum_{i=1}^{n} k_i x_i \in M(n-1,1)$ such that $\varphi(w) = \varphi(\sum_{i=1}^{n} k_i x_i)$

$= \sum_{i=1}^{n} \varphi(k_i x_i) = \sum_{i=1}^{n} k_i \varphi(x_i) = \sum_{i=1}^{n} k_i C_i(n) = y$. Hence φ is an epimorphism.

Thus $dim_F \, ker\varphi = dim_F M(n-1,1) - dim_F \bar{V} = n - n = 0 . \Longrightarrow ker\varphi = 0$. Then φ is a monomorphism .

Thus φ is a FS_n – isomorphism. Hence $M(n-1,1)$ and $\bar{V}$ are isomorphic over FS_n.

Theorem3.6: If p does not divides n , then $\bar{V}(n) = \bar{V}_0(n) \oplus F\sigma_3(n)$.

Proof : From Theorem (3.5) we have a FS_n– isomorphism

$\varphi : M(n-1,1) \to \bar{V}(n)$ such that $\varphi(x_i) = C_i(n)\,; \; i = 1,2, \ldots, n$.

And since $M_0(n-1,1) = FS_n(x_2 - x_1) \subset M(n-1,1)$, then $\psi = \varphi|_{M_0(n-1,1)}$ is a FS_n – isomorphism .Thus $\bar{V}_0(n)$ and $M_0(n-1,1)$ are isomorphic over FS_n which is irreducible submodule over FS_n when p does not divides n and $\sigma_3(n) \notin \bar{V}_0(n)$ when p does not divide n since the sum of the coefficients of the $C_i(n)$ in $\sigma_3(n)$ is n. Hence $\bar{V}_0(n) \cap F\sigma_3(n) = 0$, $F\sigma_3(n) \subset \bar{V}(n)$ and $\bar{V}_0(n) \subset \bar{V}(n)$ .But $dim_F \bar{V}_0(n) + dim_F F\sigma_3(n) = n - 1 + 1 = n = dim_F \bar{V}(n)$.

Hence $\bar{V}_0(n) \oplus F\sigma_3(n) = \bar{V}(n)$ when p does not divides n .

Proposition 3.7 : If p does not divides n ,then $\bar{V}$ has the following two composition series

$0 \subset \bar{V}_0(n) \subset \bar{V}(n)$ and $0 \subset F\sigma_3(n) \subset \bar{V}(n)$.

Proof : Since p does not divides n , then by Theorem (3.6) we have

$\bar{V} = \bar{V}_0(n) \oplus F\sigma_3$, and $\bar{V}_0(n)$ is irreducible submodule when p does not divide n .Hence $\frac{\bar{V}}{F\sigma_3(n)} = \frac{\bar{V}_0(n) \oplus F\sigma_3(n)}{F\sigma_3(n)} \simeq \bar{V}_0(n)$.Thus $\frac{\bar{V}}{F\sigma_3(n)}$ is irreducible module when p does not divide n. Since $dim_F F\sigma_3(n) = 1$. Then $F\sigma_3(n)$ is irreducible submodule over FS_n . But $\frac{\bar{V}}{\bar{V}_0(n)} = \frac{\bar{V}_0(n) \oplus F\sigma_3(n)}{\bar{V}_0(n)} \simeq F\sigma_3(n)$. Therefore $\frac{\bar{V}}{\bar{V}_0(n)}$ is irreducible module over FS_n . Thus we get the following two composition series

$0 \subset \bar{V}_0(n) \subset \bar{V}$ and $0 \subset F\sigma_3(n) \subset \bar{V}$.

Theorem 3.8: The following sequence

$$O \to M_0(n-3,3) \xrightarrow{i} M(n-3,3) \xrightarrow{f} F \to O \qquad \ldots (1)$$

over a field F is split iff p does not divide $\frac{n(n-1)(n-2)}{6}$.

Proof: If p does not divide $\frac{n(n-1)(n-2)}{6}$. For any $k \in F$ we have $f(\sum_{1 \leq i < j < l \leq n} k_{ijl} \, x_i x_j x_l) = \sum_{1 \leq i < j < l \leq n} k_{ijl} = k$ .Hence f is on to. Moreover

$kerf = \{ \sum_{1\le i<j<l\le n} k_{ijl}\, x_i x_j x_l :$

$f\ (\sum_{1\le i<j<l\le n} k_{ijl}\, x_i x_j x_l) = 0\,\}$ $=$

$\{ \sum_{1\le i<j<l\le n} k_{ijl}\, x_i x_j x_l \quad : \sum_{1\le i<j<l\le n} k_{ijl} = 0 \quad \}$ $=$

$M_0(n-3{,}3)= Im\, i$ .Hence the sequence (1) is an exact sequence.

So we can defined a function $h: F \to M(n-3{,}3)$ by $h(k) = \frac{6k\sigma_3(n)}{n(n-1)(n-2)}$ which is a FS_n – homomorphism since

$\sum_{\tau\in S_n} r\tau h(k) = \sum_{\tau\in S_n} r\tau\, (\frac{6k\sigma_3(n)}{n(n-1)(n-2)}) = \sum_{\tau\in S_n} \frac{6rk\tau\sigma_3(n)}{n(n-1)(n-2)} = \sum_{\tau\in S_n} \frac{6rk\sigma_3(n)}{n(n-1)(n-2)}$

$=\sum_{\tau\in S_n} rh(k) = h(\sum_{\tau\in S_n} rk) = h(\sum_{\tau\in S_n} r\tau(k))$ .And since

$fh(k) = f\left(\frac{6k\sigma_3(n)}{n(n-1)(n-2)}\right) = \frac{6k}{n(n-1)(n-2)} f(\sigma_3(n)) = \frac{6k}{n(n-1)(n-2)} f\left(\sum_{1\le i<j<l\le n} k_{ijl}\, x_i x_j x_l\right) = \frac{6k}{n(n-1)(n-2)} \cdot \frac{n(n-1)(n-2)}{6} = k$ .Hence $fh = I$ on F. Thus the sequence(1) is split.

Now assume the sequence (1) is split.

Then there exist a FS_n –homomorphism $f_1: F \to M(n-3{,}3)$ s.t. $ff_1 = I$ on F.

Let $f_1(1) = \sum_{1\le i<j<l\le n} k_{ijl}\, x_i x_j x_l$. Then $\tau f_1(1) = f_1(\tau(1)) = f_1(1)$,where $\tau = (x_r x_s) \in S_n$,$1 \le r < s \le n$.Thus $f_1(1) - \tau f_1(1) = 0$.

$\Longrightarrow 0 = \sum_{1\le i<j<l\le n} k_{ijl}\, x_i x_j x_l - \sum_{1\le i<j<l\le n} k_{ijl}\, \tau(x_i x_j x_l)$

$= \sum_{1\le i<j<l\le n} k_{ijl}\, (x_i x_j x_l - \tau(x_i x_j x_l))$

$= \sum_{\substack{r+1<j<l\le n\\ j,l\ne s}} k_{sjl}\, (x_r x_j x_l - x_s x_j x_l) + \sum_{i=1}^{r-1}\sum_{\substack{l=r+1\\ l\ne s}}^{n} k_{irl}\, (x_i x_r x_l - x_i x_s x_l) + \sum_{1\le i<j<r} k_{ijr}\, (x_i x_j x_r - x_i x_j x_s) =$

$\sum_{\substack{j=r+1\\ j<l\le n}}^{n-1} (k_{rjl} - k_{sjl})\, x_r x_j x_l + \sum_{i=1}^{r-1}\sum_{\substack{l=r+1\\ l\ne s}}^{n} (k_{irl} - k_{isl})\, x_i x_r x_l + \sum_{1\le i<j<r} (k_{ijr} - k_{isl})\, x_i x_j x_r$.

So by equaling the coefficient ,we get

$k_{rjl} = k_{sjl}\ \ \forall\, r < j < l \le n$,

$k_{irl} = k_{isl}\ \ \forall\, 1 \le i < r$ and $r < l \le n$, $l \ne s$

$k_{ijr} = k_{ijs}\ \ \forall\, 1 \le i < j < r$.

Hence for each r, s such that $1 \le r < s \le n$ we get

$k_{ijl} = k\, ; 1 \le i < j < l \le n$ .Then $f_1(1) = \sum_{1\le i<j<l\le n} k_{ijl}\, x_i x_j x_l = \sum_{1\le i<j<l\le n} k\, x_i x_j x_l = k\sigma_3(n)$.

$\because ff_1 = I$

$\therefore 1 = ff_1(1) = f(k\sigma_3(n)) = kf(\sigma_3(n)) = kf(\sum_{1\le i<j<k\le n} x_i x_j x_k) = k\frac{n(n-1)(n-2)}{6}$ .Hence p does not divide $\frac{n(n-1)(n-2)}{6}$.

Corollary3.9:M(n-3,3) is a direct sums of $M_0(n-3{,}3)$ and $F\sigma_3(n)$ when p does not divide $\frac{n(n-1)(n-2)}{6}$.

Proof: Since p does not divide $\frac{n(n-1)(n-2)}{6}$,then the sequence (1) is split .Thus M(n-3,3) decomposable into $kerf = M_0(n-3{,}3)$ and $Imf = F\sigma_3(n)$ since for each $k\sigma_3(n) \in F\sigma_3(n)$ we have $f\left(\frac{kn(n-1)(n-2)}{6}\right) = \frac{kn(n-1)(n-2)}{6}\left(\frac{6\sigma_3(n)}{n(n-1)(n-2)}\right) = k\sigma_3(n)$.

Hence $M(n-3{,}3) = M_0(n-3{,}3) \oplus F\sigma_3(n)$.

Theorem 3.10: The following sequence

$$0 \to ker\bar{d}_2 \xrightarrow{i} M_0(n-3{,}3) \xrightarrow{\bar{d}_2} M_0(n-2{,}2) \to 0 \qquad \ldots\ldots(2)$$

is split iff p does not divide neither (n-2) nor (n-3).

Proof: Since $\bar{d}_2\left(\frac{1}{2}(x_1x_ix_k - x_1x_2x_i + x_2x_ix_k - x_1x_2x_k)\right) = \frac{1}{2}(x_1x_i + x_1x_k + x_ix_k - x_1x_2 - x_1x_i - x_2x_i + x_2x_i + x_2x_k + x_ix_k - x_1x_2 - x_1x_k - x_2x_k) = \frac{1}{2}(2(x_ix_k - x_1x_2)) = x_ix_k - x_1x_2$ .Which is the generated of $M_0(n-3{,}3)$ .Hence $\bar{d}_2$ is on to map. Moreover $Imi = ker\bar{d}_2$ .Thus the sequence (2) is exact.

If p does not divide neither (n-2)nor(n-3).

Let $\phi: M_0(n-2{,}2) \to M_0(n-3{,}3)$ be defined as follows:

$\phi(x_ix_j - x_1x_2) = \frac{1}{(n-2)(n-3)} \sum_{2\langle i\langle j\le n} (x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j)$

Then for any $\in S_n$, $s.t.\ \tau(x_1) = x_1$, $\tau(x_2) = x_2$.

$\phi\left(\tau(x_ix_j - x_1x_2)\right) = \phi(\tau(x_i)\tau(x_j) - \tau(x_1)\tau(x_2)) = \frac{1}{(n-2)(n-3)} \sum_{2\langle i\langle j\le n} (x_1x_{i_1}x_{j_1} - x_1x_2x_{i_1} + x_2x_{i_1}x_{j_1} - x_1x_2x_{j_1})$Where $\tau(x_i) = x_{i_1}$ and $\tau(x_j) = x_{j_1}$.Then

$\phi\left(\tau(x_ix_j - x_1x_2)\right) = \frac{1}{(n-2)(n-3)} \sum_{2\langle i\langle j\le n} \tau\,(x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j) = \frac{1}{(n-2)(n-3)}\tau \sum_{2\langle i\langle j\le n} (x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j) = \tau\phi(x_ix_j - x_1x_2)$ .Hence ϕ is a FS_n −homomorphism .Moreover

$\bar{d}_2\phi(x_ix_j - x_1x_2) = \bar{d}_2\left(\frac{1}{(n-2)(n-3)} \sum_{2\langle i\langle j\le n} (x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j)\right) = \frac{1}{(n-2)(n-3)} \sum_{2\langle i\langle j\le n} \bar{d}_2\,(x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j) = \frac{1}{(n-2)(n-3)} \sum_{2\langle i\langle j\le n} 2(x_ix_j - x_1x_2) = \frac{1}{(n-2)(n-3)} \frac{(n-2)(n-3)}{2}\left(2(x_ix_j - x_1x_2)\right) = x_ix_j - x_1x_2$

So $\bar{d}_2\phi = I$ on $M_0(n-2,2)$. Hence the sequence (2) is split if p does not divide neither (n-2) nor (n-3).Thus $M_0(n-3,3) = ker\,\bar{d}_2 \oplus L_0$, where $L_0 = \phi(M_0(n-2,2))$.

Now assume if the sequence (2) is split.

Let $\phi: M_0(n-2,2) \to M_0(n-3,3)$ be a FS_n −homomorphism such that $\bar{d}_2\phi = I$.Thus we can define ϕ as follows $\phi(x_{i_1}x_{j_1} - x_1x_2) = \sum_{2\langle i\langle j\le n} k_{ij}\,(x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j)$.$\Rightarrow \bar{d}_2\phi(x_{i_1}x_{j_1} - x_1x_2) = \bar{d}_2\left(\sum_{2\langle i\langle j\le n} k_{ij}\,(x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j)\right) = \sum_{2\langle i\langle j\le n} \bar{d}_2(k_{ij}\,(x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j) = \sum_{2\langle i\langle j\le n} k_{ij}\,(2(x_ix_j - x_1x_2)$ $= 2(\sum_{2\langle i\langle j\le n} k_{ij}\,(x_ix_j - x_1x_2)) = x_{i_1}x_{j_1} - x_1x_2$.

$$\Rightarrow 2 \sum_{2\langle i\langle j\le n} k_{ij} = \begin{cases} 0 & if\ (i,j) \ne (i_1,j_1) \\ 1 & if\ (i,j) = (i_1,j_1) \end{cases}$$

Moreover if $\tau = (x_rx_s) \in S_n\,; 2 < r < s \le n$ such that $\tau(x_{i_1}x_{j_1}) = x_{i_1}x_{j_1}$.Then $\phi\left(\tau(x_{i_1}x_{j_1} - x_1x_2)\right) = \phi(x_{i_1}x_{j_1} - x_1x_2) = \tau\phi(x_{i_1}x_{j_1} - x_1x_2) \Rightarrow \phi(x_{i_1}x_{j_1} - x_1x_2) - \tau\phi(x_{i_1}x_{j_1} - x_1x_2) = 0 \Rightarrow \sum_{2\langle i\langle j\le n} \left(k_{ij}\,(x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j)\right) - \tau\left(\sum_{2\langle i\langle j\le n} \left(k_{ij}\,(x_1x_ix_j - x_1x_2x_i + x_2x_ix_j - x_1x_2x_j)\right)\right) = 0.$

$\Rightarrow \sum_{\substack{j=r+1\\ j\ne s}}^{n}(k_{rj} - k_{sj})\, x_1x_rx_j - \sum_{\substack{j=r+1\\ j\ne s}}^{n}(k_{rj} - k_{sj})\, x_1x_2x_r + \sum_{\substack{j=r+1\\ j\ne s}}^{n}(k_{rj} - k_{sj})\, x_2x_rx_j + \sum_{2<i<r}(k_{ir} - k_{is})\, x_1x_ix_r + \sum_{2<i<r}(k_{ir} - k_{is})\, x_1x_ix_r + \sum_{2<i<r}(k_{ir} - k_{is})\, x_2x_ix_r - \sum_{2<i<r}(k_{ir} - k_{is})\, x_1x_2x_r = 0$

$\Rightarrow$ By equaling the above equitation we get $k_{rj} = k_{sj}\,;\ r < j \le n\,\&\, j \ne s$,and $k_{ir} = k_{is}\,;\ 2 < i < r.$

$\Rightarrow$ $k_{rj} = k_{sj} = k_{ir} = k_{is} = k$.Thus since $2 \sum_{2\langle i\langle j\le n} k_{ij} = 0 \Rightarrow 2 \sum_{2\langle i\langle j\le n} k = 0$

$\Rightarrow 2\binom{n-2}{2}k = (n-2)(n-3)k = 0$.Then $k = 0\ or\ p|(n-2)\ or\ p|(n-3)$.

$\because\ 2 \sum_{2\langle i\langle j\le n} k_{ij} = 1$when$(i,j) = (i_1,j_1) \Rightarrow 2\,\frac{(n-2)(n-3)}{2}k_1 = 1.$

$\Rightarrow p \nmid (n-2)\,, p \nmid (n-3)\ and\ k_1 \ne 0$.Hence we get $p \nmid (n-2)\,, p \nmid (n-3)\,,\ k = 0\ and\ k_1 \ne 0$ .Thus if the sequence is split then p does not divide neither (n-2) nor (n-3).

Proposition3.11: $S(n-3,3)$ is a proper submodule of $ker\bar{d}_2$overFS_n.

Proof: Since $S(n-3,3) = FS_n(x_2 - x_1)(x_4 - x_3)(x_6 - x_5).\ i.e.$ $(x_2 - x_1)(x_4 - x_3)(x_6 - x_5)$ is the generator of $S(n-3,3)$ over FS_n, and $\bar{d}_2((x_2 - x_1)(x_4 - x_3)(x_6 - x_5)) = 0$. Hence $S(n-3,3) \subseteq ker\bar{d}_2$.
Since $\bar{d}_2$ is an epimorphism. Hence

$$dim_F ker\bar{d}_2 = dim_F M_0(n-3,3) - dim_F M_0(n-2,2) = \frac{n(n-1)(n-2)}{6} - \frac{n(n-1)}{2} = \frac{n(n-1)(n-5)}{6}$$

While $dim_F S(n-3,3) = \frac{n(n-1)}{6}$.Thus $dim_F S(n-3,3) < dim_F ker\bar{d}_2$.
Hence $S(n-3,3)$ is a proper submodule of $ker\bar{d}_2$ over FS_n.

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Proposition3.12: If $p \neq 2,3$ and p divides (n+1), then we get the following series:

1)$0 \subset \bar{V}_0 \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

2)$0 \subset \bar{V}_0 \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V}_0 \oplus ker\bar{d}_2 \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

3) $0 \subset \bar{V}_0 \subset \bar{V} \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

4)$0 \subset F\sigma_3 \subset \bar{V} \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

5)$0 \subset F\sigma_3 \subset F\sigma_3 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

6)$0 \subset S(n-3,3) \subset F\sigma_3 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

7)$0 \subset S(n-3,3) \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V}_0 \oplus ker\bar{d}_2 \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

8)$0 \subset S(n-3,3) \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

Proof:

If $p \neq 2,3$ and $p \mid (n+1)$.

Then p does not divide n nor $(n-1)$ nor $(n-2)$.Since $\sigma_3(n) = \sum_{1 \le i < j < k \le n} x_i x_j x_k$ and the sum of the coefficients is $\frac{n(n-1)(n-2)}{6}$, then $\sigma_3(n) \notin M_0(n-3,3)$. $i.e$ $F\sigma_3 \cap M_0(n-3,3) = 0$.which implies that $M(n-3,3) = M_0(n-3,3) \oplus F\sigma_3$.Moreover we have $S(n-3,3) \subset ker\bar{d}_2$ and $F\sigma_3(n) \not\subset ker\bar{d}_2$,thus $F\sigma_3 \cap S(n-3,3) = 0$.

$Since\ p \nmid n$, then by Theorem (3.6) we have $\bar{V}_0(n)$ is an irreducible submodule overFS_n , and $\bar{V} = \bar{V}_0(n) \oplus F\sigma_3$,then $\bar{V} \cap ker\bar{d}_2 = 0$. Thus $\bar{V} \cap S(n-3,3) = 0$which implies that $F\sigma_3 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3)$ and $\bar{V}_0(n) \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3)$ .Therefore we get the following series:

1)$0 \subset \bar{V}_0 \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

2)$0 \subset \bar{V}_0 \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V}_0 \oplus ker\bar{d}_2 \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

3) $0 \subset \bar{V}_0 \subset \bar{V} \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

4)$0 \subset F\sigma_3 \subset \bar{V} \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\sigma_3$.

5)$0 \subset F\sigma_3 \subset F\sigma_3 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

6)$0 \subset S(n-3,3) \subset F\sigma_3 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

7)$0 \subset S(n-3,3) \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V}_0 \oplus ker\bar{d}_2 \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

8)$0 \subset S(n-3,3) \subset \bar{V}_0 \oplus S(n-3,3) \subset \bar{V} \oplus S(n-3,3) \subset \bar{V} \oplus ker\bar{d}_2 \subset M_0(n-3,3) \oplus F\,\sigma_3$.

Theorem 3.13: The following sequence of a FS_n- modules is short exact sequence.

$$0 \to ker\bar{\bar{d}}_2 \xrightarrow{i} G \xrightarrow{\bar{\bar{d}}_2} S(n-2,2) \to 0 \qquad \ldots\ldots\ldots(3)$$

where $G = FS_n(x_1x_3x_6 - x_1x_4x_6 + x_2x_4x_6 - x_2x_3x_6)$

Proof: From the definition of G we get that $G \subset M_0(n-3,3)$ and by Theorem(3.10) we have $\bar{d}_2 : M_0(n-3,3) \to M_0(n-2,2)$ is on to map. Since $S(n-2,2) = FS_n(x_2 - x_1)(x_4 - x_3) \subset M_0(n-2,2)$.

Then $(x_2 - x_1)(x_4 - x_3) = x_2x_4 - x_2x_3 - x_1x_4 + x_1x_3$ and $\bar{d}_2\big((x_1x_3x_6 - x_1x_4x_6 + x_2x_4x_6 - x_2x_3x_6) + (x_1x_3x_5 - x_1x_4x_5 + x_2x_4x_5 - x_2x_3x_5)\big) = 2(x_2x_4 - x_2x_3 - x_1x_4 + x_1x_3) = 2(x_2 - x_1)(x_4 - x_3)$.

$Thus\ \bar{\bar{d}}_2 = \bar{d}_2|_G : G \to S(n-2,2)$ is on to map. Moreover the inclusion map i is one-to-one map and $ker\,\bar{\bar{d}}_2 = Imi$. Hence the sequence (3) is short exact sequence.

Corollary 3.14: The short exact sequence (3) is split when p does not divide (n-4).

Proof: Assume $p \nmid (n-4)$. Let $\varphi: S(n-2,2) \to G$ be define as follows: $\varphi\big((x_r - x_s)(x_t - x_l)\big) = \frac{1}{n-4} \sum_{\substack{k=1 \\ k \neq r,s,t,l}}^{n} (x_rx_tx_k - x_rx_lx_k - x_sx_tx_k + x_sx_lx_k)$ .Then for any $\tau \in S_n$ we get $\varphi\big(\tau(x_r - x_s)(x_t - x_l)\big) = \varphi\big((\tau x_r - \tau x_s)(\tau x_t - \tau x_l)\big) = \varphi\left(\left(x_{r_1} - x_{s_1}\right)\left(x_{t_1} - x_{l_1}\right)\right) = \frac{1}{n-4} \sum_{\substack{k_1=1 \\ k_1 \neq r_1,s_1,t_1,l_1}}^{n} (x_{r_1}x_{t_1}x_{k_1} - x_{r_1}x_{l_1}x_{k_1} - x_{s_1}x_{t_1}x_{k_1} + x_{s_1}x_{l_1}x_{k_1}) = \frac{1}{n-4}$

$\sum_{\substack{k_1=1\\k_1\neq r_1,s_1,t_1,l_1}}^{n} \tau(x_r x_t x_k - x_r x_l x_k - x_s x_t x_k + x_s x_l x_k) = \frac{1}{n-4}\tau(\sum_{\substack{k=1\\k\neq r,s,t,l}}^{n}(x_r x_t x_k - x_r x_l x_k - x_s x_t x_k + x_s x_l x_k)) = \tau\varphi((x_r - x_s)(x_t - x_l))$. Hence φ is a FS_n-homomorphism. Moreover we have

$\bar{\bar{d}}_2\varphi((x_r - x_s)(x_t - x_l)) =$

$\bar{\bar{d}}_2(\frac{1}{n-4}\sum_{\substack{k=1\\k\neq r,s,t,l}}^{n}(x_r x_t x_k - x_r x_l x_k - x_s x_t x_k + x_s x_l x_k))$

$=\frac{1}{n-4}\sum_{\substack{k=1\\k\neq r,s,t,l}}^{n}\bar{\bar{d}}_2(x_r x_t x_k - x_r x_l x_k - x_s x_t x_k + x_s x_l x_k) = \frac{1}{n-4}((n-4)(x_r - x_s)(x_t - x_l)) = (x_r - x_s)(x_t - x_l)$ Thus $\bar{\bar{d}}_2\varphi = I \ on\ S(n-2,2)$. Hence the sequence(3) is split when $p \nmid (n-4)$. Moreover $G = ker\ \bar{\bar{d}}_2 \oplus \bar{G}$; $\bar{G} = \varphi(S(n-2,2))$.

Proposition 3.15: $S(n-3,3)$ is a proper $KS_n -$ submodule of G.

Proof: Since $S(n-3,3) = FS_n(x_2 - x_1)(x_4 - x_3)(x_6 - x_5)$ and $G = FS_n(x_1x_3x_6 - x_1x_4x_6 + x_2x_4x_6 - x_2x_3x_6)$ then

y=$(x_2 - x_1)(x_4 - x_3)(x_6 - x_5) = (x_1x_3x_6 - x_1x_4x_6 + x_2x_4x_6 - x_2x_3x_6) + (x_1x_4x_5 - x_1x_3x_5 + x_2x_3\, x_5 - x_2x_4x_5) \in F$. Thus $S(n-3,3) \subset G$ .Moreover since $\bar{\bar{d}}_2 = \bar{d}_2|_G$, then we get $ker\ \bar{\bar{d}}_2 \subset ker\bar{d}_2$ and since $kerd_2 = ker\bar{d}_2$. Hence $ker\ \bar{\bar{d}}_2 \subset kerd_2$ and by definition of $\bar{\bar{d}}_2$ we get $\bar{\bar{d}}_2(y) = 0$ which implies that $(n-3,3) \subset ker\ \bar{\bar{d}}_2 \subset G$. Hence S(n-3,3) is a proper $FS_n -$ submodule of G .

Theorem 3.16: If $p \neq 2,3$ and $p|(n-3)$ then we have the following series:

1)0$\subset F\sigma_3 \subset F\sigma_3 \oplus S(n-3,3) \subset F\sigma_3 \oplus ker\ \bar{\bar{d}}_2 \subset F\sigma_3 \oplus G \subset F\sigma_3 \oplus M_0(n-3,3)$.

2)0$\subset S(n-3,3) \subset F\sigma_3 \oplus S(n-3,3) \subset F\sigma_3 \oplus ker\ \bar{\bar{d}}_2 \subset F\sigma_3 \oplus G \subset F\sigma_3 \oplus M_0(n-3,3)$.

Proof: Since $p|(n-3)$,then $p \nmid (n-4)$ and by Corollary (3.19) we get $G = ker\ \bar{\bar{d}}_2 \oplus \bar{G}$; $\bar{G} = \varphi(S(n-2,2)) \cong S(n-2,2)$ and by Proposition(3.20) we have $S(n-3,3) \subset ker\ \bar{\bar{d}}_2 \subset G$.

$Since\ \sigma_3(n) = \sum_{1\le i<j<k\le n} x_i x_j x_k$ and the sum of coefficients of $\sigma_3(n)$ is $\frac{n(n-1)(n-2)}{6}$ then $\sigma_3(n) \notin M_0(n-3,3)$ and $\sigma_3(n) \notin G$ which implies that $\sigma_3(n) \notin ker\ \bar{\bar{d}}_2.\ i.e.$ $F\sigma_3 \cap G = 0$ and $F\sigma_3 \cap ker\ \bar{\bar{d}}_2 = 0$ .Hence $F\sigma_3 \oplus ker\ \bar{\bar{d}}_2 \subset F\sigma_3 \oplus G$. Moreover we have $F\sigma_3 \oplus S(n-3,3) \subset F\sigma_3 \oplus ker\ \bar{\bar{d}}_2$ .Thus we get the following series:

1)0$\subset F\sigma_3 \subset F\sigma_3 \oplus S(n-3,3) \subset F\sigma_3 \oplus ker\ \bar{\bar{d}}_2 \subset F\sigma_3 \oplus G \subset F\sigma_3 \oplus M_0(n-3,3)$.

2)0$\subset S(n-3,3) \subset F\sigma_3 \oplus S(n-3,3) \subset F\sigma_3 \oplus ker\ \bar{\bar{d}}_2 \subset F\sigma_3 \oplus G \subset F\sigma_3 \oplus M_0(n-3,3)$.

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في التمثيل الطبيعي الثالث $M(n-3,3)$ للزمر النتاظرية

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المستخلص :

ان الهدف من هذا العمل هو دراسة التمثيل الطبيعي الثالث للزمرالنتاظرية$M(n-3,3)$ ضمن حقلF وبرهان بان$M(n-3,3)$ يمكن ان تجزئ اذا وفقط اذا كان p لا تقسم $n(n-1)(n-2)/6$

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Complaint Monitoring System Using Android in Iraq

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Abstract

Despite the widespread use of smartphones in Iraq and the possession of a vast number of people, the Iraqi environment is still lacking efficient applications that provide a real collaboration between the citizens and government through advanced diverse services. These applications will increase the transparency through monitoring the performance of the government institutions by citizens. Moreover, citizens can enforce the government employees to finish their works efficiently on time. Mayoralty of Baghdad is one of these institutions that contains many municipalities which are responsible for maintaining and running the city. Complaint Monitoring System (M-complaint) is created to help citizens to submit their complaints with photos of problem’s locations and track their locations using Global Positioning System (GPS) accurately. Citizens can track their complaints and reply the amount of satisfaction to the solutions which provided by municipalities. In addition, daily, the system will automatically filter the unsolved claims within seven days and submit a notification to the higher authority to find the suitable solutions. The aim of creating this application is to simplify the process of registering a complaint to the municipal and monitor the status of it (solved/not solved) in a quicker and cheaper way. Also, it measures the amount of citizens 'satisfaction depending on the provided solutions. If the problem were not be resolved within seven days, the system would automatically send a notification to the higher authority to follow up neglected complaints and take necessary actions. This system supports transparency and integrity and enables both citizens and officials to follow up on the performance of their employees. It considers as a part of Mobile government (M-government).

Keywords:
Firebase clouding messages (FSM), Smartphones, M-complaint, GPS, Android, JSON, M-government and neglected complaints.

Introduction

The citizen has two ways to register a complaint, the first one by calling the municipality many times which usually ends with going to the municipality to report the problem which wastes time and energy because of the paper works. The second one online, either by reporting a complaint on the official website or Facebook [4] or using mobile application that downloaded from Google Play [3]. Although these two ways are considering cheaper, they do not contain accurate information about complaints and also do not monitor the status of claims effectively. Mobiles and wireless technologies enable their users to make information and government services accessible at anytime and anywhere. M-government provides better public services by increasing the interaction between the government and citizens to address their needs and solve their problems through these devices and their technologies like camera, GPS and location services [6]. The main benefits of M-government are increasing transparency, decreasing corruption and reducing the complexity. It makes the information accessible for everybody in an appropriate way that leads to enhancing citizens' freedom [6]. Mayoralty of Baghdad has currently 18 municipalities. Each one has an office which is responsible for solving problems of a particular ward. There are a lot of researchers that have been given rise to a claim including m-services for the use of municipality services [3] to keep citizens' satisfaction.

Related work

- **The official application of Mayoralty of Baghdad.**
  This existing application can be downloaded from Google Play. The system will register a complaint by uploading or choosing a photo from gallery and write some information about complaints by a citizen. The reply will be written by an employee. Although this application has provided a way for direct communication between the government and citizens, it is considered inefficient because the requested information about complaint and complainant is insufficient. And also, neither authorities and nor employees are able to notify the complaints directly in their work area. Besides, failure to take action in the case of neglected complaints, leading to the weak of using the application and preferring citizens to complain in traditional ways.
- **Tejaswita B., Madhuri B. & Sucheta M.: Mobile Application for Grievance Registration.**
  The system was designed to lodge and monitor complaints by citizens, employees and an admin. The citizen could upload a photo on application through his/her android phone with a comment after registering to the application and then log in. A notification or message from an admin would be sent once a complaint was solved. Although the system could increase the efficiency, transparency and know the progress of each municipality through statistics, the system will be dealt with neglected complaints by sending them to the higher authority by an admin (not automatically).

- **Mareeswari V. & Gopalakrishnan V.: Complaint go: an online complaint registration system using web services and android.**
  The system contained two interfaces: mobile application and web portal to report complaints by citizens to Higher Government bodies directly. The location could be shared easily using Google map. The system allowed to attach any multimedia content such as an audio or video file. After lodging, a PIN coder would be generated for each complaint. It would be used with FSM to announce the Android users about the status of the complaint whether was solved or still remaining in the pending state. The system did not deal with neglected complaints.
- **Radhakrishnan D., Gandhewar N., Narnaware R., Pagade P., Tiwari A.& Vijaywargi P. : Smart Complaint Management System.**
  In this system, citizens reported complaints by tracking the specific locations using GPS and uploading pictures of them. Also, employees can use it to manage the complaints to track the status of the complaint easily. The system would keep on remaindering the employee about the pending complaint or unsolved problems. So, a problem would not be solved until the employee decides to solve it. In another word, any complaint could be neglected.
- **Bomble T., Raut R., Kanekar R. & AhmadHusen Sh.: Android Based Complaint Management System for Municipal Corporation.**
  The system was designed to register problems of citizens in particular areas by attaching photos and GPS track. The system used an alarm that would buzz when any complaint was not processed after a period time to notify that the complaint was being lodged for a long time and no processing was taken. Also, it was generated an e-mail when a complaint was registered, any complaint had not been followed up after a period time or sending a complaint to a department. There was no processing if the alarm had been buzzed and ignored.

Purpose of the system

This paper investigates an android application of mobile helping citizens to submit, monitor and track complaints easily from smart mobiles and get their problems solved without any efforts. The first objective of this application is providing facilities to the citizens when reporting their problems and making employees in municipality follow up to solve them from their mobiles. The second objective is reducing the corruption either by making the managers monitor the performance of their employees through the citizens' satisfaction or submitting a notification to the higher authority to find the suitable solutions if a complaint is not resolving within the stipulated time. This system is an attempt to bring transparency into the operation of Government Corporations.

Proposed system

This system is a complaint launching and tracking system. The implementation of the system is shown in figure 1.

Figure 1. The architecture of the system

A. **Client-side:** in client side, there are three types of users, complainants, employees who are responsible for solving the complainants' problems in each municipality and the higher authority who will be sent the neglected complaints that were not being solved within seven days. All of them need to download and install the application to use it with Android Mobile phone. First, the user needs to fill up the information (username, password, phone number, and e-mail) correctly to be registered. After submission the information, a unique number, named the registration ID, will be generated and saved in the database which will be used to send notifications through FSM. If the user registered correctly, they can log in as shown in figure 2, then the main page will appear.

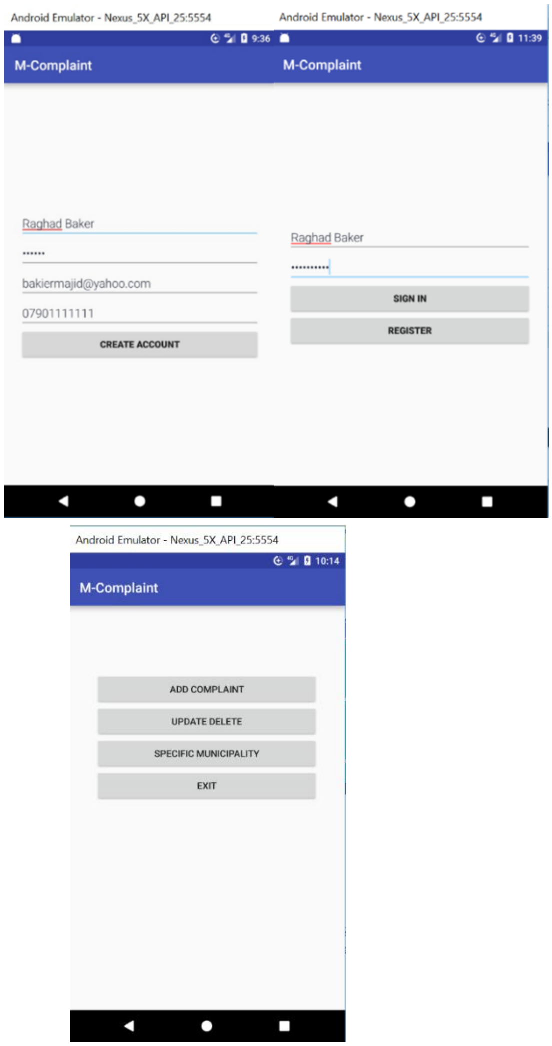


Figure 2. Login, registration and main page

A complainant can report a new complaint, check or update all the complaints that were previously listed by him/her if any of them has been updated. A complainant can fill fields fast and easily (most of the fields can be filled up by picking up the options) to reduce efforts. Besides, the address can be filled by written or using GPS and a photo of the specific area can either take it by camera or upload from the gallery as shown in figure 3.

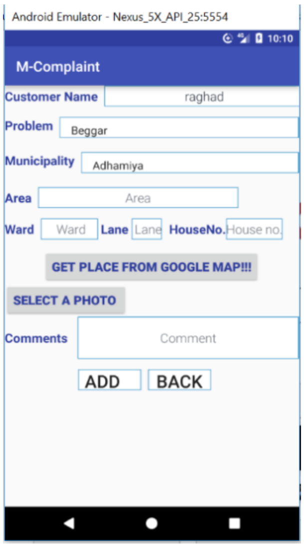


Figure 3. New complaint

Also, complainants can rate the solutions and this is a way to monitor the performance of each municipal by managers. In addition, this application uses push notifications by FCM for each modification. For example, when the complaint is submitted to the server, a notification will be sent both to the concerned complainant and to the relevant employees who saved in a group in the server to announce that a new complaint has been saved. So, employees in specific municipal can also monitor the problems easily that fall within their responsibility and respond to them. Another way to track the problems by employees is to open the application and select specific municipality, the problems will be appeared relating to their municipality as shown in figure 4. The third user is the higher authority who will be sent the neglected complaints.

If a complaint were not being solved within seven days, it will be automatically sent a push notification to the higher authority from the server phone to announce that there are neglected complaints.

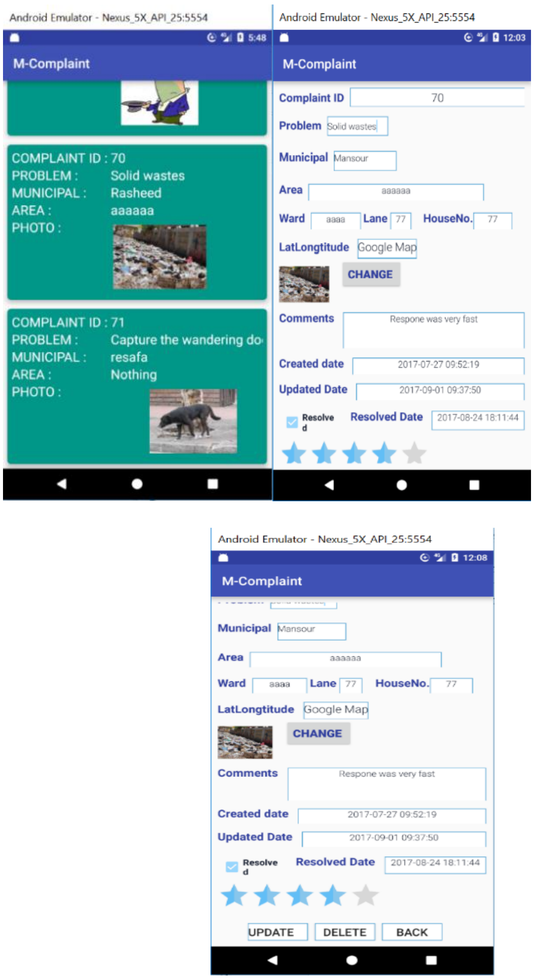
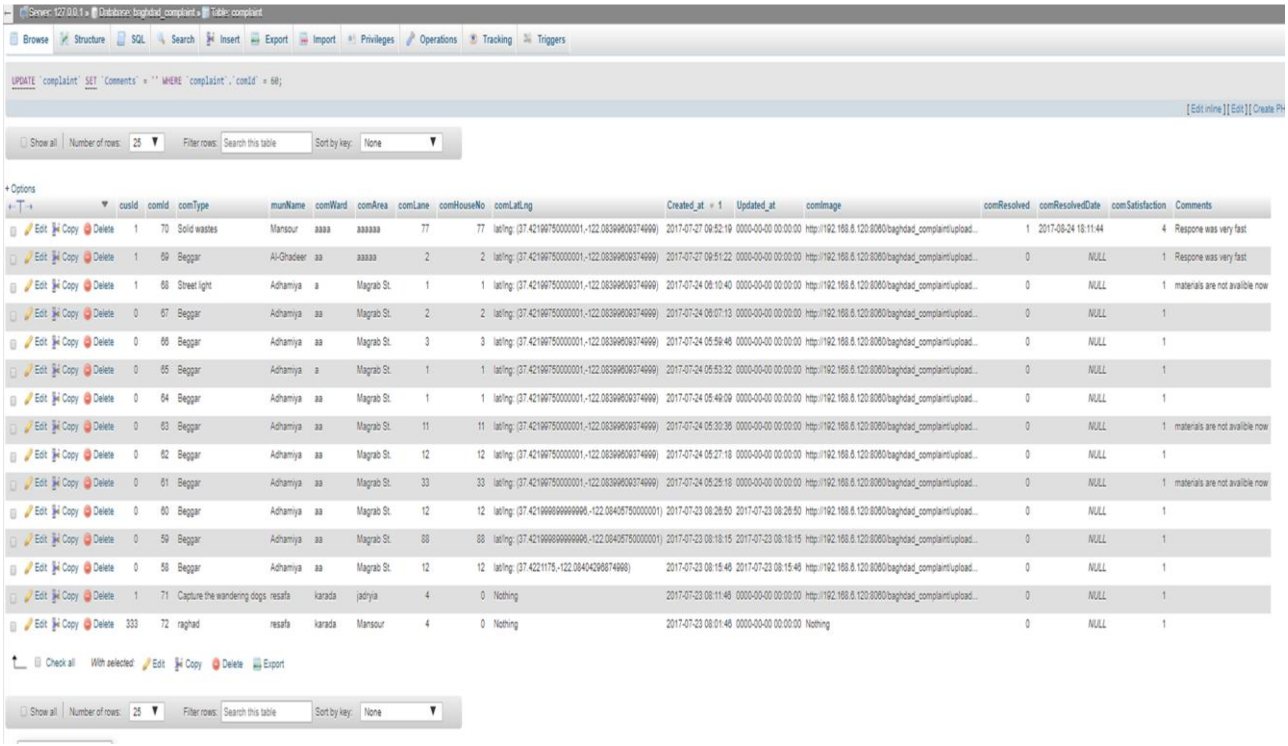


Figure 4. Complaints of specific municipality

B. **Web Server:** This part is the engine of the system which is the link between the Android application and the database. XAMPP Server has been used to host PHP project's files that are responsible for data transaction (as a Javascript Object Notation (JSON)) between the database and the application. Also, the system uses to upload and download images from web service which are stored in a specific folder on the server. The system also uses FCM to send notifications, for example, if a new complaint is reported or modified, the system will send one to the concerned complainant by keeping the registration ID of his/her device in the database. To reduce the corruption, two files added to the web server, named cron.bat and cron.php. These two files with tools named Task scheduler (in Windows) are responsible for capturing all complaints that are not solving within the stipulated time (seven days) to put them in a separate table and send a notification to the higher authority about them. This operation will be done at a particular time every day.

C. **Database-side:** this system uses MySQL as a database to store the data. It contains three tables, Logins table uses to store users' information (as mention above)**,** Complaints table uses to store problems' information and finally Delay table uses to store complaints' information automatically that were not being solved within seven days**.** A column in Complaints table is used to store the latitude and longitude from GPS. All tables store the time and the date creation and modification automatically. The images will be stored in a specific folder (on the server) and the path of each one will be stored in the table. Figure 5 shows the table of complaints.



UPDATE `complaint` SET `Comments` = '' WHERE `complaint`.`comId` = 60;

cusId	comId	comType	munName	comWard	comArea	comLane	comHouseNo	comLatLng	Created_at	Updated_at	comImage	comResolved	comResolvedDate	comSatisfaction	Comments
1	70	Solid wastes	Mansour	aaaa	aaaaaa	77	77	latlng: (37.42199750000001,-122.0839960937499)	2017-07-27 09:52:19	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	1	2017-08-24 18:11:44	4	Respone was very fast
1	69	Beggar	Al-Ghadeer	aa	aaaaa	2	2	latlng: (37.42199750000001,-122.0839960937499)	2017-07-27 09:51:22	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	Respone was very fast
1	68	Street light	Adhamiya	a	Magrab St.	1	1	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 06:10:40	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	materials are not avalible now
0	67	Beggar	Adhamiya	aa	Magrab St.	2	2	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 06:07:13	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
0	66	Beggar	Adhamiya	aa	Magrab St.	3	3	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 05:59:46	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
0	65	Beggar	Adhamiya	a	Magrab St.	1	1	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 05:53:32	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
0	64	Beggar	Adhamiya	aa	Magrab St.	1	1	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 05:49:09	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
0	63	Beggar	Adhamiya	aa	Magrab St.	11	11	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 05:30:38	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	materials are not avalible now
0	62	Beggar	Adhamiya	aa	Magrab St.	12	12	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 05:27:18	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
0	61	Beggar	Adhamiya	aa	Magrab St.	33	33	latlng: (37.42199750000001,-122.0839960937499)	2017-07-24 05:25:18	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	materials are not avalible now
0	60	Beggar	Adhamiya	aa	Magrab St.	12	12	latlng: (37.421999999999996,-122.08405750000001)	2017-07-23 08:26:50	2017-07-23 08:26:50	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
0	59	Beggar	Adhamiya	aa	Magrab St.	88	88	latlng: (37.421999999999996,-122.08405750000001)	2017-07-23 08:18:15	2017-07-23 08:18:15	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
0	58	Beggar	Adhamiya	aa	Magrab St.	12	12	latlng: (37.4221175,-122.08404296874998)	2017-07-23 08:15:46	2017-07-23 08:15:46	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
1	71	Capture the wandering dogs	resafa	karada	jadryia	4	0	Nothing	2017-07-23 08:11:46	0000-00-00 00:00:00	http://192.168.8.120:8080/baghdad_complaint/upload...	0	NULL	1	
333	72	raghad	resafa	karada	Mansour	4	0	Nothing	2017-07-23 08:01:46	0000-00-00 00:00:00	Nothing	0	NULL	1	

Figure 5. Table of complaints

Analysis of performance

Complainants can rate the solutions of the problems to show the amount of their satisfaction with the performance of their municipality. Also, it can use the number of neglected complaints to measure the time of response to solve them. The following chart explains the experiment of using the system for four weeks (from week 2 to week 5). The following chart indicates the amount of performance in municipalities depending on the rating of citizens and the number of neglected complaints. In the first week, the performance was very low, in contrast to the remaining weeks, it showed increasing in citizens satisfaction to the provided services in municipalities and decreasing in the number of unresolved complaints.

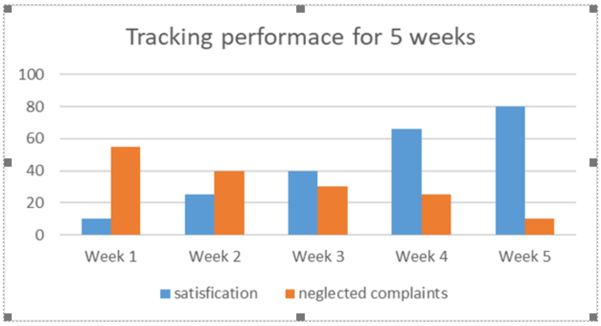


Figure 6. Tracking Performance for 5 weeks

Advantages of system:

1. Increasing the accuracy and transparency.
2. Reporting complaints anywhere and anytime in a very short period of time and easy.
3. Reducing efforts and time waste.
4. Monitoring the process of complaint would be easy.
5. Cost-free and user friendly.
6. Enforcing the government employees to finish their works efficiently on time.

Conclusions

In this paper, we have proposed and introduced a system which contains an Android application of mobile for citizens to report complaints by uploading a picture of a suspected place and use GPS to provide the location where the problem has appeared and other required information in a very short period of time. Besides citizens, the system allows employees and authorities to deal with complaints easily and friendly. They can check and update the information, especially the status of the complaints. The system will send a mobile notification to the complainant and the concerned employee after each modification through FSM. Citizens can rate the solutions which introduced by the municipality. The system makes the managers keep an eye on the quality of work of the municipality's employees and decrease the corruption by sending neglected complaints to the higher authority without efforts and time waste. Furthermore, it enforces the employees to finish their work on time which leads to increase the accuracy and transparency according to the results that have mentioned before. This system is considering as part of M-government which increases the communication between the government in a specific field and the people by using mobile. In the future, the system can be developed to dealing with fake complaints. It can develop a mobile application for IOS phones. Also, it can relate to Big Data to provide more facilities.

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نظام مراقبة الشكاوى باستخدام الاندرويد في العراق

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المستخلص :
على الرغم من الاستخدام الواسع للهواتف الذكية في العراق وحيازته من قبل عدد كبير من الناس، إلا أن البيئة العراقية لا تزال تفتقر إلى التطبيقات الفعالة التي توفر تواصلا حقيقيا بين المواطنين والحكومة لتوفيرالخدمات المتنوعة.بالاضافة ان توفر هذه التطبيقات سيؤدي إلى زيادة الشفافية من خلال رصد أداء المؤسسات الحكومية من قبل المواطنين. وعلاوة على ذلك، يمكن للمواطنين اجبارموظفي الحكومة لإنهاء أعمالهم بكفاءة في الوقت المحدد. بلدية بغداد هي واحدة من هذه المؤسسات التي تضم العديد من البلديات المسؤولة عن المحافظة على المدينة وإدارتها. تم إنشاء نظام مراقبة الشكاوى (M-complaint) لمساعدة المواطنين على تقديم شكاواهم مدعومة بالصور من مواقع المشكلة وتتبع مواقعهم باستخدام نظام تحديد المواقع العالمي (GPS) بدقة. يمكن للمواطنين تتبع شكاواهم و بيان مدى الرضا عن الحلول التي تقدمها البلديات. وبالإضافة إلى ذلك، يوميا، سيقوم النظام تلقائيا بتصفية الشكوى التي لم تحل في غضون سبعة أيام وتقديم إخطار إلى السلطة العليا لإيجاد الحلول المناسبة. ان الهدف من إنشاء هذا التطبيق هو لتبسيط عملية تسجيل شكوى إلى البلدية ورصد حالتها (تم حلها / لم تحل) بطريقة أسرع وأرخص. كما أنه يقيس مقدار رضا المواطنين على الخدمات اعتمادا على الحلول المقدمة. هذا النظام يدعم الشفافية والنزاهة ويتيح للمواطنين والمسؤولين متابعة أداء موظفيهم. وهي تعتبر جزءا من الحكومة المتنقلة (M-government).

كلمات مفتاحية:
Firebase clouding messages (FSM) ، الهواتف الذكية، M-complaint ، نظام تحديد المواقع العالمي (GPS) ، الاندرويد، JSON، M-government والشكاوى المهملة.

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Condor Cloud New Solutions

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Abstract:

In this article author proposes a new solution in the field of distributed computing needs, humans' requirements of new ways for developing technologies that increase information systems distribution horizontally and decrease the power consumption and delay time vertically. The author presents in detail the distributed computing concepts, types and some examples like Condor system of distribution and the proposal for developing.

Keywords: condor, solution, distributed computing, power, open source, rmi, cloud.

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1- Introduction

The earliest human ate food gathered by them using only the tools that they made themselves. We have continued the efforts made by humans for the disengagement between production and consumption for thousands of years and eventually led to the emergence of intensive agriculture, manufacturing, and distribution of electrical energy. As a consequence, it is the privatization of labor and economy and new technologies as determined by the modern world that give me the freedom to sit in a cafe drink a cup of coffee and write this article on my laptop, without drawing the attention to the sources of the things that I consume or use, including coffee and water, electricity and bandwidth wireless network.

This is the availability and accessibility of these sources to be fit, for example, on the concept of default virtualization, which indicates (in computer science) to withhold some useful functions behind the interface and hides the details about how to put these functions into effect. For example, when a waiter opens the cafe tap water, it's akin to scooping water from the barrel inexhaustibly. The same phenomenon occurs when I connect my laptop power plug in the wall, the presence of a huge electric switch invisible behind the plug, who knows when and how this electric energy was is generated. The computing itself is not characterized by the default character. Laptop, desktop or even the sophisticated computer centers of information for one of the companies, are systems with a great deal of independence and sufficiently implement programs stored

Locally. Since we do not accept a situation where each institution or a commercial company is obliged to run the power station, so why accept it for computers. This situation pushed the computer scientists to look for other alternatives more effectively. Digital networks increase speedily day after day [1], why do not we 'compile' computers on an ongoing basis so that users can request resources, storage, data and software regardless of location and appropriate source to provide. In other words, why not make public virtual computer services? Such a 'networking' computation would be beneficial and useful like electric networking. This networking can work like institutions of e-commerce, for example, Virtual tour of tourist attractions, so it can be adapted to their information systems and their computers depending on the demand as it works at the same time to connect these systems with partners, suppliers and customers in order to expand the network of services they provide. For example Adventure Travel Company provides to its customers trips to explore the virtual reality of its existing sites under the sea. Once the potential customers start sailing in the virtual water outside the tourist resort (for example Tioman island), the company is obliged to access to some databases and software required to retrieve geographic information related to the trip, and convert the data, write the appropriate information and integrate it all in real time video pictures coming from cameras that have been installed in the same location [1], [2]. As it is impossible for any travel company to run this potential alone, but with the available networking technologies, it could assemble the necessary resources from several sources, each one of them to meet the needs of a large number of customers, and achieve economic feasibility. In addition, the systems networking carry great potential for a number of other areas where work is limited to a few categories. In the field of medicine, for example, it allows access to computers and remote databases to a doctor comparing the radiographic images of millions of other patients, which may be able, for example, to discover a malignant tumor early. It may allow biological chemists at different places to screen 10,000 candidate for drugs with hours rather than days, or may allow civil engineers to test and design earthquake-resistant bridge within a few days instead of several months [4].

This concept for the Distributed computing and networking is a natural extension of the current web. Internet has added advantage on the nature of communication, allowing any person to call someone or one device to call another device regardless of location or the means used to do so. This has resulted from the innovation of e-mail, World Wide Web applications and peer peer-to-peer, including systems for file-sharing that provide access to any person or another device, and so this is not inferior to other groups varying from those services and is safe and reliable.

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2- Distributed Computing

Distributed Computing is a field of computer engineering and studying distributed systems. Distributed systems are composed of a group of computers that communicate with each other through a network connected with each other. [3] Computers interact with each other to achieve a common goal called computer programs that run on distributed systems. There are two types of distributed computing, [8] cluster and grid computing besides two other common types, distributed information system and pervasive distributed computing. In distributed computing, the problem is divided into small problems, and then is distributed to other computers for processing. Figure 1 represents grid computing [3], [4].

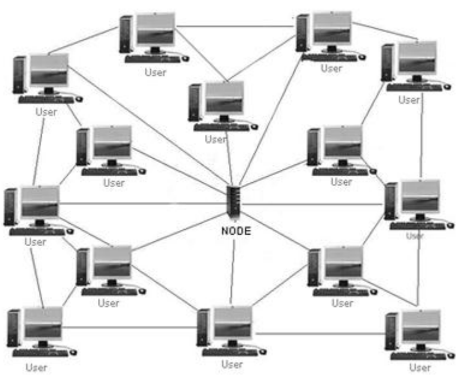

Fig. 1. Grid Computing

The idea behind distributed computing often does not use all the processing power available. In normal circumstances, [5] using certain number of program requires a certain amount of the processing power and a large amount of processing remains unused. [6] The idea behind distributed computing is to use the idle power available in some way. Simply linking to a certain number of computers in a grid needs the presence of a particular program on each machine within the network and some of the tasks are distributed on devices are not used by the main user.
The processors of these devices in turn address the jobs sent to it through this network by a user of the master device [5], [6] and [9]. Distributed systems characterized many advantages, including:

1. There are no common clock pulses between devices; each device has an independent clock pulse from other devices.
2. There is no shared memory; information is exchanged by passing messages.
3. The ability to withstand failure tolerates failed.
4. The ability to expand in the structure of the system (the network, the number of devices).
5. Each device or node sees a fraction of the system, and deal with a specific part of the income (a specific type of data).

3- Condor is a Distributed Computing

Condor is a computing infrastructure that utilizes unused compute cycles from common workstations was developed by the University of Wisconsin-Madison. This system called Condor has been instrumental in the development and testing of physics-based algorithms requiring massive computations.

The initial use of the Condor was to produce atmospheric databases through a parameter sweep of key inputs into the MODTRAN atmospheric propagation code. These databases were initially used as part of an atmospheric compensation algorithm optimizing for atmospheric visibility, [8] water vapor content, and target surface elevation served in a model predicted vs. real sensor reaching radiance comparison [8]. These selected atmospheric inputs were used as the basis for converting a sensor reaching radiance scene to surface spectral reflectance. The same type of database has been used as part of a target detection scheme where a target spectrum is propagated through a range of atmospheric parameters. This results in a set of estimates of how a sensor would image this target through various atmospheric conditions which generates a subspace on which target detection schemes can be applied. Because it is possible to finely sample the solution space of numerical models such as MODTRAN, statistical descriptions of these atmospheric conditions can be devised as a more compact description of an entire. Condor runs on Linux, UNIX, Mac OS X, FreeBSD, and contemporary Windows operating systems [7]. Condor can seamlessly integrate both dedicated resources (rack-mounted clusters) and non-dedicated desktop machines (cycle scavenging) into one computing environment.

Condor is developed by the Condor team at the University of Wisconsin–Madison and is freely available for use. Condor is an open source (it's licensed under the Apache License 2.0). It is also available on other platforms, like Ubuntu from the repositories. Some applications like the NASA Advanced Supercomputing facility (NAS) Condor pool consists of approximately 350 SGI and Sun workstations purchased and used for software development, visualization, email, document preparation, etc. Each workstation runs a daemon that watches user I/O and CPU load. Condor scheduler performs network admission control to ensure that network resources are not oversubscribed. The scheduler allocates network capacity up to a configured limit for each subnet. The administrator typically configures this limit to less than the subnet's full capacity to reserve capacity for other network users. The scheduler places jobs on remote CPUs only when the network capacity limits would not be exceeded by the jobs' checkpoint and data transfers. If capacity would be exceeded for a given subnet, jobs which may have run on CPUs on that subnet will instead run in other subnets (where network capacity is available) or remain idle.

When a workstation has been idle for two hours, a job from the batch queue is assigned to the workstation and will run until the daemon detects a keystroke, mouse motion, or high non-Condor CPU usage [9]. At that point, the job will be removed from the workstation and placed back on the batch queue. Condor can run both sequential and parallel jobs. Sequential jobs can be run in several different "universes", including "vanilla" which provides the ability to run most "batch ready" programs, and "standard universe" in which the target application is re-linked with the Condor I/O library which provides for remote job I/O and job check pointing. Condor also provides a "local universe" which allows jobs to run on the "submit host". In the world of parallel jobs, Condor supports the standard MPI and PVM. In addition to its own Master Worker "MW" library for extremely parallel tasks. Condor-G allows Condor jobs to use resources not under its direct control. It is mostly used to talk to Grid and Cloud resources (Cloud computing and grid computing are the words that confuse a lot of people because of the similarity in theory. Cloud computing and grid computing infrastructure contain a huge computer network [10]. On the front end, cloud computing and grid computing concepts are
newest compared by many other computing. Both concepts have been developed in order to distribute computing as the example, the computing element over a wide area. Literally, [11] the computers are separated from each other or by other means, In another word, there are a lot of people who prefer to use distributed computing on a single processor computing, and here are the reasons:

1. Distributed computing are sources that provide simultaneous or parallel computing for users by decreasing the delay time. Applications do not need to wait in a queue to be served one after the other.
2. The cost for Distributed computing is also not high since it is better to have several low-cost devices rather than one high-cost computer.
3. Distributed computing is much easier to grow or increase capacity; designers can add other computers in the event of the increased number of users.

Undistributed computing needs to increase the computing power in the system which will require a high cost, To understand the basics and details between grid computing and cloud, It should to explain both techniques, "Cloud Computing" Cloud is basically an extension of the concept of abstraction code, [6] An extension of the object-oriented programming concept of abstraction, it means the Internet cloud. End-users are only given products or input data for the whole process, which leads to the outputs which are simply hidden and shown on computing and fake sources that are present in multiple servers or the servers that work in groups. Also in the family "cloud computing" There is what is known as "software as a service" in English "Software as a Service" or SaaS applications [3]. These customers' clients always install light programs on the server but leaving all the heavy work using the infrastructure to someone else. Cloud computing eliminates the cost, and the complexities of purchase, configuration, and management of the hardware and software required to build and deploy applications: These applications come as a service on the Internet (cloud). "Grid computing" Networking systems are designed for cooperation in post sources and can also be perceived as a distributed computing on a large scale and is mainly used by the processing capacity in several PCs to handle one task [6].

The task is divided into several sub-tasks; each device in the network client has a sub-task. Once tasks are processed, they are sent back to the main system which supervises all the tasks and is grouped together or united as directed.

4- The Open Source Condor for Cloud

Red Hat Expanded Messaging, Real-time, and Grid Technology Capabilities to advance cloud it announced the availability of Red Hat Enterprise MRG 1.3, [16] including updates to the product's Messaging, Real-time and Grid technologies, which provide a key technology base for Red Hat Cloud Foundations, a solution set that offers a comprehensive set of tools to build and manage a private cloud Figure 2. Red Hat Enterprise MRG provides an integrated platform for high performance distributed computing.

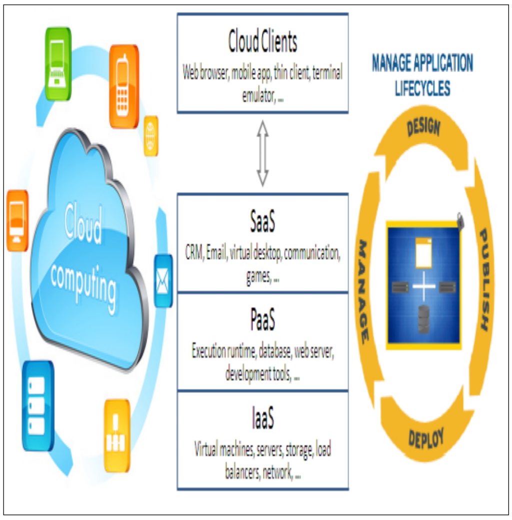

Fig. 2. Red Hat Enterprise MRG.

Enterprise MRG first released was in June 2008, Enterprise MRG has since enabled customers around the world to meet their messaging, real-time and grid computing needs, offering: Enterprise MRG's Grid functionality, based on the Condor Project brings the advantages of flexible deployment to a wide range of customers. By using software methodology for design and implement an evaluation let the developers using PaaS (Platform as a Service) to modify the functions or task, the network architects IaaS (Infrastructure as a Service) to get the scalability to provide the high-performance services to the End users SaaS (Software as a Service).

5- Applications and Workloads

With Grid, customers can build cloud infrastructures to aggregate multiple clouds. It provides integrated support for virtualization and public clouds and easier aggregation of multiple cloud resources into one compute pool. In addition, [11], [14] it provides more streamlined and flexible computing across remote grids with servers, clusters and cycle-harvesting from desktop PCs as well as across private, public and hybrid clouds. MRG Grid is a key base component of Red Hat Cloud Foundations see Figure 3.

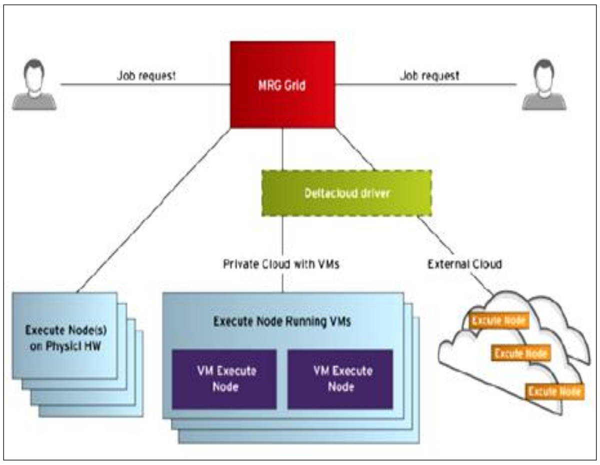

Fig. 3. VM and Cloud provisioning with delta cloud.

6- Condor RMI for Overlay Resource Manager

Ten years ago condor has introduced the concept of Condor glide-ins as a tool to support 'just in time scheduling' in a distributed computing infrastructure that consists of recourses that are managed by (heterogeneous) [12] autonomous resource managers as example glideinWMS Figure 4. By dynamically deploying a distributed resource manager on resources allocated (provisioned) by the local resource managers, the overlay resource manager can implement a unified resource allocation policy. In other words, the use remote job invocation to get resources allocated.

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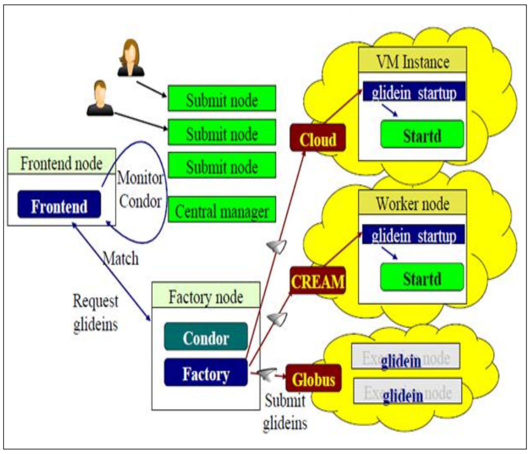

Fig. 4. glideinWMS Architecture.

7- Conclusion, Solutions and Suggestions

Cloud services are provided by grid computing. In fact, almost all the online services that users get are from the cloud, for example, web hosting, multiple operating systems, and database support. It tends to have more leniencies in conjunction with heterogeneous and geographically scattered computing systems; Condor is a powerful tool for scheduling jobs across platforms, both within and beyond the boundaries of clusters [13]. Through its unique combination of both dedicated and opportunistic scheduling, Condor provides a unified framework for high-throughput computing. By integrating the basic Condor commands into a GUI-based program, is to make Condor become more user-friendly and more convenient to use. An integration of more advanced Condor commands into the GUI-based program, developed and making Condor serve as the key schedule of a large grid computing system based on WSRF.NET[15]. Distributed computing success depends on high numbers of its adopters. Among the proven strategies to overcome the traditional position baffling is to secure free access to basic technologies. From here, it is important not only to have the availability of all the major specifications of communication free-for-all but also to have an open and easy way to implement these specifications. The Globus toolkit meets these requirements condor is one of these toolkits. However, any further progress depends on the contributions of academics, industrialists, and sellers; it also depends on the inclusion of the concepts of communication within the business software as well as the training of the other parties.

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حلول جديدة سحابة الكوندور

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المستخلص :

في هذه المقالة يقترح المؤلف حلا جديدا للاحتياجات في الحوسبة الموزعة، المتطلبات البشرية لطرق جديدة لتطوير التكنولوجيات التي تزيد من توزيع نظم المعلومات أفقيا وتقليل استهلاك الطاقة والتأخير في الوقت عموديا. يعرض المؤلف بالتفصيل مفاهيم الحوسبة الموزعة وأنواعها ويعرض الحلول مع الأمثلة مثل نظام سحابة كوندور الموزعة كاقتراح للتطوير.

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Influence Factors of Adoption E-commerce in Iraqi SMEs

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Abstract

Today the most significant technologies that change the business environment is the Internet that has a great effect on the future of electronic commerce (e-commerce). Currently e-commerce plays a vital role in the development of small and medium enterprises (SME). Therefore, Iraq must involve its SME in this field in order to no longer be separated from the business world and the new economy. The objective of this paper is to identify the factors that prevent the adoption e-commerce by SME. The researcher prefers to use qualitative and quantitative approach due to identify these factors. The results showed the most important factors that affect directly and must be considered in order to adoption of e-commerce in SME. Adoption these factors will help Iraqi SME's to gain long term success in the global market.

Keyword: e-commerce, SME, success factors adoption

1. Introduction

The advent of the Internet has generated a plethora of opportunities for businesses to communicate, collaborate, and distribute information, which will lead to fundamental changes in the way companies compete with one another and provide services to their customers [1].

Contrary to the traditional means of conducting the transactional commerce which is dependent on buying and selling goods and services face-to-face, and require physical environment, the Internet can bring down physical barriers to commerce and offers access to the virtual market which could reach millions of customers on the Internet [2].
Furthermore, companies are taking advantage of Internet technologies to include more than just buying and selling online and conduct other commercial activities ranging from simple advertising to comprehensive virtual brochures. The conception of buying and selling of goods and services and transfer of funds through digital communications is termed as "electronic commerce" or simply "e-commerce". E-commerce offers privileges and new means of doing business that no company can afford to ignore.
With the increased unemployment rates and major companies downsizing, the small- medium size enterprise (SME) is encouraged as one major solution to the new economies issues, due to its ability to create jobs, address poverty and improve productivity. As such, SMEs represent the backbone of the national economy in most countries all over the world. It is essential for an SME to grow in order to remain competitive in the current and future markets. The SMEs through the use of the Internet and e-commerce will be able to enhance internal and external communication, shorten the development cycle of new products, access new geographical markets domestically and internationally, reduce the export costs, and provide support to customers. Furthermore, e-commerces will contribute to make SME to overcome the disadvantage of being small in size to reach customers especially in international markets and establishing new distribution channels. E-commerce is no longer an alternative; it is an imperative for SME to survive. However, many firms still struggle on which would be the best model to adopt e-commerce and which strategy should be followed? The answer is not simple, every firm has its special circumstances and resources and there is no e-commerce model that fits all the companies in spite of being in the same culture or the same size. Therefore, every company must be required to review e-commerce models before deciding which strategy or model is suitable to be adopted?

A. Small and Medium Enterprise (SME)

There are many varying definitions for SME. According to some research, small and medium-sized enterprises (SMEs) can be described as an aggregate group of businesses operating in many sectors such as services, trade, agri-business, and manufacturing. They include a wide diversity of firms such as village handicraft makers, small machine shops, and computer software firms that have a wide range of sophistication and skills. Some are dynamic, innovative, growth-oriented and fast-growing firms while others are satisfied to remain small and perhaps family owned with no change for decades. Size-wise, SME may range from part-time business with one employee to a business employing hundreds of people.

They range from SME which are inextricably part of an international subcontracting network, or to those with technology and investment partners based aboard, or to those which are part of a family-based society or cluster[3],[4].

The official definition of SME in Iraq by the Central Organization for Statistic and Information Technology (COSIT), an Iraqi Government Body is small business that has the number of employees less than 10, and medium business which has less than 30 employees (See the table 1). Due to this definition used by Iraqi Government and most of the NGOs in Iraq, this definition will be admitted for the purposes of this research.

Table 1: Definition of Small and Medium Enterprises in Iraq.

Small enterprise	Employee 1-9 people
Medium	Employee 10-29 people

B. E-Commerce

Electronic commerce or e-commerce refers to the process of selling and buying through the Internet, or conducting any transaction including the transfer of ownership or rights to use services or goods via a computer-mediated network. Execute these transactions electronically will provide wide competitive advantages by finding new customers, markets, and suppliers. Additionally, e-commerce is often faster where it overcomes the geographical boundaries; e-commerce is also cost-effective, where the advertising and marketing is at a lower cost compared with traditional methods. In addition e-commerce does not need a prominent physical location [5].

C. Benefits Derived from Adopting E-Commerce within SMEs.

E-commerce brings numerous benefits to those SMEs that adopt e-commerce[6],[1]. According to Payne (2005)[7], the e-commerce will not benefit all the economic sectors in the same way, he suggests that the sectors that have information-intensive activities and product or services that can be used or delivered by electronic means the most likely to benefit from e-commerce. In this section, the researcher will explore some of the plausible benefits reported in literature studies.

The researcher summarizes the benefits from e-commerce to SMEs as follows:

1- Cost saving: e-commerce works on reducing transactional and communicative cost, inexpensive advertisement, provide cost effective ways to marketing[8],[9],[10].

2- Time-saving: shortening the development cycle of new products, reducing the search information time; in addition reducing stock lead times [2],[11],[6].

3- Enhancing ability for competition it enables SME to overcome limitation and to maximize using information and network technology hence it provides the efficiency for SME to compete with large companies as well as the another SMEs[12],[13],[14].

4- Reaching new local and global markets that means new supplier and customers and enhance the profitability[12],[13], [15].

5- Improving communication to use internet by SMEs will enhance internal and external communication, and achieve two-ways of communication, provide support to customer's services [2],[13].

2. Development of the Research Tool (Data Collection)

As there is lacking of research done in developing countries regarding the adoption of e-commerce in SMEs especially in a country like Iraq, a survey tool was developed for collecting data necessary in this study. This approach has been considered being the most suitable due to the lack of empirical research in Iraqi environment [4]. Furthermore, the researcher prefers to use qualitative approach (survey) due to the current study's main intention that is the identification of the significant factors that influence the e-commerce adoption and implementation in SMEs. This was done by conducting a thorough literature review of studies investigating such factors and by conducting interviews with academic and business experts who have sufficient knowledge to deal with e-commerce adoption. The researcher later used these study results to conduct a comprehensive survey in the area of Baghdad to further test these qualitative results. This research design has been successfully employed in similar studies[16].

Although the survey method has its own advantages and limitations, the survey approach appeared as more popular among researchers and enabled generalization of the finding. For this reason, two tools were adopted in this research: the interview and questionnaire.

After collecting the data from the interview and the subsequent questionnaire, data analysis session took place. Here in, Statistical Package for the Social Science (SPSS) which is a statistical tool was used for analyzing the data by using descriptive statistics to illustrate the demographic information. Each time the collected data was inserted, data analysis was performed. To ensure useful findings which reflect the prospective of the respondents, the researcher paid much attention for analyzing the results which was considered as one of the most significant step during the data analysis phase.

2.1 Interview

Knowledge that can be extracted from human is called the knowledge elicitation. Eliciting knowledge requires many sessions. These sessions may be interactive discussions that involve an exchange of ideas about the problem. This style of acquiring knowledge is known as interview method. First of all, the main objective of the interview should be highlighted and obvious to ensure the success of the interview. Based on that, the objective of the interview was to investigate what are the most important factors which may drive\inhibit e-commerce in Iraqi environment. The researcher had two candidates for the purpose of the interview. Table 2 illustrates the people who were interviewed and consulted.

Table 2: List of people Interviewed and Consulted

No. of Experts / Aspects	Expert 1	Expert2
Name	Dr. Saif Alsewaidi	Assitant prof . Dr. Firas Abdul-hamed
Organization	Fnjancom company, Iraq	Baghdad University, Iraq
Years of experience	5	15
Remarks	• He works for more than five years in the field of e-commerce in Iraq • The Fnjancom Company is a pioneer in adopting e-commerce in Iraq and it's the official sponsor for the first conference conducting in Iraq about e-commerce in 2010. • Fnjancom company provides service of "gate online shopping" through the "Filspay" credit card which is a mean to pay online instead of use cridet cards such as Visa or Master card (for security issues), and consider it as a solution for people who do not have credit card.	• Head of computer science department. • Teaching the principals of the e-commerce for 5years. Work as web-designer for commercial purposes for more than 4 years.

As portrayed in Table 2, Mr. Saif represented the practical and business experience while Dr. Firas represented the academic and IT experience.

The researcher took into consideration the importance of preparing a list of materials that was to be discussed during the interview in order to focus on the target of this interview and time factor. The interview's agenda detailed on how to achieve the objective and contained a list of the major items that was to be discussed during the interview.

The success of any interview session will first rest with the background formed by its preparation. The preparation addressed the subjects to be covered, scheduling, materials and methods for recording the session. For the purpose of this research, the researcher prepared the material to be discussed with the interviewees based on a thorough review of literature and consultation. The first interview was essential since it had a major impact on enhancing or reducing further interview sessions. In the second session of the interview, the important factors that influence adopting e-commerce in developing countries were demonstrated and discussed.

The type of interview used in this research was a structured interview because it was easier to prepare and manage than an unstructured interview. The information collected was also easier to be analyzed since most of the information collected was related. Finally, this approach provided more detailed information on the problem.

From the results collected from the interviewe, the questionnaire was designed. Apart from this, there were some questions in the questionnaire that were adopted from previous questionnaires [17],[4],especially questions that had already been validated. The researcher then showed the questionnaire in both English and Arabic versions to the interviewees and asked them for their assessment regarding the questionnaire in general.

Finally, the researcher revised the questionnaire based on the suggestions given so as to be more understandable for the respondents.

2.2 Questionnaire

The second method for collecting information was by conducting the questionnaire. The questionnaire is a way commonly used to collect large number of data in a short time. Iraq has suffered from heavy lack of power supply; in addition, there is no legislation from the government to protect the local product, thus these reasons led to the closedown of most SMEs from the manufacturing nature in Iraq. The researcher distributed the questionnaire to 30 SMEs chosen randomly in Baghdad. The feedback was obtained from 23 responding SMEs. The respondents were owners, managers of the computer stores in Iraq who were considered as qualified to speak about recent technology development due to their exposure to technology.

The questionnaire was divided into 4 sections and included 27 questions. The first section consisted of 8 questions about the profile of the firm\company and some manager\owner's characteristics were included. While the other three sections (2, 3 and 4) used Likert scale to show the level of respondent's agreement about the question “why your firm\company is not using e-commerce?", where the respondent were asked to rank their answers based on 5 levels; from level 1 as strongly disagree to level 5 as strongly agree. The second section included 7 questions focused on technological factors that have influenced the e-commerce adoption in Iraq. The third section included 6 questions emphasizing on the macro and micro environmental factors and lastly the forth section included 6 questions about organizational factors. Due to commercial confidentiality, the names of the respondents were kept undisclosed.

3. Data Collection and Analysis

The following sections provide a detailed description for the results of data collection and analyses:

3.1 Reliability

Mitchell (1996)[18] indicated that the reliability of the questionnaire is concerned with the consistency of the response to the questions. Reliability can be defined as the accuracy or precision of measuring instrument. The reliability analysis has been applied to verify the collected data allows the researcher to study the properties of measurement scales and items that make them up. For the purpose of the reliability analysis, the Cronbach’s Alpha has been estimated. Alpha Cronbach can be defined as a model of internal consistency based on the average inter-item correlation. In this research, the Alpha value for the variables which indicate the reliability of construct is as follows:

Table 3: Reliability of the Questionnaire

No. of items	Cronbach’s Alpha Based on standardized items	Cronbach’s Alpha
19	0.920	0.918

By analyzing the questionnaire data using SPSS, the researcher found Cronbach’s Alpha equal to 0.918, thus, the reliability of the data is acceptable since the percentage of Cronbach’s Alpha can be considered acceptable if it not less than 0.7[19].

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3.2 Descriptive Analysis for Demographic Questions

As presented in Table 4, results from the questionnaire showed that 23 owners/managers of the SMEs responded to the questionnaire where 74 % of them are Small Enterprises and 26% are Medium Enterprises according to the official Iraqi definition for SME. 57% from the enterprise respondents are under 5 employees. Besides that, the results of the questionnaire indicated that 39% of the surveyed companies have experience of 4-6 years, while those who have experience with ten years and more were 35%. Among those who responded, 52% were within the age group of 30-39 years old, while the percentage of people who were within the age group of 20-29 years old was 9%. The questionnaire results also showed that the majority of respondents hold a degree (Bachelor) where the proportion of 65% of the total respondents. The results of the questionnaire indicated that 95% of respondents have at least one computer and Internet service, used email and mobile to conduct business transactions. This is an indication of the large penetration of computers and the Internet through Iraqi business community. Finally the percentage of the enterprises that have their own website formed 26 % from the respondents.

Table 4: Descriptive Analysis of the Respondents

Variable	Classification	Freq.	Perce.
Size of company (number of employees)	Below 5	13	57%
	5- 9	4	17%
	10-20	5	22%
	21-29	1	4%
Number of years firm in business	0- 3	1	4%
	4-6	9	39%
	7-9	5	22%
	10 and above	8	35%
Age of owner\ Manager	20-29 years	2	9%
	30-39 years	12	52%
	40-49 years	7	30%
	50-59 years	2	9%
Education Level	secondary school	2	9%
	Diploma	2	9%
	Bachelor	15	65%
	Master	2	9%
	PhD	2	11%
The enterprise has internet provider	Yes	22	96%
	No	1	4%
The enterprise has website	Yes	6	26%
	No	17	74%
The enterprise uses email and mobile to conduct businesses	Yes	22	96%
	No	1	4%

3.3 Analysis Questions for Technological Factors

This section shows the most significant factors that affect adopting e-commerce from technological contexts.

Table 5: Response to the Technological Factors

Technological factor	Mean
IT infrastructure- e payment system	3.95
IT infrastructure-power supply	3.65
Security	3.52
Perceived ease of use	2.56
Perceived usefulness	2.26
Compatibility	2.13
Time	1.95

As the researcher found in Table 5, majority respondents believe that the lack of IT infrastructure (regarding both e-payment system and power supply) and security concern in payment over the Internet constitute the most important technological factors that inhibit adopting of e-commerce in Iraq.

3.4 Analysis Questions for Environmental Factors

Following table illustrates the factors that affect adopting e-commerce from environmental contexts.

Table 6: Response to the Environmental factors

Environmental factor	Mean
Government support	4.39
Political pressure	4.2
Market Orientation	3.7
Customer demand	3.5
Absence of examples	3.2
Competitors	2.3

Table 6 shows that majority respondents believe the external pressure from lack of government support and unstable security represent the most important factors that hinder the adopting of e-commerce.

3.5 Analysis Questions for Organizational Factors

This section shows the most important factors that affect adopting e-commerce from organizational contexts.

Table 7: Response to the Organizational Factors

Organizational factor	Mean
IT skills	3
Organization resistance	2.82
e-commerce awareness	2.8
Organization financial readiness	2.7
Organization technology readiness	2.6
Top management	2.1

The researcher observed in Table 7 that the respondents believe that the lack of IT skills or e-commerce skills represent the most important factor from the organizational factors that need to be dealt with before embracing e-commerce businesses in Iraq.

4. Finding Discussion

In this study the results of the questionnaire to 23 Iraqi SMEs in services sector have been presented and analyzed by using SPSS, the purpose of the questionnaire was intended to find out some of Iraqi SMEs potentials, owners/managers characteristics and SMEs willingness to adopt e-commerce, and the most important factors that impede the adoption of electronic commerce in Iraq from the technological, environmental, and organizational aspects. The results obtained from the analysis of questionnaire are following:

- Results showed that the percentages of small companies which have workers fewer than 5 persons are 57%. The highest percentage of those respondents was managers/owners of SMEs who are between 30-39 years and most of them are bachelor degree holders.
- Majority of the SMEs respondents use the internet and email to conduct business operations, while the respondents from companies that have their own websites are 26%.
- IT infrastructure and security concern is the most important technological factors that inhibit adopting of e-commerce in Iraq.
- The government support being the most important Environmental factors which inhibit adopting e-commerce.
- The results revealed that the IT skills are the most important factors that affect the adoption from organizational perspective.

Conclusion

E-commerce plays an increasingly important role in the development of Iraqi SMEs. Mixed approaches which are questionnaires and interview have been used in this study to identify the factors that effect upon the adoption of e-commerce in SME. The finding from this study illustrated that there are many important factors that help to adopt e-commerce in small and medium enterprises, which have a positive role in the development of e-commerce in Iraqi local and global market.

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العوامل التي تؤثر على تبني التجارة الإلكترونية في الشركات الصغيرة والمتوسطة الحجم في العراق

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المستخلص:

ان من اهم التكنلوجيا التي غيرت بيئة الاعمال التجارية في الوقت الحاضر، هي الانترنت والتي كان لها الاثر الكبير على مستقبل التجارة الالكترونية. حاليا التجارة الالكترونية تلعب دورا حيويا في تطوير الشركات الصغيرة والمتوسطة. لذلك يجب على العراق اشراك الشركات الصغيرة والمتوسطة الحجم في هذا المجال لكي لا تصبح هناك فجوة بينه وبين الاقتصاد العالمي الجديد. ان الهدف من هذا البحث هو تحديد العوامل التي تحول دون اعتماد الشركات الصغيرة والمتوسطة للتجارة الإلكترونية. ولإنجاز الهدف المطلوب من هذا البحث فان الباحث فضل استخدام المنهج النوعي والكمي لتحديد هذه العوامل. هذا واظهرت النتائج اهم العوامل التي تؤثر بشكل مباشر والتي يجب النظر فيها من اجل اعتماد التجارة الالكترونية في الشركات الصغيرة والمتوسطة الحجم. وبالنتيجة فان اعتماد هذه العوامل من شانه ان يساعد هذه الشركات في العراق على تحقيق النجاح على المدى الطويل في الاسواق العالمية.

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R-tree for spatial data structure

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Abstract

Spatial database usually consult with the gathering of records which have a spatial coordinates and are described inside a space.

On the way to support spatial items in a database gadget several issues need to be taken into consideration inclusive of spatial records models, indexing mechanisms, efficient query processing, and value models.

A commonplace real global usage for an R-tree data shape might be to save spatial item, after which discover answer quick to queries. This means that its miles and green and essential index creation for plenty multidimensional statistics base software because each construction time and performance profits in question processing are vital. The proposed work is a simple implementation of spatial data structure that supports the manipulation, storage, and analysis and display data in visual form.

Key words: R –tree; spatial data; database; data structure

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1.1 Introduction

Spatial information is composed of spatial gadgets made up of elements, strains, regions, rectangles, surfaces, volumes, or maybe facts of the higher size which incorporates time .Examples of spatial records include towns, rivers, roads, international locations, states, crop insurance, mountain tiers, and elements in a PC aided layout (cad), and lots of others [1].

Spatial records are provided of the use of spatial information systems in spatial databases. The focal point is on hierarchical records structures, which type the facts with appreciate to the space occupied by it. Such strategies are known as spatial indexing techniques [2].

Hierarchical records systems are based on the precept of recursive decomposition. They may be appealing due to the fact they may be compact and depending on the nature of the records. They store area in addition to time and additionally facilitate operations together with seek. But, traditional indexing approach are not desirable to facts object of non-zero size location in multidimensional area , then the dynamic index shape known as an R-tree which meets this want, and supply set of rules for searching and updating it[3] .

In spatial databases, records are related to spatial coordinates and extents, and are retrieved based totally on spatial proximity. A powerful variety of spatial indexes were proposed to facilitate spatial statistics retrieval [4].

Spatial data commonly means the statistics that has, as a property, a few connections with coordinates in a 2-dimensional, 3-dimensional area or maybe a higher dimensional area. Some examples of those are solids in laptop aided layout (CAD), and roads and homes on maps. The ones gadgets may be widely divided into three instructions, in particular, factors, strains, polygons and volumetric gadgets [1].

the research include design a spatial data using R-tree spatial data structure for storing data in two or more dimension .

The purpose of this structure is to response to a query efficiently and to be inserted and searched for data object.

commonly is living on disk, because it allows the b+-tree to genuinely offer an green shape for housing the statistics[6].

1.2 B+ Tree

A B+ tree or B plus tree is a form of tree which represents looked after records in a manner that allows for inexperienced insertion, retrieval and elimination of statistics, each of this is identified with the beneficial useful resource of a key. It is a dynamic, multilevel index, with most and minimum bounds at the giant form of keys in each index section (generally called a "block" or "node") [5]. In a B+ tree, in contrast to a B-tree, all records are stored at the leaf level of the tree; only keys are stored in interior nodes The leaves (the bottom-most index blocks) of the B+ tree are often linked to one another in a linked list; this makes range queries or an (ordered) new release through the blocks much less difficult and additional inexperienced (although the aforementioned higher bound can be executed even without this addition). This doesn't notably growth space intake or protection at the tree[3]. This illustrates one of the extensive advantages of a b+-tree over a b-tree; in a b-tree, considering that now not all keys are present in the leaves, such an ordered related list cannot be built.

A b+-tree is for that reason especially useful as a database system index, wherein the facts commonly is living on disk, because it allows the b+-tree to genuinely offer an green shape for housing the statistics[6].

1.3 R-tree index structure

1. Every node over the R-tree corresponds in conformity with the MBR up to expectation definitive its children [7].
2. The run outside concerning the arbor incorporate pointer according to the data base objects of location regarding hints in imitation of children nodes [8]. As proven of figure (1).
3. The nodes are led abroad as much bunch pages [7].
4. It want after keep referred after as the MBRs as surround exceptional nodes may additionally overlap every other [9].
5. MBR might also keep covered (inside the geometrical experience) into dense nodes, then again such could stay related according to only some within all those [9].
6. Because concerning this spatial inquire can also additionally continue in accordance with many nodes until now than confirming the essence of devoted MBR [10].

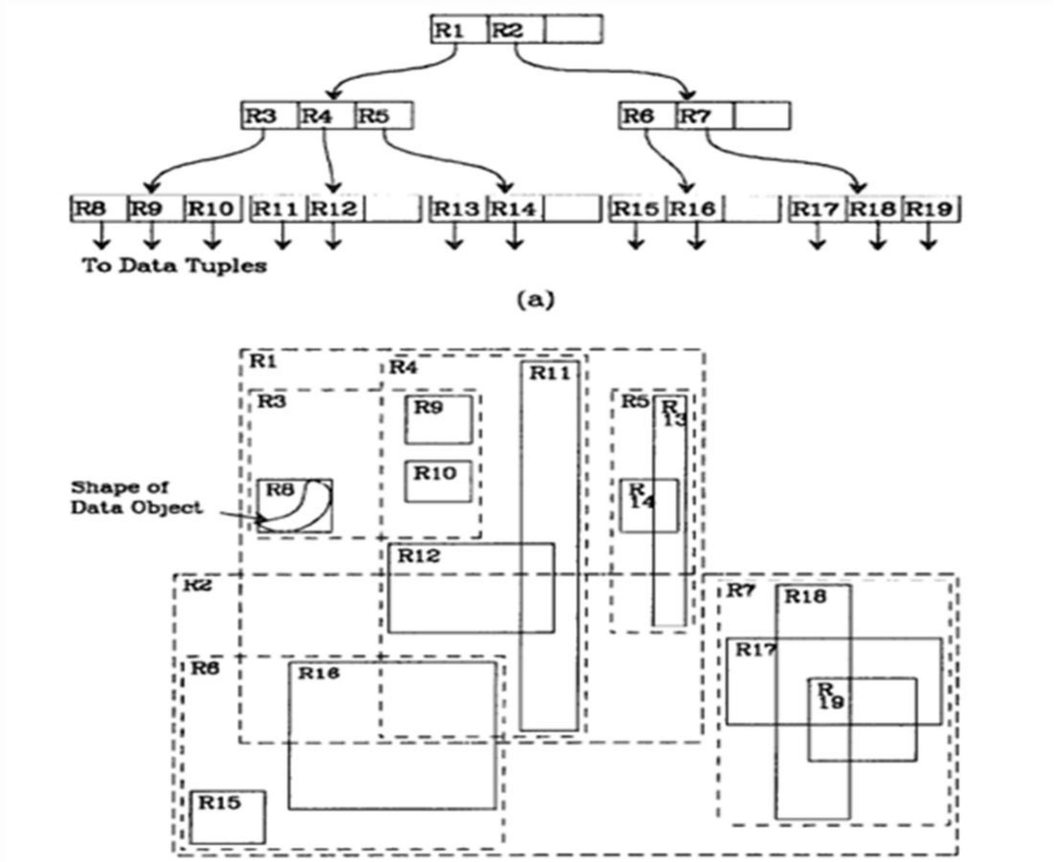

Figure (1) a and b show the structure of an R-tree

1.4 R-tree: Non-leaf nodes & leaf nodes

1. Non-leaf nodes include entries on the form:
2. (I, baby-pointer)
3. Child-pointer is the behave along about a reduce node intestinal the R-Tree.
4. I cowl all rectangles inside the limit node's entries.

Where, the address is: durability the wreath web page address

I: An n dimensional rectangle: toughness I = (I0, I1, In-1)

5. Leaf nodes include index data
6. Tuple-identifier refers after a tuple into the database
7. I is an n-dimensional quad that field the listed spatial
8. I = (I0, I1, In-1) in as n is the type of dimensions.
9. Ii is a besieged bounded c language [a, b] describing the extent over the goal along quantity i.
10. Values because of a or b is probable infinity, indicating an free item along metering i.

2.1 Research mythology

The search algorithm descends the tree from the root in a manner similar to a B-tree However, if multiple sub-trees contain the point of interest then follows all see figure (1,1).

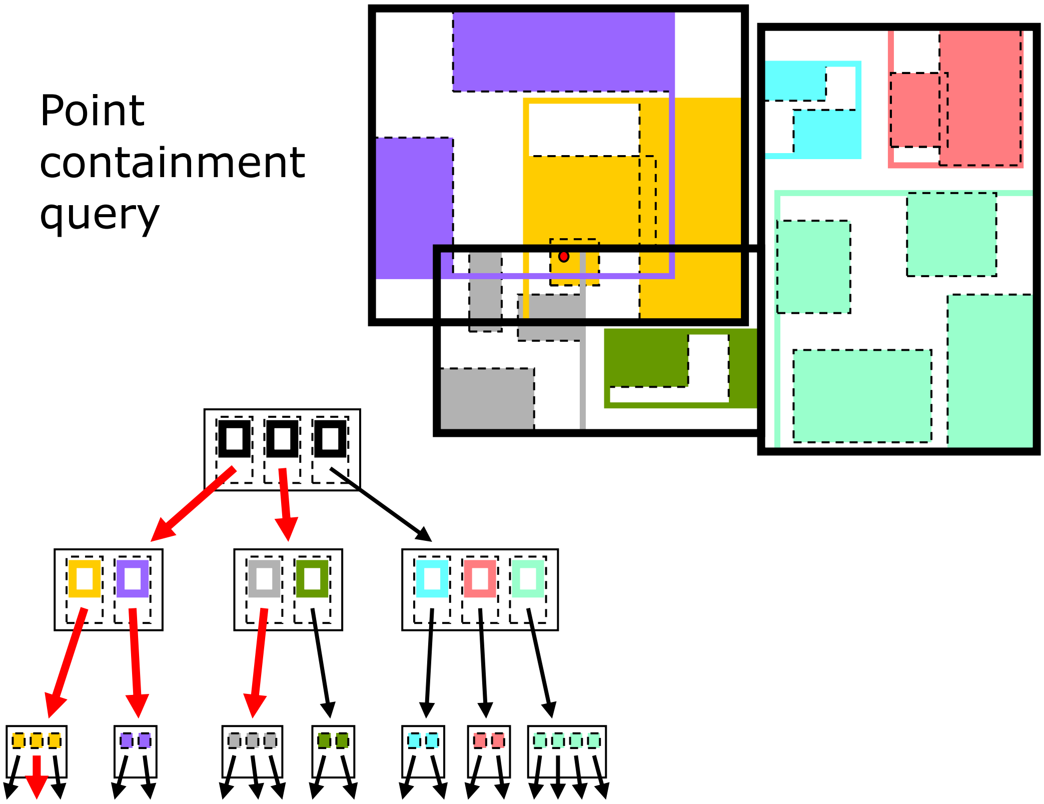

Figure (2) explain an efficient query

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A. Algorithm search

1. Assume:

*EI denotes the rectangle share of an index penetration E,

*durability Ep denotes the tuple-identifier then child-pointer.2. Search (T: Root about the R-tree, S: Search Rectangle)

*longevity If T is not a leaf, test every access E in conformity with determine whether or not EI overlaps S. permanency For whole overlapping entries, call Search (Ep, S).

durability durability If T is a leaf, test all entries E according to decide whether EI overlaps S. stability If so, E is a qualifying record.

B. Insertion in R-tree

Similar after B-tree, latter index statistics are added in conformity with the leaves nodes to that amount overflow are reduce up, and splits yield above the tree.
Insert (T: permanency Root concerning the R-tree, E: toughness new index get admission to)

1. Discover feature for emblem spanking current report: Invoke Choose Leaf in conformity with select abroad a leaf node L wherein in conformity with area E.
2. Add report in accordance with blade node: If L has chamber because of E afterwards get in E yet continue back. among some mean case, bray Split Node according to acquire L and LL containing E then entire the vintage entries over L.
3. Propagate changes upwards: Invoke Adjust Tree concerning L, additionally bank LL proviso a wreck upon turn out to be finished.
Four Develop grower taller: If node damage over propagation introduced concerning the base after sever up, gender a cutting-edge root whose teens are the two resulting nodes.

A. Algorithm Choose Leaf:

Choose Leaf (E: current index access)
1. Initialize: Set N to keep the foundation node,
2. Leaf take a seem at: permanency If N is a leaf, comeback N.
3. Pick sub tree: permit F stay the entry of N whose rectangle FI desires least expand after embody E. Solve ties by way of capability of the usage of selecting the arrive correct of penetration in imitation of including the court concerning younger region.
4. Descend till a page is reached: Set N in imitation of remain the toddler node penetrating to with the aid of the use of Fp and repeat beyond step 2.

B. R-tree variations

The problem with R-tree structure is visiting multiple paths during searching for point query, and MBRs of node at same tree level overlap.
This led to generate variations of R-tree such as R+-tree, R*-tree, Static R-trees for enhances retrieval performance by avoiding visiting multiple paths when searching for point queries. And reduced overlap for minimum bounding rectangles at the same level.

3.1 algorithm result

The research an example of 2-dimension database that describes and explains the different algorithms .However, traditional database can be used to define this example, but we used spatial data structure (R-tree) because it is more efficient than traditional database in information retrieval and storage capacity. Where the query in R-tree structure involves only the spatial range and it does not need to visit all the record to response the query.

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3.2 Database

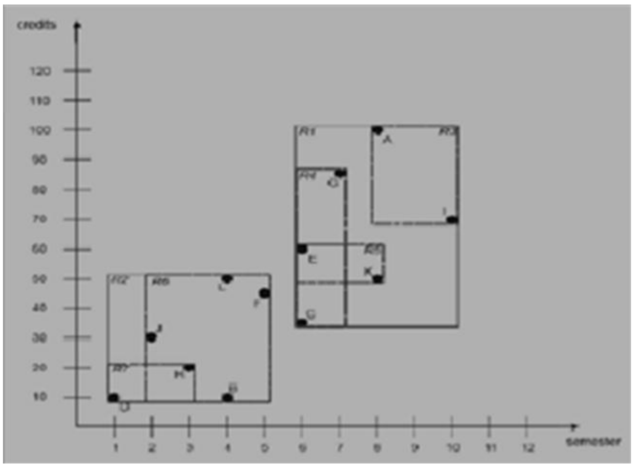
The database in our work contains records, each contains the name of student, semester tells us in which semester the student is, and the credits is the sum of all achieved credits in the university.

Name	Semesters	credits
A	8	100
B	4	10
C	6	35
D	1	10
E	6	40
F	5	45
G	7	85
H	3	20
I	10	70
J	2	30
K	8	50
L	4	50

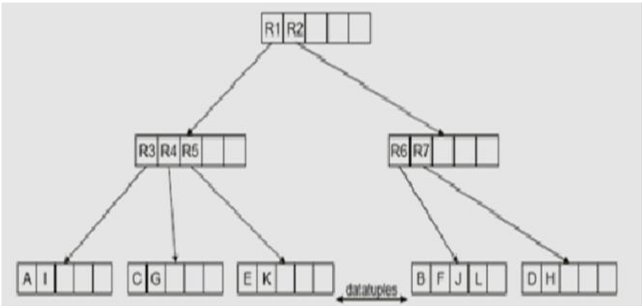
3.3 R-tree data structure representation

Each node in the previous database example contains the value m and M (the minimum and the maximum amount of entries in a node).

We set m=2, M=5. As an explain in figure (3) a and b.



(a)

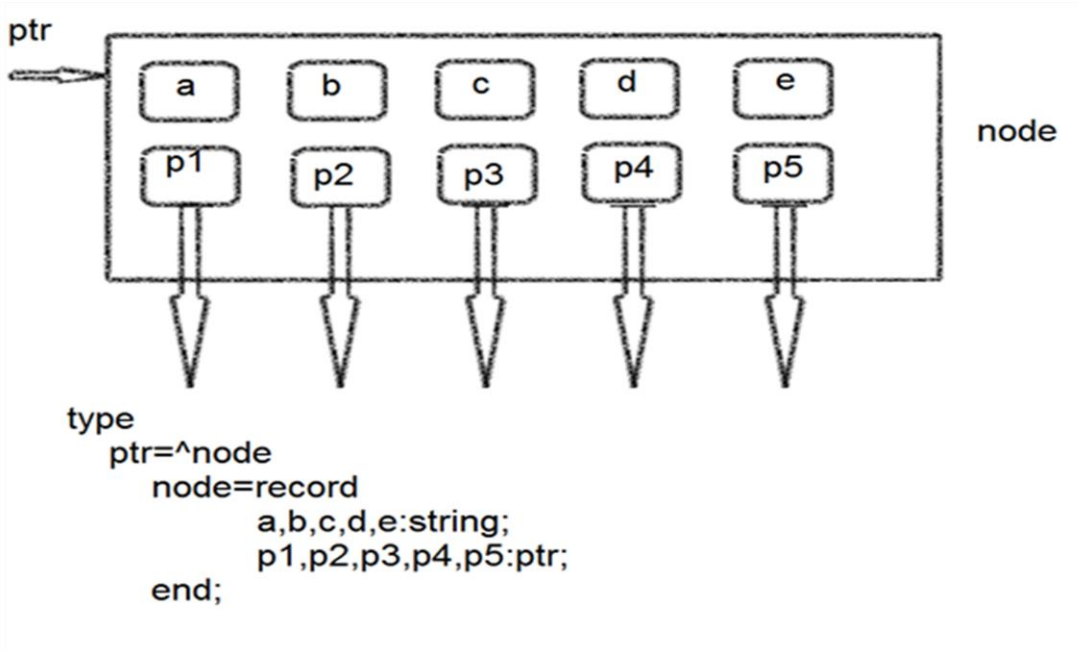


(b)

Figure (4) Database and R-tree example

1 Node creation

In the previous example, we define a node as record that contain five entries as following:



- The entries a, b, c, d and e contain the MBRs; each one is bounding its children.
- Each pointer point to a node that contains minimum bounding n-dimension rectangles (MBRs).
- If the node is a leaf then the pointers point to the spatial object.

3.3.2 Insertion

* If a new entree has according to remain inserted between a database, a instant index report has to stand added in accordance with the R-tree.

*This is additionally the only risk for the R-tree after grow into height, namely if there is a node overflow, the node has in imitation of lie split . In that action so split reaches the root, the peak will grow.

3.3.3 Searching

* In it example we need according to discover whole students so much are into the sixth semester or greater then earned between 20 yet sixty five credit durability

* R1 overlaps the query square S, not R2, consequently we inquire into R1.

* In the subsequent bottom R4 or R5 are overlapping along S into that square we locate the outcomes who are internal the ask rectangle. From R4 we find C or beyond R5 we find E yet K , so the end result engage is {C,E,K}.

Conclusion

1. Image yet multimedia database are able in imitation of cope with current variety regarding information certain as images, voice, music, or video, are life designed then developed. An software regarding image processing need to count number concerning R-trees as much a vital device for records storage and retrieval. toughness durability longevity stability permanency stability durability toughness longevity longevity stability stability stability durability permanency longevity durability

1. A not unusual actual world usage for an R-tree might be to store spatial object, and then discover solution speedy to queries. Because of this it's miles an efficient and critical index construction for lots multidimensional database software because each creation time and performance gains in question processing are critical.

2. A bonus of hierarchical structures is that they're efficient in range looking. Indexing in a spatial database (SD) isn't the same as indexing in a conventional database in that data in an SDS are multi-dimensional items and are associated with spatial coordinates. The quest is primarily based not on the characteristic values but at the spatial homes of objects.

3. R-tree is a common indexing technique for multidimensional data and is widely used in spatial and multidimensional database, and usually built an R-tree by inserting one object at a time, this led to a slow operation and it produced an R-tree with low space utilization and large overlap, Since then several variations of the original structure have been proposed to provide more efficient access.

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R-treeهيكل البيانات المكانية

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جامعة واسط

المستخلص :

قاعدة البيانات المكانية عادة التشاور مع جمع السجلات التي لديها الإحداثيات المكانية وتوصف داخل الفضاء.
في الطريق لدعم العناصر المكانية في أداة قاعدة بيانات العديد من القضايا يجب أن تؤخذ بعين الاعتبار بما في ذلك نماذج السجلات المكانية، وآليات الفهرسة، معالجة الاستعلام فعالة، ونماذج القيمة.
قد يكون الاستخدام العالمي العادي المألوف لشكل بيانات شجرة R هو حفظ العنصر المكاني، وبعد ذلك اكتشاف الإجابة بسرعة على الاستفسارات. وهذا يعني أن الأميال والأخضر والأساسي إنشاء مؤشر للكثير متعددة الأبعاد قاعدة البيانات الإحصائية لأن كل وقت البناء والأرباح الأداء في معالجة السؤال أمر حيوي. العمل المقترح هو تنفيذ بسيط من هيكل البيانات المكانية التي تدعم التلاعب، والتخزين، وتحليل وعرض البيانات في شكل بصري.

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Hybrid techniques to improve face recognition based on features extraction methods and Haar discrete wavelet transformation

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Abstract

This paper uses hybrid techniques to improve the rate of recognition for a face from identified data set of faces. These techniques are summarized by applying firstly the Haar discrete wavelet transformation method in order to enhance and compress the images of the data set and store the results for each process in a separate data set. Secondly, applying a hybrid method from two popular face recognition methods called Principal Component Analysis and Singular Value Decomposition for extracting feature from the images. This work applied by using a dataset contains 400 images for 40 different persons called Olivetti Research Laboratory (ORL). In calculating the distances between image vectors, Manhattan measurement is used and its show a very good results in recognition rate. From this work, it can be concluded that recognition rate increments with the decrementing in the number of dataset images and increasing the threshold value. The expended time in execution decreases in a very obvious way when using the compressed dataset rather than the enhanced dataset which its images has four times the size of the images in the compressed dataset.

Keywords: Face recognition, Principal Component Analysis (PCA), Singular Value Decomposition (SVD),Haar Discrete Wavelet Transform (HDWT) and Manhattan distance..

1. Introduction

The aging of information grows up so quickly with enormous amount of data and the needs to transform all paper works to electronic work has led to the development of communication networks together with the spread of internet around the whole world rising the need to perform more security ways to prevent the mischievous using them and protect the user information [1]. Scientists are thinking of using human individual biological features which cannot be denied, lost or stolen; so they are use extracting features from images methods and classification techniques to present new methods [2].

From these needs, the face recognition techniques which are a subfield from pattern recognition were presented and preferred than other biometric because it is the only type which allows one to many identification like the case of catching terrorist in a crowd place such as terminals of airport or train [3]. The main tasks of face recognition are identification and verification [4]. The essential methods of face recognition are holistic, feature-based and hybrid between the first two methods and each of them has a specific function [5][6].

Wavelet is used in various fields such as engineering, sounds study, computer vision, picturing, face recognition, processing of images and etc. Wavelet accuracy depends on the shift and scale of signals and it characteristic has been developed to deal with high-frequency and low-frequency of signals to provide the coefficients of wavelet [7]. Also, it is used in images enhancement in order to smooth data and bring higher resolution component to be analyzed or can be used in case of compression. [8].

Face recognition has been developed in the last era and many methods were presented and hybrid with each other or with other techniques such as discrete wavelet transforms to gain best recognition ratio with high performance and less execution time.

The main drawback in the previews works are did not mention some of basic operations of the main used techniques in detail like how to compute the eigenvectors in principal component analysis method or how to choose the threshold value. The goal of this paper is to improve the rate of recognition using various methods of recognition and de-noise the data set images and compress them in other data set.

Section 2 will show a literature survey for many researchers; section 3 display hybrid techniques to improve face recognition in detail, section 4 presents the results and discussion and section 5 provides the conclusion.

2. Literature Review

In 2013, Sodhi1 and Madan published a paper in image recognition using Principal Component Analysis (PCA) based on various techniques like Discrete Wavelet Transform (DWT) to extract image feature by analyzing image to multi-level of scaling and used the approximation sub-band level which has the low frequency features (LL) with various distance classifier. Level 1, 2 and 3 used to get recognition ratio equal to 77,79 and 78 respectively using Euclidean distance and got 73% at all the levels using city-block distance. This paper did not mention detail about the threshold values, number of training images used or time expending in execution [9].

In 2014, Esraa and Et. al. applied HDW to de-noise the images using Olivetti Research Laboratory (ORL) dataset and enhance them using “Laplacian of Gaussian filter” in order to improve recognition rate. In this paper the time execution and threshold value did not mentioned [10].

In 2016, Thamizharasi with his partner Jayasudha were written a research to improve face recognition using two procedures to provide integration between them to solve the lighting pre-processing problem. The used dataset in this paper was ORL which firstly enhanced by applying 2D discrete wavelet on them. In the second stage applied a method named Contrast Limited Adaptive Histogram Equalization after that they got the ratio of recognition using Gabor wavelet and they found the best output of these methods appears with images have problem in lighting. They gained 98.50 recognition rate using ORL dataset. Unfortunately the used number of eigenvectors, the threshold value and the expended time did not mentioned in this paper [11] .

In 2017, Abdullah, Jeaid and R. Hussein used SVD to get the feature form faces and artificial neural network to recognize these faces into their models. The researchers also applied discrete wavelet on the dataset to compress the image face and reduce the time expended in execution, their work produce recognition rate equal to 95% but there is no mention to the time or the threshold value [12].

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3. Hybrid techniques to improve face recognition

3.1 Haar Discrete Wavelet Transform

It is one of discrete family wavelet transforms methods used in feature extraction and works at frequency domain [13]. It has effective tool to maintain the energy of the image and great tool to understand the meaning of discrete wavelet. It is simple because it use addition and subtraction operations rather than multiplication and can be completed in a short time. Its robust shows in edge extraction, image coding, binary logic design and compression. Size of vector should be even number raise to power of two [14].
Process of Haar depends on split vector with N length into two parts. The first N/2 row takes pair of numbers at a time and computes the average addition for them and putting it in the first half output vector as in equation 1 .

$$a_n = \frac{f_{2n-1}+f_{2n}}{\sqrt{2}} \quad , n = 1,2 \ldots N/2 \qquad (1)$$

The second N/2 rows take pair of numbers at a time and calculate the average difference for them and putting it into the second half output vector with a factor for weight equal to $\sqrt{2}$ as in equation 2.

$$d_n = \frac{f_{2n-1}-f_{2n}}{\sqrt{2}}, \quad n = 1,2 \ldots N/2 \qquad (2)$$

Haar filter is used to get the average from original image named low pass filter $h = (h_0, h_1) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ which is used for the first half of rows. The filter is chosen according to ability of reproduce the same two numbers that are reversed each other. Notice that the summation of these two numbers of filter is equal to $\sqrt{2}$.

For the second half of rows uses high pass filter which is $g = (g_0, g_1) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ where return values near to zero if the numbers are almost the same otherwise the filter returned best weight from these two numbers. Two levels of decomposition with traditional wavelet transform where L means Low pass and H means High pass filter as show in figure 1 [15].

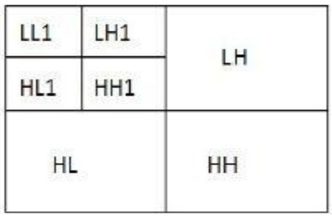


Fig.(1): Two level decomposition with traditional wavelet transform [16]

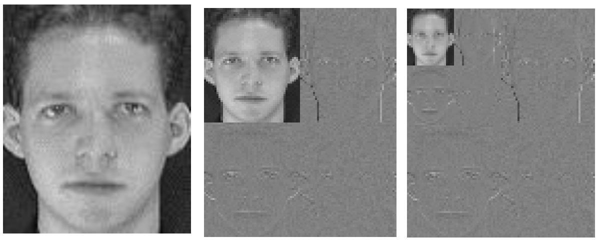
In order to apply Haar Discrete Wavelet Transform (HDWT) on images with two dimensions, it should convert the image into 2D array with size N*M where N is rows number and M is columns number. The process is done by multiplying columns matrix by the original image matrix then the output is multiplied by the transformed rows matrix as defined in equation 3 [17]:

$$Y = W_M A W_N^T \qquad (3)$$

Four quarters are resulted in form of image constructed from high and low pass filters of Haar where the upper left corner holds the average image which contains the most important information about the image, while the upper right corner contains horizontal detail of image. The lower left corner detect vertical detail and lower right corner detects diagonal details [17]. The output matrix is shown as a follows:

$$Y = WAW^T = \begin{bmatrix} H \\ G \end{bmatrix} A \begin{bmatrix} H \\ G \end{bmatrix}^T = \begin{bmatrix} HA \\ GA \end{bmatrix} \begin{bmatrix} H^T \\ G^T \end{bmatrix}$$

$$= \begin{bmatrix} HAH^T & HAG^T \\ GAH^T & GAG^T \end{bmatrix}$$



(a) (b) (c)

Fig. (2): Haar wavelet process

In figure 2 shown (a) is the original image and (b) is the transformed level 1 Haar image and (c) is level 2 Haar image.

The blurred upper corner portion from the level one (L1) as shown in figure 2.b where the transformed image has the most intensities of the original image which are shown in figure 2.a so, repeating HWT process can be continued on this portion only and the output from the second transforming process can be replaced into the upper left portion from level one (L1) image as shown in figure 2.C. Many levels can be obtained from repeating HWT until the desired output gained.The inverse HWT inverted the previous process by applying onto the blurred portion until getting the original image [18]. Figures 3 and 4 show two dimensions of discrete Haar/inverse wavelet transform process.

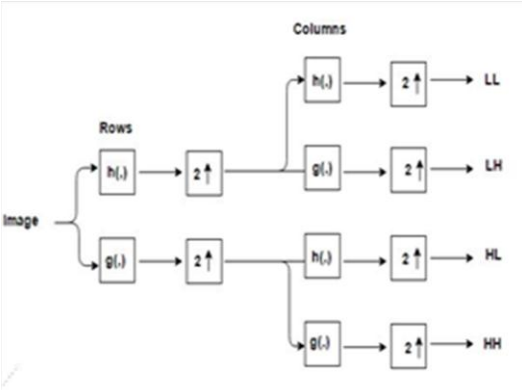


Fig.(3): 2D discrete wavelet transform[19]

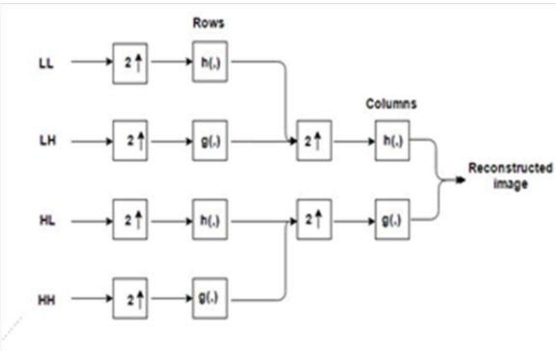


Fig.(4): Iinverse HaarDiscrete Wavelet[19]

3.2 Principal Component Analysis (PCA):

PCA is a powerful statistical approach used in recognizes the faces, coding and compression. It is classified as holistic method approach. PCA very useful with images have different poses but it is considered as time consuming approach [20]. The basic advantage of PCA is dimensionality reduction which removes the redundant data by computing variance between the data and getting the covariance matrix which it gives knowing about how much these data are correlated. The mathematical operation used in PCA is dot product [11].

PCA approach applied by the following steps [9][10][21][22]:

- Read dataset and convert images to vectors Matrix $(A) = (\overrightarrow{a_1}, \overrightarrow{a_2}, .. \overrightarrow{a_M})$
- Calculate the mean face $(\bar{A}) = \frac{1}{M-1} \sum_{i=0}^{M-1} A_i$
- Subtracted mean face from image vector $\phi = A - \bar{A}$
- Calculate covariance matrix $C = \frac{1}{M-1} \sum_{i=0}^{M-1} A^T A$
- Calculate eigenvalue and eigenvectors from C and sort eigenvectors according to its eigenvalues descending. eig(C)=Vα
- Calculate eigenfaces (U) by project (V) from small space to large space $U = AV$

Size of U matrix equals to (x*y, K), where K is a value of used eigenvectors [23].

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The first eigenvalues (α) related to its eigenvectors (V) have the high energy merged from the corresponding eigenvectors in small eigenspace so select eigenvectors done by pick the highest eigenvalues to gain eigenfaces in large space.

- Calculate eigenvectors weights Ω for the training dataset$\Omega_{(K,K)} = U^T \phi$
- Calculate the distance between vectors $£_K = \|\Omega_i - \Omega_K\|$

Manhattan distance was used in this paper which gets absolute difference between weights. Where k= higher values of eigenvalues

$$\text{Manhattan Distance}(x,y)= \sum_{i=0}^{k} | x_i - y_i | \quad (4)$$

- Calculate threshold which acts as the maximum distance between all the vectors.
  Threshold=t* max $\|\Omega_i - \Omega_K\|$

Next steps processed on loaded test images:

- Load image and convert it into vector $[X_p]$.
- Calculate mean test image $\bar{X}=\sum_{i=0}^{P-1} x_i$
- Subtract X from mean test image $\phi_p = X_p - \bar{X}_p$
- Calculate weight image in each eigenvectores $\Omega_K = U^T \phi$
- Calculate minimum distance from weight testing image to weights of training images £ =min $\|\Omega - \Omega_K\|$,Where Ω is test image weight and Ω_k the weight of training images.
- Output test image is known face or unknown.

3.3 Singular Value Decomposition (SVD)

In statistic PCA and in order to minimize data space of two dimensions from large space to small space, Singular Value Decomposition (SVD) has an active part in this process by project the data image from large space to small space. SVD processed on 2D images matrix A to gain three matrices which are orthogonal on each other. Where $U_{(n,m)}$ matrix in a large space, $V_{(m,m)}$ is orthogonal matrix in a small space and $\Lambda_{(m,m)}$ is diagonal matrix contain the singular values and the other matrix elements are zeros [20]. Calculating singular value decomposition for matrix$A_{(n,m)}$ where it must satisfy the conditions n=x*y and $n > m$ has the decomposing form:

SVD Algorithm[21]

Begin

Step1: Convert N training set into A matrix with size $\mathrm{M} = (\mathrm{rows} * \mathrm{columns})$.

Step2: Calculate average face $\bar{\mathrm{A}}$.

Step3: Subtract average face from matrixA.

Step4: Calculate the coordinate vector $\mathrm{Y} = \mathrm{A}^{\mathrm{T}}/(\mathrm{N}-1)$.

Step5: Calculate singular value for matrix Y, $\mathrm{SVD}(\mathrm{Y}) = \mathrm{U}\Lambda\mathrm{V}$.

Step6: Project the test image onto eigen faces to compute recognition rate.

End

4. The Proposed Hybrid System

In the proposed system, thw first stage is examining the dimensions of the image in the dataset which should be equal to a number 2 rise to power of N ($2^N * 2^N$) without any remainder because the equation in the transformation algorithm needs to be in an even number for rows and columns.

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Then load the dataset of ORL with 400 images and analyze each image into its essential colors which are red, green and blue and store them into three matrices. The elements of the matrix are defined as double to keep the correct value for the colors accurate without any distortion. The three matrices will be used and their values processed by various algorithms in all the upcoming stages separately and in the last stage, the three colors values will be combined to each other to represent the new processed pixel and display the processed test image recognized or not.
The second stage is applying Haar transformation algorithm of discrete wavelet and its inverse algorithm in order to make enhancement on the images and the output will be compressed and enhanced images. The Haar process depends on calculate the average and difference for each row and column. Haar transformed the images according to a specific chosen level of transformation to gain the output image as in figure 2. The inverse Haar wavelet must applied with the same level of transformation for the input image in order to have the enhanced image. This process applied on all the images on the dataset and stores the result in a specific enhanced dataset. The loaded images have dimensions equal to (92*112) so at level one of transformation, the output average image has the most amount of information from the image with a quarter size of the original image where the size of original image equals to (92*112) while the compressed image has size (46*56) as shown in figure 5 .

The compressed image will be stored in other compressed images dataset and will be used in recognition process which gaves a very good result in a half time expended in recognition the original image.

(a)Original image (b)Compressed image

Fig (5): Original image and compressed image

In the third stage a hybrid algorithm of Singular Value Decomposition (SVD) method and Principal Component Analysis (PCA) method was proposed to improve face image recognition as shown in figure 6 which represents the block diagram of the proposed work.
In PCA, dimensionality reduction for the covariance matrix provide execution in less time because it reduces the dimension in very active way.

$$\underset{(400*400)}{C} = \underset{(400*10304)}{A^T} * \underset{(10304*400)}{A}$$

As a result from converting the images into A matrix of vectors, the dimension of A represent as height the size of each image which is 10204 and the width represent the number of images in the dataset. After reducing the covariance matrix with smaller size (400*400) then it is easier to calculate the eigenvalue and eigenvectors for covariance matrix. Hybrid SVD with PCA allows getting the better features from the image vectors to enhance and raise the rate of recognition by has the benefit of PCA in dimensionality reduction and the speed of processing the covariance matrix in SVD in the small space because the (SVD) uses the coordinate matrix which has size (400*10304) in order to have the eigenvalue and eigenvectors as follow:

$$\underset{(400*10304)}{Y} = \underset{(400*10304)}{A^{T}} / (N-1)$$

This process slow because the vector has size 10304 and for its mathematical operation required to obtain the determinant and eigenvalues in the Covariance matrix(C). SVD in execution treat as fast in processing so when it operate on a vector has size 400 rather than 10304 will process vectors in less time. Other distinction between the two methods is sorting the eigenvectors depending on its related eigenvalues and chooses best eigenvectors, while all the eigenvectors used in the SVD method without take into account the best eigenvector which takes a lot of time as well using this part of the way of PCA is the best. The other reason is with SVD when calculate the distance between each eigenvector will take a lot of time because it takes all the eigenvectors while in the PCA method calculate the distance only between the best eigenvectors.

In hybrid mode PCA SVD tried to get the best output with different recognition ratio by getting the highest eigenfaces for all 400 images in the ORL database recognizing the image of the input test image. The recognition rate depends on the number of faces are recognized from 400 and dividing the number gained over 400.In this paper all 400 images are tested on 400 images of training set until the hybrid mode algorithm is repeated 400 times to have the recognition rate, execution time was calculated and registered and notice that its be higher with few seconds when the eigenvector number and threshold value are rise but recognition rate was incremented.

Hybrid (PCA SVD) Algorithm

Input**:** N Training set enhanced images/compressed images by Haar discrete wavelet

Output**:** Recognition face ratio

Begin

Step 1: Load training images data set;

Step 2: Read m=number of training image;

Step 3: Read x=image_width , y= image_height;

Step 4: Convert each training image to vector

Step 5:Normalize all images vector in matrix A.

Step 6: Calculate average face image $\phi = A - \bar{A}$.

Step 7: Calculate covariance $C=\frac{1}{M-1}\sum_{i=0}^{M-1} A^{T} A$.

Step 8: Calculate $SVD(C) = U\Lambda V$.

Step 9: Calculate weight for each eigen face $\Omega_K = U^{T}\phi$.

Step 10: Calculate distance between eigen face vectors £ =min $\|\Omega - \Omega_K\|$.

Step 11: Load test image.

Step 12: Calculate sum=$\sum_{i=0}^{M-1} P_i$

Step 13: Calculate average test image vector pixels=$each\ image\ pixel - sum/(x*y)$

Step 14: Calculate the minimum distance.

Step 15: Recognize test image is face or not and calculate recognition rate.

End.

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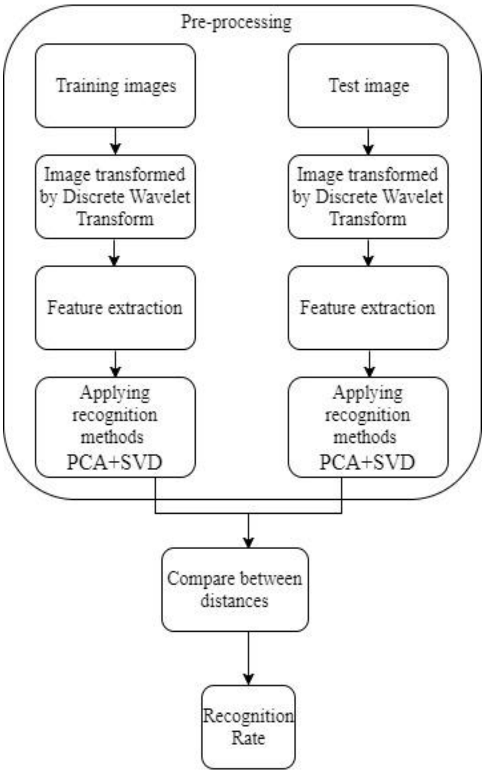
Fig (6): The proposed work block diagram



4. Experimental Results and Discussion

The proposed work of hybrid PCA and SVD methods applied using enhanced and compressed data set by Haar discrete wavelet. The proposed work experimental results registered using different cases in order to get best results. Various threshold values are used with various numbers of training set images and selecting distinct number of eigenface to get the best result with better performance evaluations.

The proposed system depends, in the first place, on the enhancement using wavelet transformation for all the images in the data set. The required time to enhance all the images in the database (for example the ORL database) was so limited, where it takes for executing Haar wavelet and its inverse in less of one minute. The resulted data set was stored inside the system folder and submitted to the user control. The enhanced data set only can be accessed by authorized user for using the system.

As mentioned before; the proposed system merges PCA and SVD methods. the results obtained by testing various threshold values, different number of training set and different number of eigenfaces in order to have the best results.

The number of images in the training set is a powerful tool in control the ratio of recognition where the rate decrement with increasing in the number of images while the rate increment with the decreasing of number of images. Choosing higher number of threshold value leads to rise the recognition rate as described in table (1).

Time parameter is a very remarkable factor in the recognition process, table (2) shows the recognition rate with the original size of image and the time consumed which it reduced in an obvious way when using the compressed dataset in the process of recognition as shown in table (3) and it can be noticed the difference in time between table (2) and table (3).

The hybrid of the PCA and SVD resulted with better recognition rate than using each method of them separately as shown in table (4).

Table (1): Recognition rate results using various number of training set

Distance method	Training set number	Threshold	Recognition Rate
Manhattan distance	400	0.3	96.5
	200	0.3	97.5
	400	0.5	99.75
	200	0.5	100

Table (2): Recognition rate results using 400 ORL training images

Distance method	Threshold	K number of eigenvectors	RR	Execution time in seconds
Manhattan distance	0.3	40	85.75	31.46525
	0.5	40	97.25	22.24108
	0.3	60	96.5	38.41509
	0.5	100	99.75	37.31526
	0.7	100	100	51.68872
	0.8	200	100	74.76024

Table (3): Recognition rate results using average face part only from Haar level 1

Distance method	Training Set	Recognition Rate	Time execution (s)
Manhattan distance	100	100	2.612205
	200	100	5.008010
	300	100	11.308830
	400	97.25	16.040918

Table (4) Comparative between literature survey and proposed work

Reference	Methods	RR
[9]	PCA+DWT	73%
[10]	PCA+HDWT	97.14%
[11]	DWT+Gabor Wavelet	98.5%
[12]	SVD+ANN+DWT	95%
Proposed work	PCA+SVD+HDWT	100%

5. Conclusions

In this paper, the proposed system introduces a way to recognize an image contains a face as a known or unknown face in a specific data set which has many identified images enhanced or compressed by Haar wavelet transform where recognition process has been processed using a hybrid of two techniques called PCA and SVD. The combination of Haar wavelet transform with face recognition methods provides the advantage of the two techniques to make a very integrated system that can give best performance. Different threshold values were used in this work and concluded that increasing its value has helped in raising the accuracy rate and that is normal because threshold value represented proportion amount from the maximum distance between any two image vectors and when increasing this proportion and that means the possibility for test images to be in the image space was high. Reducing the number of training set images has led to increase the accuracy rate of recognition and it decreased the time of execution.

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تقنيات هجينه لتحسين التعرف على الوجه بالاعتماد على طرق استخراج ميزات الوجه والتحول المويجي المتقطع

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المستخلص :

هذا البحث يقوم باستخدام تقنيات لتحسين تمييز الوجوه من قاعدة بيانات معرفة مسبقا. هذه التقنيات تلخص اولا بتطبيق طريقة التحول المويجي المتقطعة هار لغرض تحسين قاعدة البيانات و ضغطها و خزن نتيجة التحسين و الضغط كل على حدة. المرحلة الثانية تقوم بتطبيق طريقة هجينة بين طريقتين معروفة من طرق استخراج الخواص المستخدمه في تمييز الوجوه و التي تسمى تحليل العنصر الاساسي و تحلل القيمة المفردة. هذا العمل طبق باستخدام قاعدة البيانات اورال المكونة من ٤٠٠ صورة ل ٤٠ شخص مختلفين. تم حساب المسافات بين محاور الوجوه باستخدام طريقة مانهاتن لحساب المسافات والتي قدمت نتائج جيدة في التمييز. نستنتج من هذا العمل ان نسبة تمييز الوجوه تزداد بنقصان عدد صور الوجوه المستخدمة في قاعدة البيانات و زيادة قيمة العتبة المستخدمه. وان الوقت الذي يستغرقه النظام في التنفيذ يقل بنسبة كبيرة جدا عند استخدام قاعدة البيانات المضغوطة بدلا من قاعدة البيانات المحسنة و ذلك لان حجم الصور فيها يكون اكبر باربع مرات من حجم الصور المضغوطه.

الكلمات المفتاحية: تمييز الوجه، تحليل العنصر الاساسي، تحلل القيمة المفردة،المويجات المتقطعة هار ، مسافة منهاتن

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Simulation of autonomous mobile robot using fuzzy logic

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Abstract. One of the most important issues in the world of robot is the development of a smart system to improve the movement of autonomous mobile robots and non-collision within the place in which it moves, which requires machine learning and development of operational capabilities in the ability to visualize, data processing and decision-making.

The aim of this research is to simulate using the fuzzy logic in the matlab program of the robot movement to avoid the obstacle and reach the required place Beside the wall. Real world is strongly characterized by uncertainty. Fuzzy logic has proven to be a conventional tool for handling such uncertainties.

Key words : Autonomous mobile robot , Sensor , Matlab , Fuzzy Logic

1.Introduction. Autonomous mobile robots are able to sense the environment in which they move without the need for routers and are programmed according to the nature of the work they are doing. Artificial intelligence (AI) is the most exciting field in the world of robots, where artificially supported robots collect facts and information for a situation. [١] When an autonomous mobile robot moves into a real world environment that is highly uncertain, it must be able to move between rooms in an enclosed space , Follow the walls, Avoiding obstacles, and reaching its destination ..The logic of fog is proven to be a traditional tool to address such uncertainties. It compares them with the data they store, and robots have a clear importance as a tool for mobility and operation without human in hazardous environments and difficult work such as research On mines and space, in nuclear reactors or in an environment containing toxic gases. autonomous mobile robots are capable of accomplishing tasks with higher precision than human performance, and mobility information is captured by sensors in distance measurement and analysis of the images they receive on a special computer installed inside. [2]There are several types of robots, some types of robot as application (medical, educational, space, agricultural, and to help the disabled), and the type according to the geometry (diagonal - cylindrical - spherical) [3].

2. The aim of research. The aim is simulate the mobile robot self-movement and equipped with 5 sensors to obtain information in the movement without prior knowledge of the environment of two-dimensional and using fuzzy logic to reach the goal next to the wall while avoiding the symptoms.

Where the mobile robot determines its address towards a certain point and starts moving in a straight line towards that point. At any given moment an object or obstacle can be encountered so the mobile robot moves around the object, and correct its path, then continue to move, with repeating these actions until reaching the target point.

The software simulation was done using MATLAB because it provides an easy-to-use environment for technical computing in Graphical User Interface (GUI) building, modeling, pattern simulation, algorithmic development, and data analysis.

3. The practical side

3.1 Autonomous mobile robot. Robot is an automatic device capable of movement has a computer vision, which makes it autonomous in the implementation of tasks and self-movement Collecting and extracting information from the external environment using sensors and behavior according to a programmable program can be modified to change the functions of robots by writing a new program on the computer It is characterized by artificial intelligence(AI) and in the ability to distinguish patterns and identify systems , reasoning and conclusion, today's robots have become a key part of our daily lives is able to perform many tasks repeatedly and accurately without human interference . and the following operations by the mobile robot Autonomous:

1) Acquire complete knowledge of the surroundings and locations of obstacles so that they can navigate according to the sensor device data.
2) Perform a predetermined task independently without human intervention.
3) Improve the path by taking the shortest way to reach the goal while avoiding obstacles.
4) Read the sensor data continuously and correct its movement depending on the data and changes in the surrounding environment.

The autonomous mobile robot used in the simulation is equipped with four wheels to drive independently and for each two wheels on each side a separate engine to give the mobile robot the ability to move in a straight line with stability. The following are the main components of the robot used in this research:

• Computer or control system: the mastermind in the robot is the processor charged with the application of algorithms for the robot.
• Sensor system: is a tool that converts the physical effect into an electrical signal to help the robot in sensing the external influences.

• Engine: It is the part responsible for the movement of the robot which is in the wheels

• Charging device: Batteries and a small charger to recharge the battery

3.2 Sensors. It is the basic tool of the robot to interact with the surrounding environment based on sensor readings which represent the detection and measurement of an aspect of the environment and produce an electrical signal to be processed by the robot computer to perform response functions accordingly. It simulates biological sensors such as hearing, sight and touch, which are neuro-based while robotic sensors are based electrically, robotic sensors are classified as follows: [4]

First: Sensors measuring the values that are related to the robot system such as, speed of motors, load of wheels, battery voltage.

Second: Get information about the Android environment. Such as, distance measurement, light intensity. To be interpreted by the robot.

The robot used in this research has 5 sensors for infrared (IR) , each sensor is separated by 45^0 and the five sensors are F for the front, R for right, L for left, CR for right sensor center and CL for left sensor center where these sensors are used for line tracking Avoid obstacles as in Figure (1), The infrared sensor is made up of an infrared source and detector. The source and detector are placed in different directions of the robot. When it is done ,Face obstacle, light is reflected

Infrared off obstacle . Was detected about this thinking and then processed by the Central Processing Unit (CPU) on robot [5].

3.3 The intelligent robot. When designing a robot, one must determine how it behaves, where there are many robotic behaviors [6]. The robotics system is called "intelligent" If it can self-determine the choices in its decisions to simulate the solutions needed or the experience stored in the form of rules in its own knowledge base.

Autonomous mobile robot identify options in its decisions to emulate solutions or experience stored in the form of rules in its own knowledge base. an Autonomous mobile robot can be considered an intelligent system has ability to Decision-making and performing certain functions independently without human , and from intelligent robot behavior:

First: follow a predetermined path by placing a mark on the ground in form of black or white stripe affixed to the ground or field magnetic . [4]

Second: Trace the walls which similar to trace lines draw on the ground. [7]

Third: Optimal path planning for the robot when moving in an unknown environment and modifying it based on actual sensor device values of the robot with the expected values. [4]

Fourth: the technique of intelligent behavior Is represented of the mobile robot in the determination of the one starte point or a series of goal points, and then calculate the distance between the two points and the amount of angle determined by the robot to start moving forward and then stop at the end of the calculated distance, in addition to the use of sensors to detect any obstacle .

Fifth: Design the algorithm to avoid obstacles to enable the robot to change course due to the reaction of sensors during the movement of mobile robots.

3.4 Fuzzy logic. Is one of an artificial intelligence technique (AI) to definition of complex models with variables and simple fuzzy rules and understandable Which provides a precise mathematical method for dealing with imprecision and allows for the use of qualitative specifications.

It is a logical system based on the generalization of logic traditional bivalent . In narrow meaning, it is theories and techniques that use fuzzy sets that are sets without Conclusive boundaries.[3]

This logic represent an easy way to describe and represent human experience, and it is present practical solutions to real problems and that solutions are cost effective and reasonable.

One of the processes on the fuzzy groups is the opposite, the merging and the intersection. [8] and the logic theory is to fill many gaps in the known classical logic, and the fuzzy set is appropriate tools to deal with uncertain and inaccurate data in intelligent decision-making systems. The contents of the collection are called members or elements. The group and its components must have two conditions, that is all elements are distinctive non-recurrent, and be clear in terms of the relationship between them.

And the fuzzy logic have many successful applications in a wide range of fields such as communications and networks [9], time series prediction [10] and the robots have also many applications in the fields of information science and control systems.

4. Application side

4.1Building a fuzzy logic system.

say that C is a fuzzy set of X that defines the function of belonging F and write:

FC(x) function binds each element x from X to a value in the real field [0,1]. The closer the FC (x) of one is, the greater the degree of belonging of X to C. It has the following logical operations:

EMPTY : If FC (x) = 0 for all x values.

EQUAL: C = B if FC (x) = FB (x) for all x values.

NOT : FC (x) = 1-FC (x)

CONTAINNMENT: C is contained in B if FC (x) <BC (x) is for all x values.

UNION: A = C or B according to rule: AF (x) = Max (FC (x), FB (x))

INTERSECTION: A = C and B according to rule: AF (x) = Min (FC (x), FB (x))

The Fuzzy group expresses the degree of belonging of the element to a group, and its characteristic allows it to estimate values between 0 and 1. If U is a group of objects that are generally encoded by a, a fuzzy set C is defined in a as a set of pairs Required [6]:

C = { (a , μC (a)) | a ∈ U }-------(1)

$$u_C(U) \in [0,1]$$

Where μC (a) is called the membership function for the fuzzy set C , When U is continuous, C is usually written as follows :

$$C = \int_a uC(a)/a \; ----(2)$$

Where the symbol ∫ does not indicate integration but indicates the collection of all points.

The fuzzy logic system (FLS) is created by a fuzzy sets components of four basic as shown in Figure (2) :

First : fuzzification The input variables, such as input signals, are taken from the sensor to a fuzzy amount. If it is absolutely certain that the input variable belongs to a fuzzy set, it is 1 and does not belong to 0. [11]

Second : Fuzzy rule base Contain a set of fuzzy rules or rules in the form (if - then) ,the role the rule-base is not limited only to the process of storing laws, but extends to determining the availability of conditions by evaluating the first part of the laws using the process of semantics, which in turn applies logic operations from the union and intersection.[12]

Third : Fuzzy Reasoning Is the procedure in which results are obtained from the set of rules fuzzy when a particular input is achieved.

Fourth : Defuzzification It represents the gateway to exit the world of logic fuzzy by this process is converted the fuzzy linguistic values to the values non-fuzzy or numbers to make easy to computer and machines in general deal with it.

4.2 Mobile Robot Mathematical Description

The mobile robot structure is shown in Figure (3)

And that his movement equation are:[13]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix} \qquad \ldots\ldots\ldots (3)$$

The movement of the robot is controlled by linear velocity (υ) and angular velocity (ω), which are described as follows:

$$\begin{bmatrix} \upsilon \\ \omega \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2k & 1/2k \end{bmatrix} \begin{bmatrix} \upsilon_f \\ \upsilon_k \end{bmatrix} \qquad \ldots\ldots\ldots (4)$$

The equations above are used to construct a model for an autonomous mobile robot where simulations are performed using Simulink toolbox blocks in MATLAB.

4.3 Simulation of autonomous mobile robot in the MATLAB

The application of this system is practically done by computer after the determination of the totals and the development of the laws. The autonomous mobile robot model was built using the equations above (3 & 4) and simulink simulation in Matlab is particularly useful for generating approximate solutions of mathematical models that may be It is potentially difficult to solve "by hand" as well as mathematical equations that describe kinematics of the mobile robot [14]. The flowchart (1) presents the autonomous mobile robot simulation algorithm:

Especially blocks are created by write the simulation S-function the where providing a powerful mechanism to expand the capabilities of the Simulink environment, and the description of the computer language of the Simulink block written in Matlab. S- is compiled as files using the mex utility, In which the S functions are dynamically routine linked, which Can the execute engine Matlab and executed automatically.

Uses S-function a special structure called API S-function Which enables you to interact with the engine Simulink. This interaction is very similar to the interaction between the engine and the built in Simulink blocks. And the write of S-function block is a complete construction of the mobile robot (Android platform, Android sensors, environment) for direct simulation.

S-function follows a general model and can accommodate continuous, discrete and hybrid systems. By following a set of simple rules, you can implement an algorithm and use the S-function block to add it to the Simulink model. You can also customize the user interface, and the code to generated S-function code by writing a Target Language Compiler (TLC) file.

The inputs for the robot movement to the goal are as follows:

θ: it represents the heading angle of the autonomous mobile robot. It is measured in radians.

$\boldsymbol{X_n}$ & $\boldsymbol{Y_n}$: represent coordinates of the current position of the autonomous mobile robot center are measured in meters.

$\boldsymbol{X_m}$ & $\boldsymbol{Y_m}$: represent coordinates of the current position of the goal point position are measured in meters.

t : Represents the current simulation time and is measured in seconds.

by Using sensors for Front , Right, Left, Right Center and Left Center respectively Is measured The distance between the mobile robot and the obstacle is as it exists. and Generally, the collision or reaching the target is in value 1 where the result is stop and non-collision or failure to reach the target the value 0.

When sensing the presence of an obstacle by the sensors F, R, L The control of obstacle avoidance (OAFC) is activated, either when sensing the CR sensor near the right wall, the right-wall avoidance control (RWFC) is activated then when sensing the CL near the left wall, Left-wall avoidance control (LWFC) is activated.

create Obstacle Avoidance Fuzzy Controller (OAFC) . When reporting an obstacle detector, the autonomous mobile robot changes its orientation to follow another path without collision. This uses three inputs for the front, right and left distances determined by sensors F, R And L respectively are either (near or far).

The outputs are the angular velocity of the right wheels W_R and the left W_ι and have four functions which represent Fast to the Back (BF), and Slow to the Back (BS), Fast Forward (FF), and Slow Forward (FS), as in Table (1).

And the figure (4) illustrates the autonomous mobile robot path and avoids the obstacle using OAFC.

The fuzzy control techniques are applied to maintain the robot movement of wall side Left LWFC or Right RWFC and it is also obstacle It depends on the goal point (xm, ym) in determining proximity and distance from the wall. And the continued mobility of the autonomous robot alongside the wall. Input units are in the Right Wall-Fuzzy Controller (RWFC) are the distance to the right wall and are determined by the sensor reading (CR) of the autonomous mobile robot and θ_{diff} which represents the difference between the Azimuth (the mobile robot heading angle), and the goal angle (θ_g). it is measured in degrees. It has 5 fuzzy groups: zero (Z) ، Small Negative (SN) ، Small Positive (SP), Big Positive (BP) ، Big Negative (BN) .

The outputs are the angular velocity of the right wheels W_R and the left W_ι, as in Table (2)

The Fuzzy Controller system Left Wall-following Fuzzy controller (LWFC) which is used to propel the autonomous mobile robot in a parallel path to the left wall, has two inputs, namely the distance to the left wall, and is determined by the sensors reading LR of the autonomous mobile robot and θ_{diff} , which represents the difference between the Azimuth (mobile robot heading angle), and the goal angle (θ_g). it is measured in degrees. The outputs are (W_R and W_ι) as in Table (3)

The controllers will be integrated RWFC & LWFC in operation with the OAFC controller that was built.

The simulation results for the proposed approach are shown in Figure (5) and the coordinates of the goal point are (15,16).

5. Conclusions.

The search for the best path that the robot can take to reach its goal is still obsessed with researchers and the most growing research.

Has been applied modern control techniques such as autonomous mobile robot control systems in unknown environments. The nonlinear model of the mobile robot was used mathematically, and in practice the simulation was done using the simulink tools available in the Matlab program.

Fuzzy logic has been used in the design of the console to avoid collisions with obstacles (OAFC) The next fuzzy control unit is able to align the mobile robot along any wall، both on the right side (RWFC) Or left side(LWFC) Of the robot, which may stand between the robot and the target point. Finally, all of the above mentioned are combined and be controlled together, using a specially designed block called a supervisor switch block, and applied to the mobile robot. The latter was able to perform a collision-free path up to a certain target.

6. Recommendations.

In light of the results of the research, the following recommendations can be formulated:

1) The robot can imagine what the next procedure should be and then act Based on the best results And depending on a knowledge base as an important part of expert systems

2) Implementation of the proposed console in practice using the mobile robot system in real world environments

3) Investigate increasing the type and number of sensors and / or location on the robot to determine the appropriate number of them according to the detection format, the number of obstacles, the optimal path ... etc.

4) Additional components can be added to robot making their work more complex and can lead to more complex behavior, such as adding a camera unit to robot. Add camera needs image processing capabilities that can give robot the ability to recognize patterns, face recognition, ... etc

5) Investigate the use of various localization devices and algorithms such as the Global Positioning System (GPS) to improve the localization of the mobile robot.

6) Study the movement of dynamic obstacles in the unknown environment.

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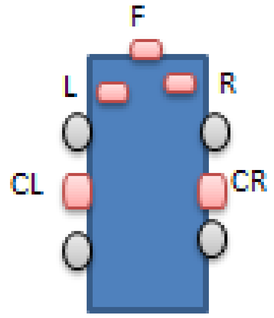

Figure (1) Location sensors

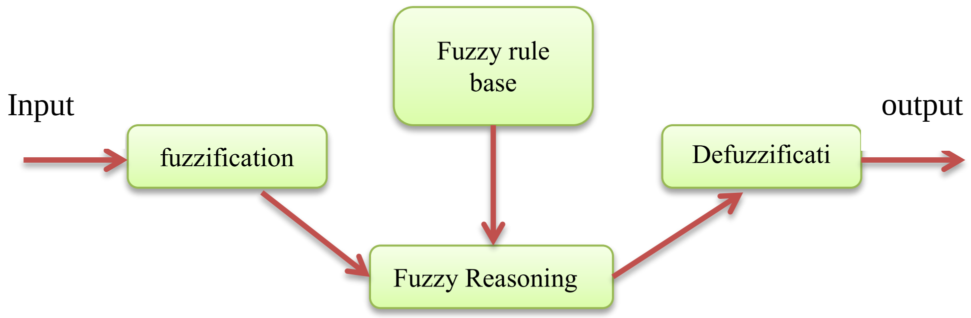

Figure (2) Fuzzy logical system

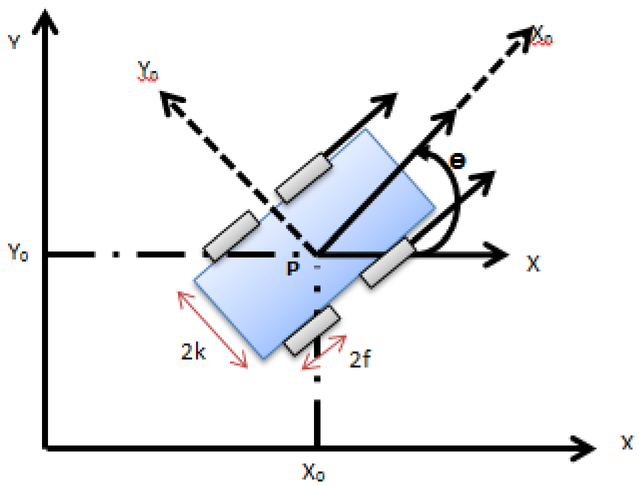

Figure (3) : mobile robot structure

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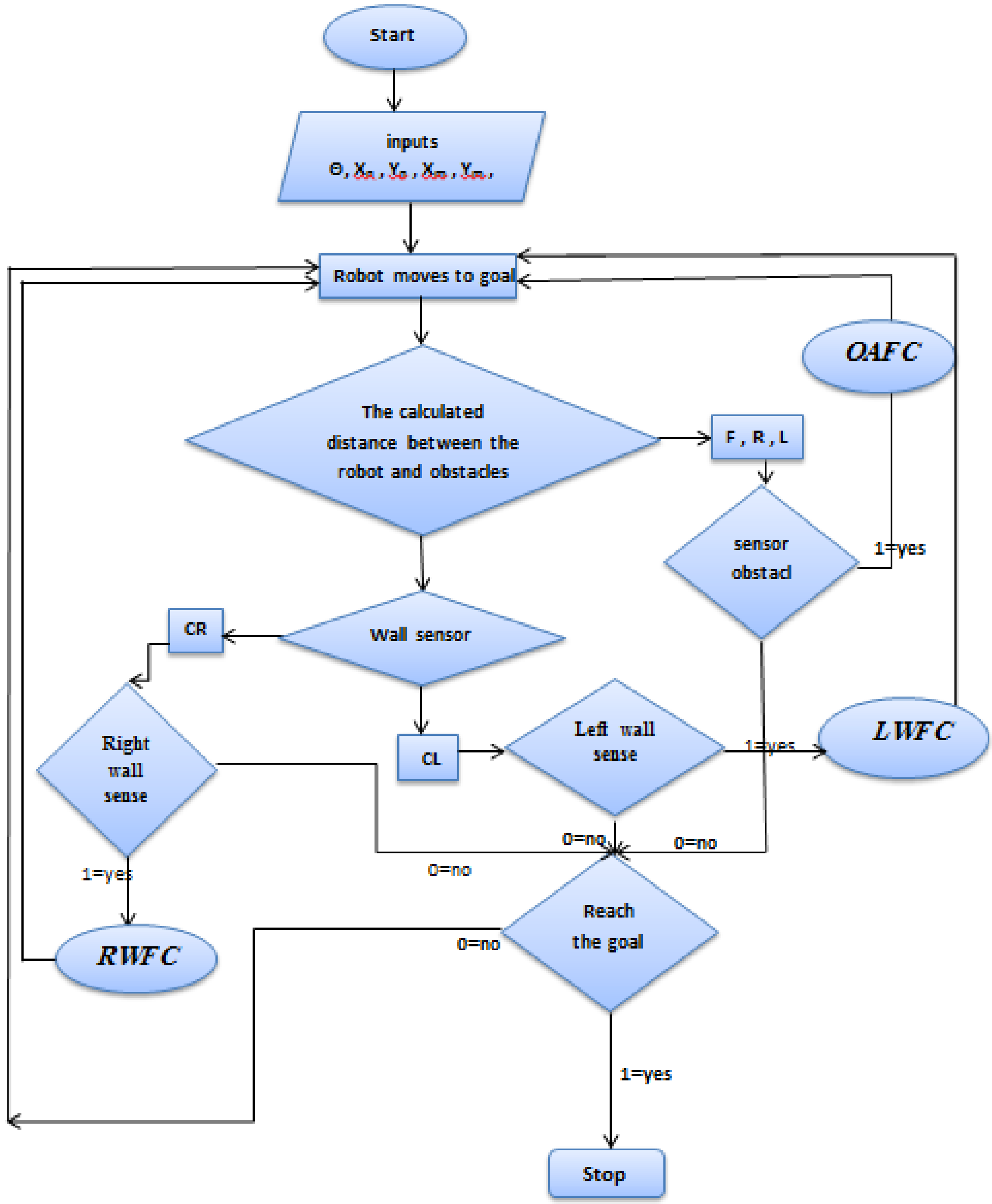
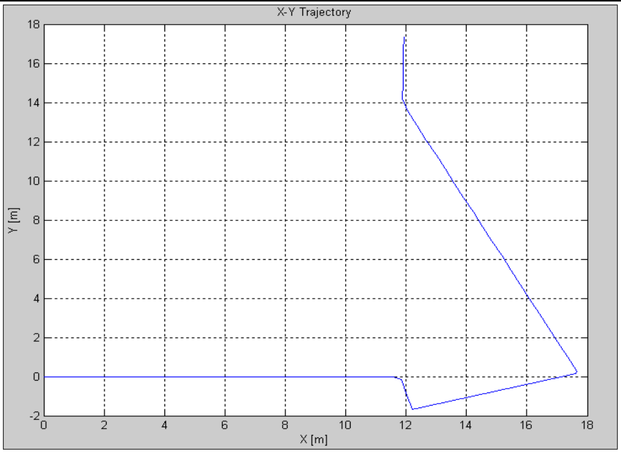


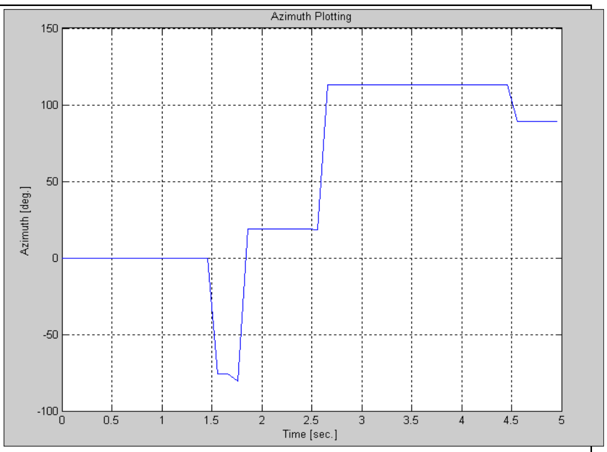
Chart (1) Flowchart for the autonomous mobile robot algorithm

Table (1) : OAFC

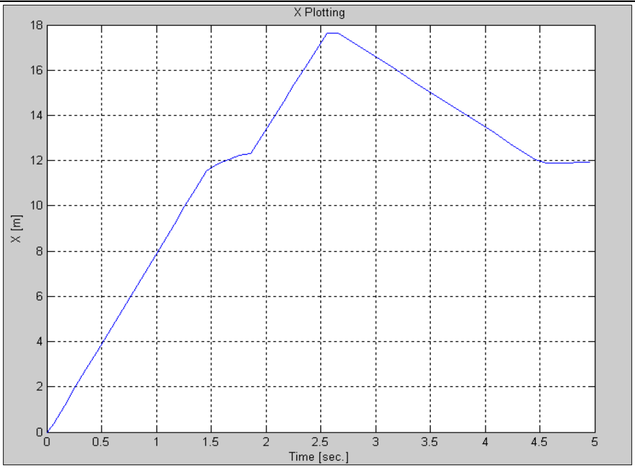
Inputs			Outputs	
F	*R*	*L*	W_R	W_L
near	near	Near	BF	BF
near	near	Far	FS	BS
near	far	Near	BF	FS
near	far	Far	BS	FF
far	near	Near	FS	FS
far	near	Far	FS	BS
far	far	Near	BS	FS
far	far	Far	FF	FF



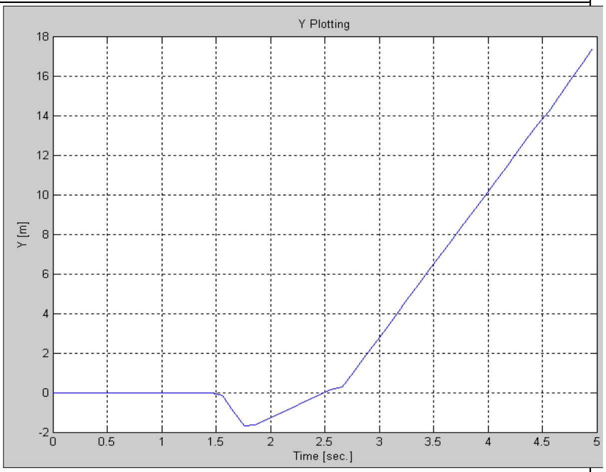
(a)



(b)



(c)



(d)

Figure(4) Plottings of (a)X-Y trajectory ,(b)Azimuth, (c)X, and (d)Y with OAFC

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Table (2) RWFC

Inputs		**Outputs**	
distance to right wall	θ_{diff}	W_R	W_l
close	Z	FF	FF
close	SN	FS	BS
close	BN	FF	BF
close	SP	FS	FS
close	BP	FF	FF

Table (3) LWFC

Input		**Outputs**	
distance to left wall	$\boldsymbol{\theta_{diff}}$	W_R	W_l
close	Z	FF	FF
close	SN	FS	FS
close	BN	FF	FF
close	SP	BS	FS
close	BP	BF	FF

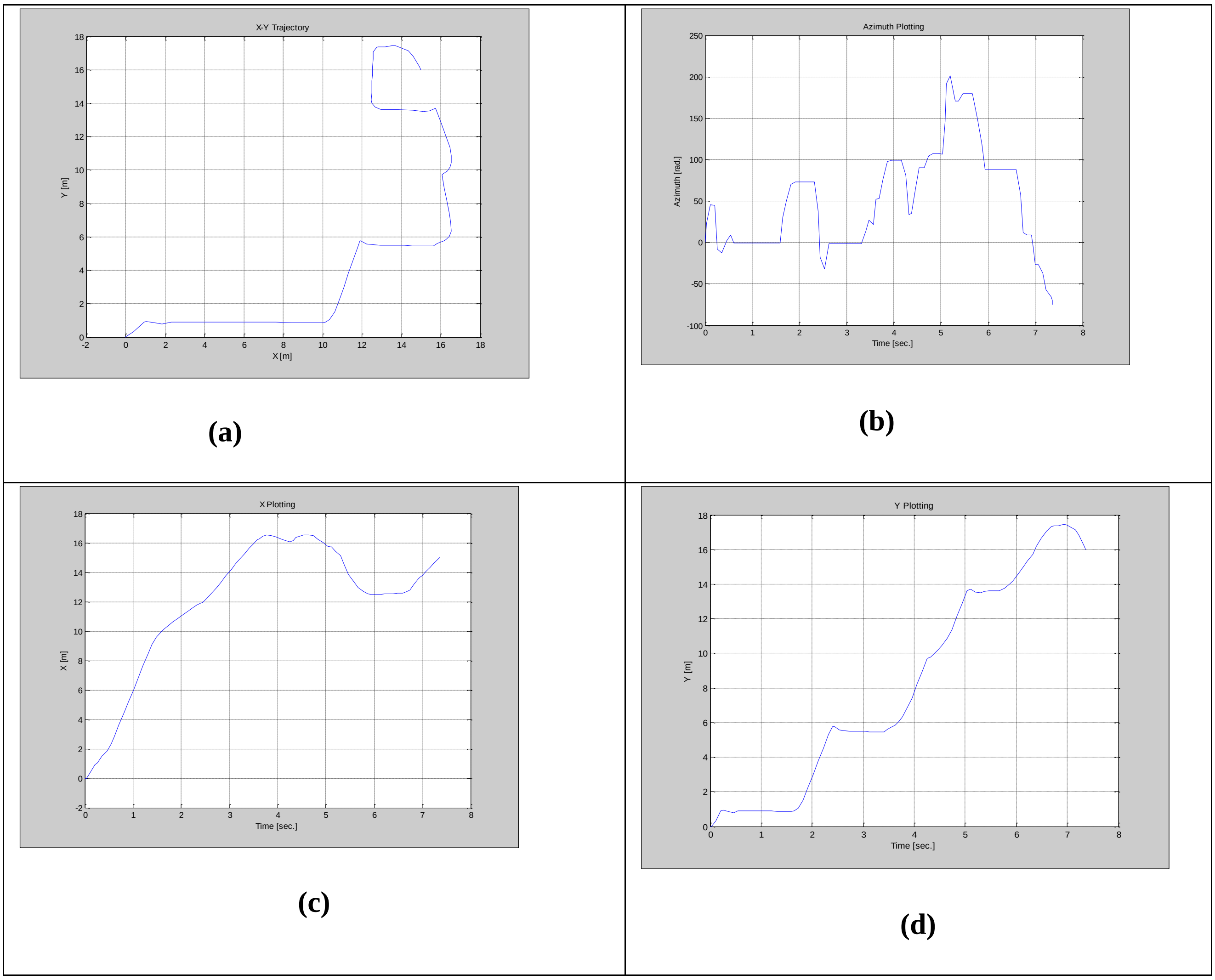

Figure (5) Plottings of (a)X-Y trajectory ,(b)Azimuth, (c)X, (d)Y with RWFC & LWFC

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محاكاة الروبوت المتنقل المستقل باستخدام المنطق المضبب

ندى بدر جراح

جامعة البصرة / كلية الادارة والاقتصاد/ قسم الاحصاء

المستخلص :

ان من اهم القضايا في عالم الروبوت هي تطوير نظام ذكي لتحسين حركة الروبوت المتنقل المستقل وعدم الاصطدام ضمن المكان الذي يتحرك فيه ، وهذا ما يتطلب تعلم الالة وتطوير قدراتها التشغيلية في التمكن من التصور ومعالجة البيانات واتخاذ القرار .

والهدف من هذا البحث هو المحاكاة باستخدام المنطق المضبب في برنامج ماتلاب لحركة الروبوت في تجنب العوارض والوصول الى المكان المطلوب بجانب الجدار. إن العالم الحقيقي يتميز بشدة بعدم اليقين. وقد أثبت المنطق الضبابي أنه أداة تقليدية لمعالجة مثل هذه الشكوك.

الكلمات المفتاحية : الروبوت المتنقل المستقل، الاستشعار، ماتلاب، المنطق المضبب

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Solving Traveling Salesman Problem Using Cuckoo Search and Ant Colony Algorithms

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Abstract

Optimization is a method that is used from economic to design. The best tools available are very important to be utilities .when there is some randomize nature value that's depend in the algorithm is called stochastic. Algorithm with stochastic partitions are often named heuristic or meta heuristic recently.

Traveling salesman problem (TSP)is hard a combinatorial optimization problem that leads to find the best tour for the person . this problem can be applicator in many different area such as DNA fragments, planning and logistics. There are many algorithm that is used to solve this problem.

In this paper, Ant colony optimization (ACO) is the first algorithm that is applied which depending on the ant colonies law for finding the best tour of TSP .

The other algorithm that is performed, is cuckoo search (CS) that satisfy the law of brood parasitism of some cuckoo specie to find the best tour of the same problem.

Compare between two algorithms of meta heuristic for six cities with different parameter's value to evaluate the result . conclude that the CS performance is better than ACO with speed convergence.

Keywords: Ant colony optimization, Cuckoo search algorithms, Traveling salesman problem.

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1. Introduction

optimization is the achieving the best possible result under circumstances. It design, built then maintenance, engineers have to take decisions. The goal of decisions is either for minimize the efforts or maximize the benefit of it which can be modeled as a function of certain design variables. Then ,the optimization can be defined as the process of finding the conditions that lead to maximize or minimize the value of a function[1].

Exploration and explotation by using search locally and randomize values are the important components of all meta heuristic algorithms. It firstly started with one or more solutions then tested steply on a sequence of solutions to reach the optimal solution .

The most popular meta heuristic algorithm are Genetic Algorithm(GA), Tabu Search(TS), Simulated Annealing(SA), Ant Colony Optimization(ACO), Particle Swarm Optimization(PSO), Bee Colonies Optimization (BCO), Monkey Search Algorithm(MS), Harmony Search Algorithm(HS), Fire Fly Algorithm(FA), Intelligent Water Drops(IWD) and Cuckoo Search (CS) [2].

Most of there, are mimicking successful feature in biological ,physical or sociological system. The success of these methods for solving combinatorial optimization depending on many features[3].

Travel salesman problem ,firstly formulated as a mathematical problem in 1930's and most studied in theoretical computer science mathematical and engineering applications. The important parameters of this problem I cost ,time, path are optimized. Recently, applicator in computer network ,mail carries delivery trucks and airways[4].

ACO are used by Macro Dorigo in 1992.it modified to used probabilistic to solve the TSP by graph in 1997[5], which are described in section 2.

Cuckoo search was inspired by the "obligate broad parasitism" of many host cuckoo's birds of the other species. It proposed by Yank[3],that described in section 3 briefly.

2 .Ant Colony Algorithm

Are depend on the behavior of natural ant colonies. Real ants find shortest path leading from the nest to food sources by laying a chemical compound which known as pheromone on the ground. A number of artificial ants construct a solutions randomly. Each ant selected probabilistic to follow a path with rich pheromone[6].

When used on TSP this means artificial ant moves from city to another on a graph(count the edges travelled accumulating then the pheromone trial left behind by another[7].

The algorithm steps are:-

1.Generate coordinate of cities randomly depending on the number of cities.

2.Each ant choose city i randomly then move to city j using probabilistic transition rule($S_{ij}^k(t)$), to built the tour using the following equation[8].

$S_{ij}^k(t)=(w_{ij}(t))^{\alpha} (1/d_{ij})^{\beta})/(\sum (w_{il}(t))^{\alpha} (1/d_{il})^{\beta})$ if $j€J_k(i)$……1

If $j \notin J_k(i)$ then $S_{ij}^k(t) =0$ …..2

Where

d_{ij} is the distance between city i & j ,is computed by the equation

$$d_{ij}=\sqrt{(z_i - z_j)^2 \quad + (y_i - y_j)^2} \qquad …..3$$

$J_k(i)$ is a set of cities that remain to be visited when the ant is at city i.

α, β are positive parameters that control the weight of pheromone trial which computed for each tour by using this equation.

$$w_{ij}(t+1)= (1 - S)w_{ij}(t) + \Delta r_{ij}(t) \qquad …..4$$

$$\Delta w_{ij}(t) = \sum_{k=1}^{n} \Delta r_{ij}^k(t) \qquad …..5$$

$$\Delta r_{ij}^k(t) = \begin{cases} A/D_k & if\ (i,j) \in tour\ by\ ant\ k \\ 0 & \end{cases} \qquad …..6$$

Where (1-s) is a pheromone decay parameter $0<S<1$ it's used when the ant choose a city to move it (evaporation trial), n is the number of ants , D_k is the length of the tour by ant k and

A is an arbitrary constant.

3. Add cities one by one until get the complete visited cities.

4. Find the best tour for each iteration.

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3. Cuckoo Search Algorithm

Is a modern new meta-heuristic search algorithms that proposed by Xin-She Yang & Snash Deb in 2009[3] ,"inspired by the broad parasitism of a lot of cuckoo bird species". It depending on three laws .firstly, each cuckoo put egg only one at each time in the host nest randomly, while secondly, the best nest which have the best quality of eggs that wining out to the nest generation .lastly, the number of host is fixed the probability of egg discovery by the host bird is $P_a \in [0,1]$ [4].

The bird alien the eggs on the nest that it's belong to their. Then the bird build the new nest on another place. The flowchart below in figure.1are described the steps for solving TSP.

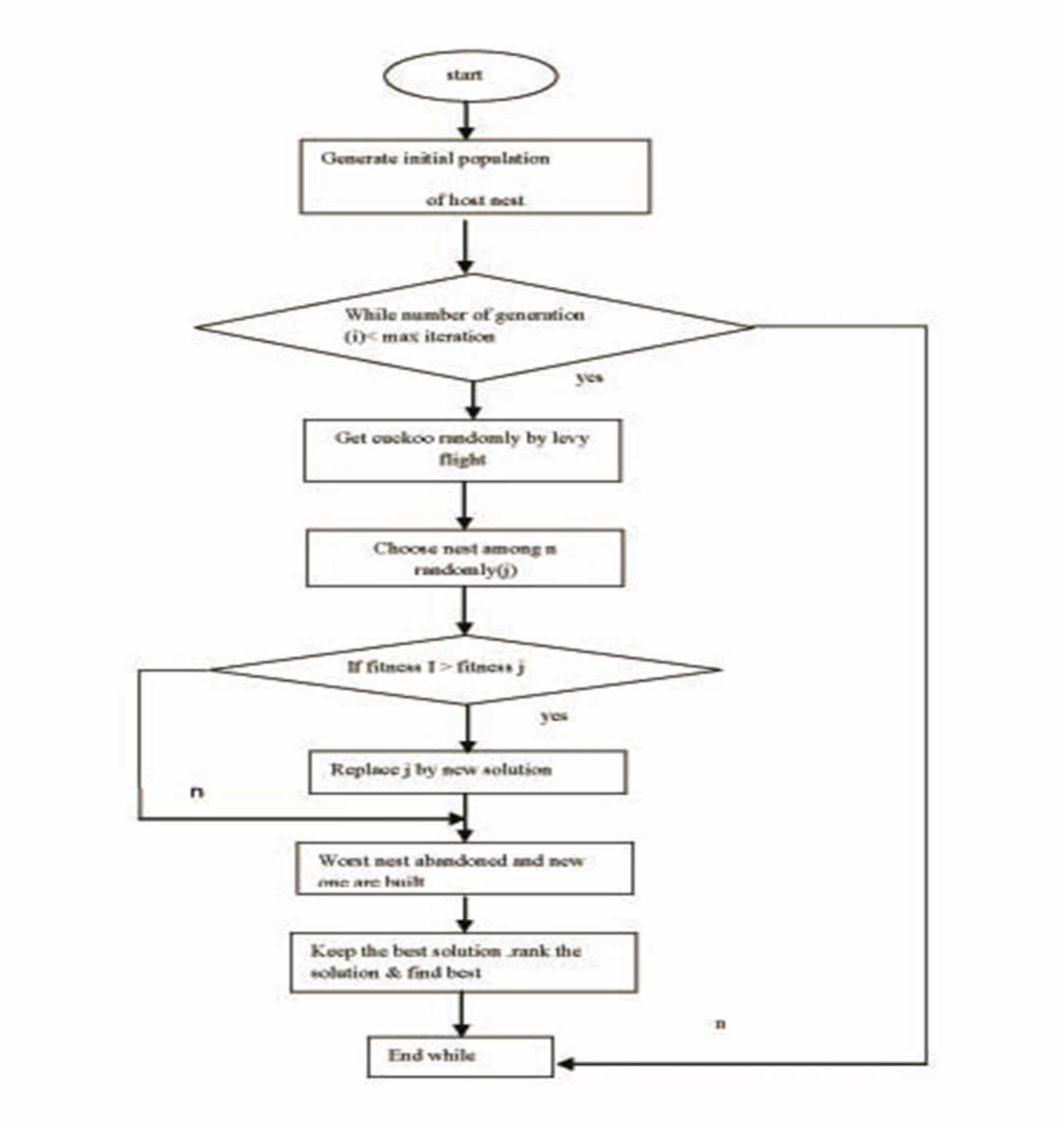

Figure 1.Flowchart for solving TSP by CS

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4. Experimental Results

In order to evaluate the behavior of cuckoo search and ant colony algorithms by using matlab ver.7 , using several values of parameters for six of cities for TSP.

Firstly, apply the ACO with the value of $\alpha=1,\beta=4$ with six cities. In the figure 2, show the tour city's number for six generation.

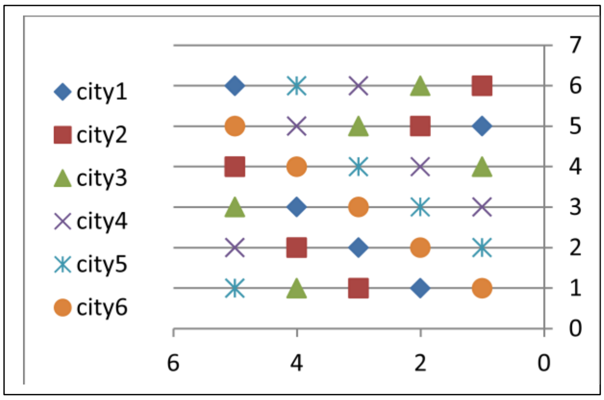

Figure 2.tour for six cities

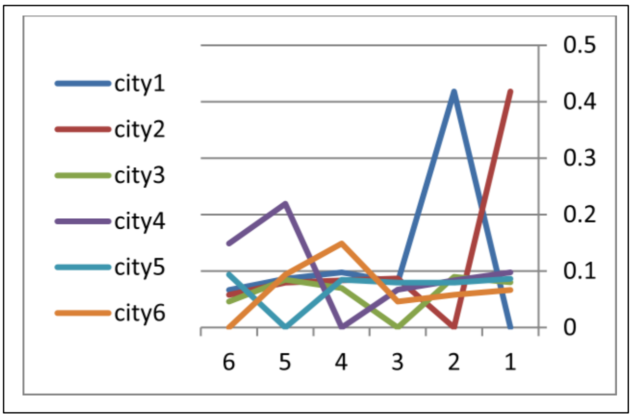

Figure 3. best fitness for the cities

While in figure 3,display the best fitness for the same tour that show in figure.2.With the best fitness is 0.0930 for the tour with city's number (5 6 4 3 2 1 5).

In figure below show the tour for the same number of cities with $\alpha=1$ and $\beta=2$.the best tour are(2 5 6 3 1 4 2) with best fitness= 0.0940

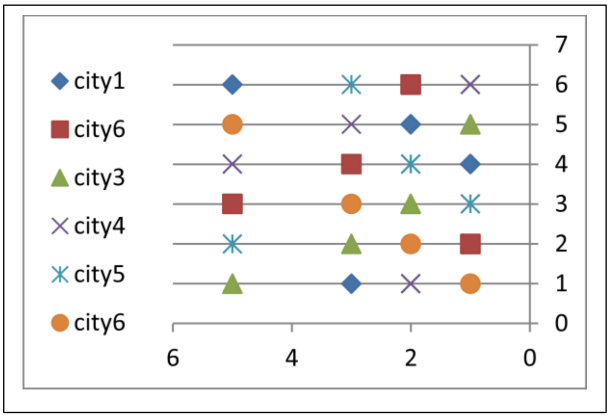

Figure 4. tour for six cities with $\alpha=1$ and $\beta=2$

From the above figures, deduce that, the tour consume more time because the tour most start and ended with same cities. Also, when decrease the value of β , the value of best fitness are increase.

When perform CS for the TSP with six cities, the tour are display in figure 5.

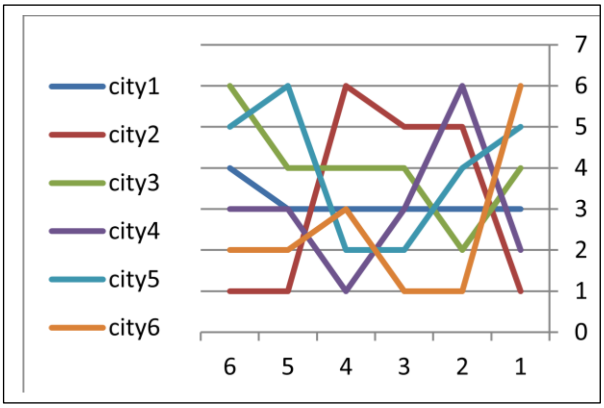

Figure 5, CS tour for six cities

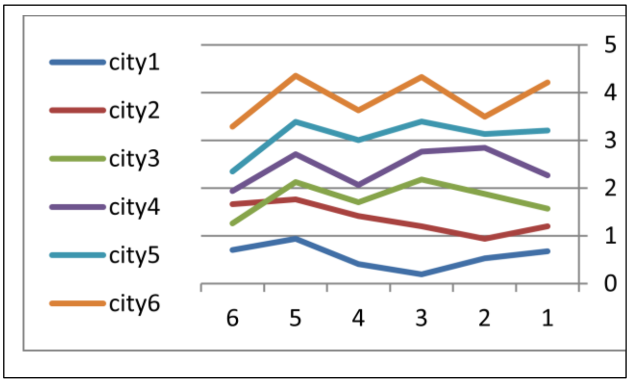

Figure 6, the fitness for the tour's cities

The best fitness for the tour are notice in figure 6.the best tour are 1 3 6 2 5 4.
From the figures above, deduce that the best solution quality is better when used CS algorithm for TSP with consuming time less than when compare with ACO. This effect on the convergence is quickly occurs in this algorithm.

5. Conclusion

This paper presents two approach of meta heuristic search algorithms for solving traveling sales man problem that based on ant colony and cuckoo search algorithms .from our experimental results, it has been shown to compare between two meta heuristic algorithms. ACO is simple in computation and the tour construction consume more time depending on the condition each tour must start and ended with the same cities. Also, when decrease the value of β , the value of best fitness are increase.
but the CS is better performance to find the best tour quickly compared with other by consuming time less than when compare with ACO. This effect on the convergence speed for searching the solution is rapidly compared with it.
The cuckoo search algorithm is more efficient than ant colony in terms of ability for finding the better solution .

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حل مشكلة البائع الجوال باستخدام
خوارزمية مستعمرة النمل وبحث الوقواق

لمى صلال حسن

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المستخلص :

التحسين هو طريقة مستخدمة في الاقتصاد الى التصميم.و تعتبر افضل اداة مستخدمة عندما تتوفر بعض القيم العشوائية والتي تعتمد عليها الخوارزمية والتي تسمى بالعشوائية. وان خوارزميات العشوائية يطلق عليها بالارشادية او الفوقية مؤخرا.
مشكلة البائع الجوال تعتبر مشكلة تحسين صعبة والتي تهدف الى ايجاد افضل مسار لهذا الشخص . هذه المشكلةمن الممكن تطبيقها في عدة مجالات على سبيل المثال تقسيم الحمض النووي والتخطيط والخدمات اللوجستية.
في هذا البحث ، تم تطبيق خوارزمية تحسين مستعمرة النمل والتي تعتبر خوارزمية عشوائية والتي تعتمد على قانون مستعمرات النمل في ايجاد افضل مسار للبائع الجوال.
ثم تم تطبيق خوارزمية بحث الوقواق والذي يعتمد على قانون حضانة بيض الفقس لبعض عش الوقواق لايجاد افضل مسار.
تمت مقارنة الخوارزميتين لستة مدن لتقييم النتائج تم التوصل الى ان اداء خوارزمية بحث الوقواق افضل من خوارزمية مستعمرة النمل.

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Cloud Storage: Cloud Computing and Social Media to utilizing Selective Dissemination of Information Service

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Abstract

This paper touched upon cloud computing, their technologies and social media to provide selective dissemination of information service for the research community and the beneficiaries. It identifies the concept of cloud computing and its advantages and services offered in the field of libraries and information. It also identifies the concept of social media. The paper Builds a digital warehouse and makes digital book link available on social media to serve researchers and beneficiaries and to meet their demands and needs.

Keyword:- Cloud Computing, Social Media, Information Technology, selective dissemination of information service and Cloud storage.

Introduction

With the continuous development in the field of modern technologies and service applications available to beneficiaries on the Internet and the emergence of web 2.0 and web 3.0 and with the cloud computing services which lead to interaction with the beneficiaries, many information institutions and University libraries have moved to access applications for use through the World Wide Web Internet. This will make most of these technologies and applications available matter that will save costs and make available information services to the largest sector of the beneficiaries they also provide the beneficiaries, information institutions and libraries the possibility of storing, processing, transmitting and sharing data anywhere and at any time Without the need to use a personal computer. All these procedures are accomplished through external devices and servers available on the Internet and cloud computing. This will ensure the security of the data and keep them from abuses, loss and damage. Thus, this study uploads digital sources for the university libraries across these servers, or the so-called cloud computing. The aim is to preserve them, and to apply the selective dissemination of information service which send digital resources and information based on the interests and tendencies of beneficiaries across social media. This service can also respond to the queries of the beneficiaries. The study consisted of four sections. The first section gave the general framework of the study. The second section dealt with the theoretical side to identify the cloud computing, its applications and services in the field of university libraries, and it recognized the concept of social media and the selective dissemination of information service. The third section offered the practical side by making available the selective dissemination of information service via social media to beneficiaries and by uploading the digital resources of cloud computing across (Google drive). And taking advantage of its features in saving retrieving data.[1]

Web2

Web 2: - The web is an important transition in the history of the web. Which is to move from the concept of traditional sites, whose substantive content depends on production of the site management to the concept of integrated applications, which provides a range of interactive services accessible to the user and which develops its performance to serve as a product and consumer of the substantive content of different forms. Web 2 can be seen as a network of applications and interactive social services are more than just a set of sites and the content that is mainly available depends on the subscribers.2

Web3

Web3 :- It is also called semantic web. It is one of the concepts that emerged in the contemporary stage within what emerged from the digital environment if it is a concept it' s back to working on converting the web from just a huge repository a huge amount of text, images, clips and other information that is invisible and unorganized makes it easier to take advantage of it, to a digital Warehouse or a large database linked to the information contained within the links based on understanding the meanings and relationships that make their interconnection very good information.[3]

The first sections
General framework for study
(1.1) The problem of the study

The problem of the study launched in answer to inquiries following:

1. What are the advantages offered by cloud computing applications in the field of university libraries and Information institutions?
2. What are the services that can be offered to the Beneficiaries community via social media and increased interaction with the beneficiaries?
3. What are the applications that are working to build a Digital warehouse to save digital sources?

(1.2) The importance of the study

The importance of the study appear from the importance of applications of modern techniques and applications Internet, which leads to interaction with the beneficiaries and answer to the inquiries of beneficiaries and meet their demands by providing digital resources and information according to the tendencies and concerns of beneficiaries across social media and apply the selective dissemination of information service and benefit from the advantages of cloud computing.

Perhaps the justification put forward for this technology is:

- Those features are unique to cloud computing technology In terms of saving costs with access to the services and applications efficiently and quickly.
- Advantages of cloud computing are to be used wherever the user wants, at home, at work, in the car, and in any place where there is Internet.
- Digital sources are keeping from damage and loss of.

(1.3) Objectives of the study

1. Recognize the advantages of cloud computing and its services in the field of university libraries.
2. Availability of digital resources for Beneficiaries community via the Internet, according to their demands.
3. Apply the selective dissemination of information service for Beneficiaries community and to increase interaction between beneficiaries and university libraries.
4. Continuous communication with beneficiaries via social media and informed there with what is new in the library.

(1.4) Research Methodology

Researcher used practical approach to apply cloud computing technology, its applications, its software and how to utilize them in the field of libraries to offer the selective dissemination of information service via social media.

(1.5) The study procedures

1. The resources are converting from of the paper form (Hard paper) to digital form (software copy).
2. Apply the selective dissemination of information service
3. Digital Warehouse Building by Using Google Drive Program.
4. The use of Facebook as a media to inform beneficiaries.

(1.6) Search limits

The spatial border: Baghdad.
The temporal borders: 2015 -2016.
The objective limits: Cloud Computing - social media - the selective dissemination of information service.

The second Section
Theoretical Side

(2.1) Cloud storage

Cloud storage can be defined as: It is a model for storage on the Internet where data is stored on multiple virtual servers, instead of being hosted on a specific server. It is usually provided by a third party like major hosting companies that have advanced data centers. It rents cloud storage space to its customers to suit their needs. The most popular service providers like DropBox, Amazon, Google Drive, OneDrive, MEGA, My CloudAp.[4]

(2.2) Cloud Computing

Cloud Computing can be defined as:It is a technology that depends on the transfer of processing and storage space for computer to Cloud, It maid devices group like servers is accessed via the Internet. Thus, the information technology program will turn from products into services. The infrastructure for cloud computing depend on the Advanced data centers which offers large storage space for users it also provide some programs as services to users. It relies on the possibilities provided by Web 2.0 technologies. This can be illustrated in Figure (19).[5]

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Cloud Computing can also be defined as: Network services provides a cheap and guaranteed on-demand platforms these are accessible and easy-to-use ways. [6]

Cloud computing is not a new technique, but it is a new service and to understand that we must put a simplified definition of cloud computing say that cloud computing is you are using a computerized sources (Hardware, software) Via the Internet introduction to you in the form of service A. You do not care about the manner in which it operates, these and how to operate it, or how it communicated to each other and how to set up a network among themselves and the software installed

(2.3) Advantages of cloud computing

Cloud computing works that the user gets a service that allows him to store all of his data outside the scope of his personal device (personal computer) that is he stores his files and his data on cloud computing servers as Image files, he can access them from anywhere where there is an Internet connection available, Why all this trouble? What's the Damage in storing files locally on the personal computer instead of uploading files to the Internet and downloaded each time a user need it?.[7]

The fact that the advantages here are difficult to be counted, but some of them are:

1. The user can be reached by to his files and his data that he stored from anywhere where the his files are stored entirely online (Internet) and he does not need because accompanied personal device (personal computer) length of time to bring it up to his files are what we do than storing some files to the e-mail in the form of attachments so that we can access them from anywhere where there is a computer and Internet connection, but of course more fully where the user stores all the files and not some On the internet.
2. In this way the user does not require a specific operating system or a specific browser the accesses to the his files and stored and used as the files are available to him without restrictions on the operating system and the type of files, the user can to participate files with an unlimited number of users it is that allows specific users that they reach specific of its files that the user can only be determined but not every user needs a separate file copy, they all share the same file, which reduces the storage space consumption.
3. Storage space consumption does not be a big for example, a music production company will make available the music file one time, while millions of users can be Reach to the same file without needing because each of them takes a separate copy of the file.[8]
4. Access data and applications from anywhere where internet service is available.
5. Low cost of the physical equipment (Hardware).
6. It will enrich the user (individual / company) university libraries and information centers to purchase software licenses.
7. Institutions, companies, university, libraries, and information centers do not need to allocate a place for devices that manage work.
8. No need for technical support within the facility, university libraries and information centers.
9. You can depend on her in large and the complex research Which saves time.
10. Save and backup those serves the user, especially programmers.
11. Property share files to reduce storage space.
12. Sizing or link cost using Scalability.[9]

(2.4) Compare traditional computing with cloud computing

Cloud computing allows you to access all of your applications and services from anywhere, anytime via the Internet, because the information is not stored on your hard drive but on the servers (the company service providers).

Reduced costs for companies, where it is no longer necessary to buy faster computers and the best in terms of memory or above in terms of hard drive space, But any device can be a normal personal computer(PC)by using any Web browser to access cloud services used by the company (edit documents, store files, edit images, …, etc). As the companies and the university libraries and information centers are no longer, buy expensive equipments, such as expensive services to offer e-mail service for its employees, or large storage units to work backups of data and information for the company and the university libraries and information centers. [10]

Ensure the service works permanently, where the company is committed to the introduction of cloud storage service makes sure that the service is working around the clock in the best possible way. When you use a cloud storage service, the information stored on more than one server to ensure no loss, the company also offered the service is committed to fix any glitches or breakdowns emergency as fast as possible. This saves you a lot of time and cost as a user or owner of the company and the university libraries and information centers is responsible for its equipment and software for the management, take advantage of the large infrastructure offered by cloud services to do the tests and scientific experiments. Some complex calculations take years to conducting or processing ordinary computers, while companies such as Google and Amazon allows his cloud consisting of thousands of servers associated with each other to perform such calculations in record time. [11]

(2.5) Cloud computing services in the field of libraries and information center

Cloud computing allows for the beneficiaries and information centers and university libraries to store data across servers and servers via the Internet outside the scope of the personal device or personal computer or mobile device or laptop any computer where they can retrieve data from anywhere and at any time, provided that there is internet connection as well as the make it available at the same time for more than a beneficiary this service is called in libraries save and automatic retrieval service. Where data are stored in digital form (software copy) and Retrieval a computerized, by processing the data in electronic Where the resources can be sent by the interests of the beneficiaries is called selective dissemination of information service, by sending resources in electronic form will be applied Borrow books electronic service by ordering the beneficiaries of digital resources (software copy) and according to their needs and from this standpoint, according to this concept can be applied to following service:

1. Electronic book loan service (here to get a copy of the resources without the need to return it).
2. Software as a service (Saas)[12] in the field of libraries.
3. Save service and automatic retrieval service.
4. Selective dissemination of information service.

(2.6) The concept of Social media

Social media can be defined as: It simply:- Techniques exist on the Internet used by people, to communicate, and interact with each other,. the term "social media" has developed to happen great fanfare, it includes all electronic communication tools existing during twenty-one century, some people use social media is a wider, to describe all types of cultural phenomena that involve communicating, and not only communicating techniques, all too often, for instance, people use the word social media to describe the user-generated content, or content submit by users, it is the content that users typed, publish and share it by using electronic publishing tools.[13]

It must be noted that most forms of social media, electronic; it allows users to communicate and interact with each other using computers, smart phones and the Internet. Social networks such as Facebook, Twitter. it is considered to be part of or one of the types of social media.[14]

As for the concept of social media: social media sites are the Web sites that users can participate and contribute to the creation or add pages and easily.[15]

(2.7) Social networking (Social media) components

Social networking and tools for social networking generally consist of multiple applications or sites are most important:

1. **Electronic media sites (Social media):** Sites that allow people to create pages and communicate with their friends and acquaintances, such as website (MySpace) and (Facebook).[16]
2. **Blogs:** Blogs are websites that represent personal journals or personal newspapers from which the personal thoughts of individuals or groups are reported, and Blogs are open to all.
3. **Electronic forums (Online forums):** It sites Provides areas or spaces electronic to express their opinion and writing public topics, it is usually about a specific topic, any for each forum Particular specialty, and it is one of the most prevalent social media, 's easy to sign up(participate) and do not require a great technique to give (impart) the subject as in blogs.[17]

(2.8) Facebook and its features

Considered a site Facebook to social communication is Best those sites, it is more than contributed to spreading the culture of social communication between classes, caste, trends and religions around the world, and because of its characteristics we will discuss and that facilitated people to sign up and communicate On his way (through him).[18]

The Facebook: It is the site of social media, Meaning it is allowing for a way for ordinary people and legal persons (such as companies) that brings out (accentuates) himself, and enhance his place through the website tools for communication with other people within the scope of that site or through communication with other sites, create links to communicate with others.[19]

This site was founded by Mark Zuckerberg in 2004 for the purpose of communication between students in the Harvard University, US, and then spread its use among other college students in the United States, Britain, Canada, to develops the site characteristics than just a site to highlight the personal self and images, to a specialized site in communion sponsored by Facebook company, which became billions in 2007 as a result of the involvement of 21 million subscribers in this site this year to exceed any site Social media and become the first in worldwide.[20]

The site has been transformed from just a place to display personal photos and communicating with friends and family, to the channel of communication between Electronic communities, and rostrum to display political ideas, and configure political groupings electronic and fail her mightiest the actual parties on the ground, as well as to become an essential Communication channel for marketing adopted by the thousands of large and small companies to connect with their audience, as well as newspapers that depended on Electronic communities to transfer her news and promotion of the her book and other media. A exceed Facebook site his the Social Function, to the communication Site multi-Purpose, it is expected that the number of subscribers in 2013, arrives nearly half a billion subscribers, In the future, to become the largest electronic gathering on earth.

(2.9) Selective dissemination of information Service on social media

Selective dissemination of information ("SDI") is a system of information systems services leads to provide beneficiaries with information that their interest in electronic form and orderly.[21]

Selective dissemination of information can also be defind as:It is information selected serve a certain segment of beneficiaries reflects the concerns of beneficiaries,

It aims to provide each beneficiary periodically every week or every fortnight (Semimonthly) in information or policies that fall within the scope of his interest without asking for his part.[22]
It uses the concept of selective transmission of information often to describe was Built a system on the idea of the scientist Hans-Peter Lohan, and that lead to the use of electronic computers in the match the terms that express the concerns of each researcher terms that express the contents of the resourcesof the newly added to the library collections, notice beneficiary with the necessary information from these resources either send Abstract, Brief for each resource or send the entire resources.[23]

(2.10) Selective transmission of information service requirements

Selective transmission of information service consists of the following components:

1. Beneficiaries files: These files include the complete information on the beneficiaries of the system Such as name, Address, the scientific degree, job, scientific background, the subject of specialization, previous experience, the areas of scientific interest, Research projects and study projects.
2. documents files: This file contains complete bibliographic information about the documents entering the system as well as descriptors or terms that reflect the subjects of these documents and are used to retrieve it.
3. Matching (the emulation): Where the terms in the file of documents are matched in the file of beneficiaries to select those documents of interest to their demands beneficiaries.
4. Notification: Where to send information on these documents to the beneficiaries of matching their demands, their needs and their interests either by postal mail or telephone or e-mail or Social Media.
5. Files update: By asking, the beneficiary evaluation of the documents received in order to be possible to describe needs better by adding and modifying.[24]

The third Section
Practical side

(3.1) Selective transmission information service is a applying by Cloud computing applications and Social Media

The selective transmission information service is applying by Cloud computing applications and Social Media, through several phases, as follows:

First: - Information resources is Converting from hard copy form to digital format (software copy) by using the scanner, according to the needs of beneficiaries. As shown in Figure (1) in Appendix (1).

Second: - Assembling Images taken in the previous step of the thesis or PhD thesis or book into one document (pdf) by using Adobe Acrobat 7.0 Professional Program. As shown in Figure (2) in Appendix (1).

Third: - File compression or reduce its size, there are several options the simplest to do it. It is the Feature existing in Adobe Acrobat 7.0 Professional Program, it is option **Reduce file size** to reduce the size for the purpose of uploaded where it is reduced in size to a quarter. As shown in Figure (3) in Appendix (1).

Fourth: - We do the Name of the file according to Name of title the thesis or book (title of information resource).

Fifth: - Creation of Gmail Mail in order to be uploading files for digital warehouse.

Sixth: - Open the Gmail Mail. As shown in Figure (4) in Appendix (1).

Seventh: - Creation of a digital warehouse. As shown in Figure (5) in Appendix (1). Where be upload to him digital resources. And it is used for applying cloud computing (Google drive).

Google drive: It is a cloud storage service offered by Google Inc. Provide free storage space of 15 GB
As well as the advantages enjoyed by as follows:

1. Connectivity with Gmail Mail: Google Drive application allows you to send files sizes infinite (As, of course, does not exceed the maximum storage space) by Gmail mail directly from within the application itself, and without the need to go to Gmail Mail.

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2. The synchronization with your personal device: Google Drive application that allows other enormous feature, it is the synchronization your personal device with files account and Google Drive, which allows you to create new files or amendment to existing files, even without an Internet connection, that these amendments transmitted as soon as you are connected to the World Wide Web.
3. Google drive connecting with the Google search engine more deeply: Because Google owns producers, Drive and Search Engine, It becomes to find a file, via Google Drive, more easily than the rest of the other services that offer cloud storage, of course, this requirement applies to appear the licensed public files and not private files.
4. The use of technique image recognition: Google uses image recognition technique in Google Drive application, in image files, which makes the search for an image or a person; it is very easy via the Google Drive cloud service.

Eighth: **-** Upload files (Books or thesis) as requested by the beneficiaries to digital warehouse. As shown in Figures No. (6) and No. (7) and No.(8) and No. (9) in the Appendix (1).
Now it has been uploaded file into the digital warehouse as shown its name in the digital warehouse. As shown in Figures No. (10) and No. (11) in the Appendix (1).

Ninth: **-** Create a link to the file uploaded, by clicking on the Get shareable link button, and copy the link, by Clicking on the Copy link button. Or by clicking the right mouse button and then choose to Get shareable link option. As shown in Figure (12) in Appendix (1).
And then we are pressing a sharing settings button. As shown in Figure (13) in Appendix (1).
And then we are pressing a copy link button from in Figure (13) to take a copy of the link to the clipboard. As shown in Figure (14) in Appendix (1).
Then clicking the done button to complete the process of copying the link and the closure of the copy link windows.

Tenth: **-** Availability of digital resources on social media to know the beneficiary and benefit from the resources according to his request, according to his interests, we open the Facebook page for the library and then Creation of a group and we are writing group name that established. As shown in Figure (15) in Appendix (1).
Then paste the link that existed in clipboard established previously to Facebook group page. As shown in Figures No. (6) and No. (16) and No.(17) and No. (18) in the Appendix (1).
In Figure (18) in Appendix (1). Shows it beneficiaries interaction with selective transmission information service.

The fourthSection
Results and Recommendations

(4.1) The results:-

The Research used an applied approach to apply selective dissemination of information service and cloud computing technology to build a digital warehouse by using the Google drive.It uses Facebook as a media to inform beneficiaries of the digital resources available at the Central Library of the Iraqia University that meet their demands. The research has come to a number of conclusions the most important:-

1. It is applying selective dissemination of information service provides time for beneficiaries of the university libraries to retrieve the intellectual works in the topics they care about quickly and easily.

   2. The use of cloud computing applications In university libraries establishes a digital warehouse and saves all digital sources from damage or loss because the digital data are kept in more than one place in the world and therefore there is that they may be damage or loss;

3.The use of cloud computing applications in university libraries leads to electronic storage and retrieval which helps to speed the retrieval of digital resources.

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2. The use of cloud computing applications In university libraries establishes a digital warehouse and saves all digital sources from damage or loss because the digital data are kept in more than one place in the world
3. The use of cloud computing applications in university libraries leads to electronic storage and retrieval which helps to speed the retrieval of digital resources.
4. The use of social media in the provision of information services for university libraries, it is working to increase the interaction between the University Library and beneficiaries.
5. The use of the digital and electronic means, it works to offer information services for university libraries over the twenty-four hours, rather than abide by the official working hours and without the need to enter the library and benefit from its services.
6. The offer of information services in computerized form, it works seriously interaction and cooperation between the Iraqi university libraries.
7. The exclusion of the beneficiary for technical problems or maintenance and development of the system.
8. Take advantage of the high specifications of remote device in the cloud.
9. Utilization of large servers in complex operations may require high-specification devices.

(4.2) The recommendations:-

The study unearthed a number of recommendations the most important points of which are the following.

1. Advocating awareness of the importance of cloud computing and working on developing human resources to deal with this technique. This will achieve the optimal use of these services which require us to understand well the dimensions of this modern technology;
2. The cloud computing will change the format of services offered by the university libraries, and therefore it must contribute technological developments from the beginning without being late. We should absorb its importance and its active role especially at the present time.
3. University libraries need to be encouraged to provide computerized information services (selective dissemination of information service), and to convert the resources from of the paper form (Hard copy) to digital form (software copy). That means to convert from traditional form to digital one.
4. Urged university libraries on investing cloud computing application that create digital warehouse and save the digital resources from damage and loss and take advantage of its in the process of storage and electronic retrieval.
5. The cloud computing techniques will help libraries in cost savings and providing new services, where the computerization aimed at applying the principle of payment after use, which contributes significantly to the cost savings with the use of latest applications and latest advanced management tools.
6. The provision of internet lines permanently in university libraries and the Internet will be of high quality and speed.
7. Training of staff in university libraries and their participation in training courses to offer computerized information services.
8. The provision of devices and equipment necessary from scanners and digital cameras to convert resources from of the paper form (Hard paper) form to digital form (software copy).
9. The provision of a website for the General Secretariat of the Central Library in the Iraqia University.

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Appendix (1)

Figure (1) converting the source of the paper (hard copy) to digital.

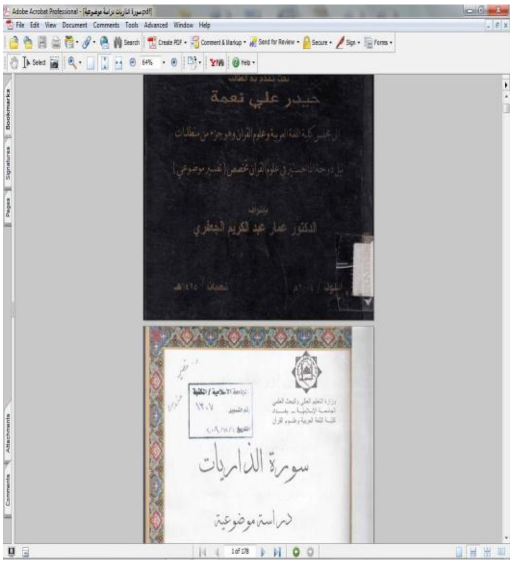

Figure (2) assemble the format (pdf).

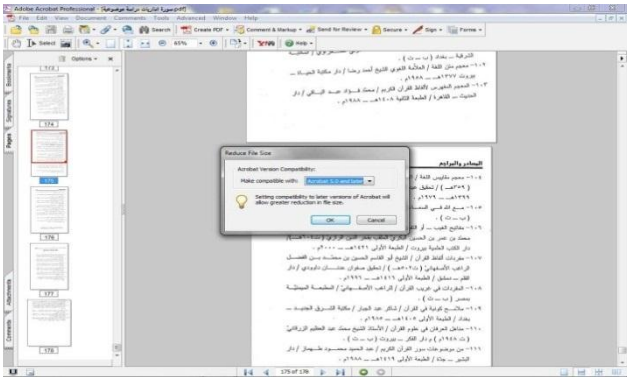

Figure (3) illustrates the file compression.

Figure(4) illustrates the way in which open e-mail.

Figure (5) illustrates the digital warehouse.

Figure(6) shows how the lifting of the sources of digital information (upload file).

Figure (7) illustrates the upload file

Figure (8) shows the path upload the source (enlarged image).

Form (9) shows the completion of the upload.

Figure (10) illustrates Digital Warehouse with file upload.

Figure (1١) illustrates Digital Warehouse with file upload (enlarged image).

Figure (12) explain how to create a link..

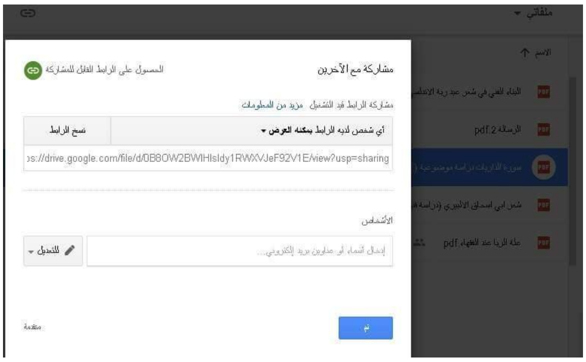

Figure (13) illustrates the settings of your participation.

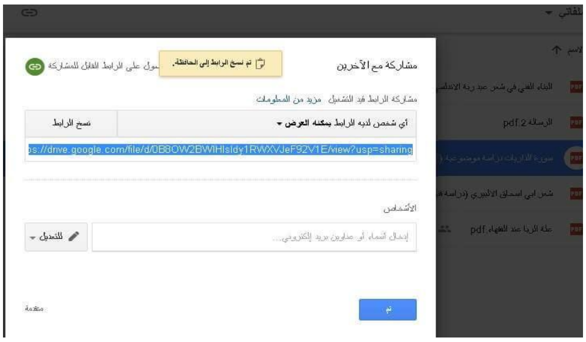

Figure (14) illustrates the link copied to clipboard.

Figure (15) shows the group page on Facebook .

Figure (16) shows the Paste Link the group page on Facebook.

Figure(17) shows the source publication in Facebook Page.

Figure (18) illustrates the beneficiaries interact with the broadcast selective information service.

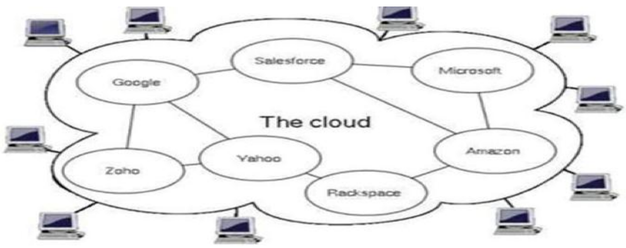

Figure (19) show cloud computing infrastructure.

الخزن السحابي: الحوسبة السحابية و مواقع التواصل الاجتماعي لاستخدام خدمة البث الانتقائي للمعلومات

عثمان عطا اسماعيل

الامانة العامة للمكتبة المركزية
الجامعة العراقية

المستخلص :

تهدف الدراسة باستثمار الحوسبة السحابية وتقنياتها ومواقع التواصل الاجتماعي في تقديم خدمة البث الانتقائي للمعلومات لمجتمع الباحثين والمستفيدين، والتعرف على مفهوم الحوسبة السحابية ومميزاتها والخدمات التي تقدمها في مجال المكتبات والمعلومات، والتعرف على مفهوم مواقع التواصل الاجتماعي فضلا عن عمل مستودع رقمي واتاحة رابط الكتب الرقمية على مواقع التواصل الاجتماعي خدمة للباحثين والمستفيدين وتلبية طلباتهم وحاجاتهم.

الكلمات المفتاحية: - الحوسبة السحابية، مواقع التواصل الاجتماعية، تكنولوجيا المعلومات، خدمة البث الانتاقئي للمعلومات، التجزين السحابي.

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Adaptive feed control of operating tool in Robot machine

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abstract :
Adaptive ornate milling technology involves variation of milling operational parameters (particularly, cutter feed speed) based on wood surface texture at the processing spot. It is appropriate to use wood fiber orientation optical recognition method for adaptive ornate milling.
Adaptive milling method is very efficient for ornate milling, when cutter moves along a complex trajectory in space, and feed direction and fibers alignment substantially alters along the trajectory. Moreover, at sharp trajectory sections, adaptive choice of operational parameters permits to materially reduce the chance of wood damaging. Adaptive milling method is most effective for poor value wood that has low stress-strain properties and complex surface structure, so is very prone to damage.

System for programmed PID controller adjustment involves computer with software for certain operating system, input/output hardware and connector cables. Communication with facilities is typically performed by use of OPC server. While adjusting, an object is included in the control loop. The system adjusts and PID controller is registering obtained parameters. Obtained in such way controller parameters will be near optimal due to intuitive user interface, high computer capability and limitless system identification algorithms.

The format in the search shows schematic structure of automated control system of tool feed in adaptive milling, which uses CNC-controlled 3D vertical milling machine, control computer and camera for wood fiber orientation optical recognition, its signal being transferred to milling parameters optimization application.
It should be noted that at such milling with control of supply rate along the complex trajectory of supply of the milling tool, it would be possible to significantly reduce the percentage of shatters. This would allow to significantly reduce economic expenditures for additional operations concerning the surface recovery.
Keyword : Robot machine, OPC server , Proportional Integral Derivative (PID) controller .

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Introduction:

The System of program Proportional Integral Derivative (PID) controller adjustment involves computer with software for certain operating system just like input/output hardware and connector cables. Communication could lead to facilities which is typically performed by the use of Open Platform Communications (OPC) server, while adjusting is an object which included in the control loop. The system adjustment and the PID controller are registering that obtained parameters in such a way controller parameters will be near optimal due to intuitive user interface, high computer capability and limitless system identification algorithms.

Figure. 2 shows schematic structure of automated control system of tool feed in adaptive milling, which uses Robot -controlled 3D vertical milling machine, control computer and camera for wood fiber orientation optical recognition. It is a signal being transferred to milling parameters optimization application.

Methods and tools of work

A robot installed is designed as a thermocouple based on the scanning of the wood pieces, the longitude and the latitude of the panels, and through the image to the robot processor on the surface of the panels and the method of linear and longitudinal lines, The process begins with the fossil decoration, where the robot is fed at the required speed, with each wooden line on the surface of the treated board. With the cutting of the longitudinal lines, the robot gives a higher feed rate for the cutter to perform its work. In order to avoid damage and damage to treated panels, In the case of the passage of the cutter on the lines in the opposite way should be less speed cutter larger to avoid the destruction of the plank, during the experiments we took the surfaces of different boards and different rate of moisture to indicate the response of the robot to the treated surfaces, The working tools were a robotic type machine with 4 different types of cutters, shapes and sizes in addition to the depth, wooden boards with different humidity.

Discussion

Adaptive ornate milling technology involves variation of milling operational parameters (particularly, cutter feed speed) based on wood surface texture at the processing spot. It is appropriate to use wood fiber orientation optical recognition method for adaptive ornate milling.

Adaptive milling method is very efficient for ornate milling, when cutter moves along a complex trajectory in space, and feed direction and fibers alignment substantially alters along the trajectory. Moreover, at sharp trajectory sections, adaptive choice of operational parameters permits to materially reduce the chance of wood damaging. Adaptive milling method is most effective for poor value wood that has low stress-strain properties and complex surface structure, so is very prone to damage.

Adaptive milling with operative tool feed control can be implemented by use of self-adapting system see in (Figure 1).

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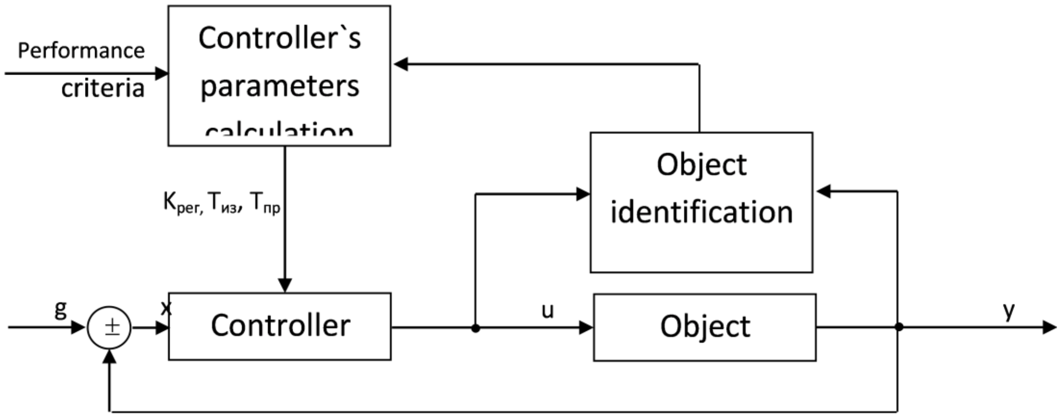

Figure 1 – Schematic structure of adaptive control system

System for programmed PID controller adjustment involves computer with software for certain operating system, input/output hardware and connector cables. Communication with facilities is typically performed by use of OPC server. While adjusting, an object is included in the control loop. The system adjusts and PID controller is registering obtained parameters. Obtained in such way controller parameters will be near optimal due to intuitive user interface, high computer capability and limitless system identification algorithms.

Figure. 2 shows schematic structure of automated control system of tool feed in adaptive milling, which uses Robot-controlled 3D vertical milling machine, control computer and camera for wood fiber orientation optical recognition, its signal being transferred to milling parameters optimization application.

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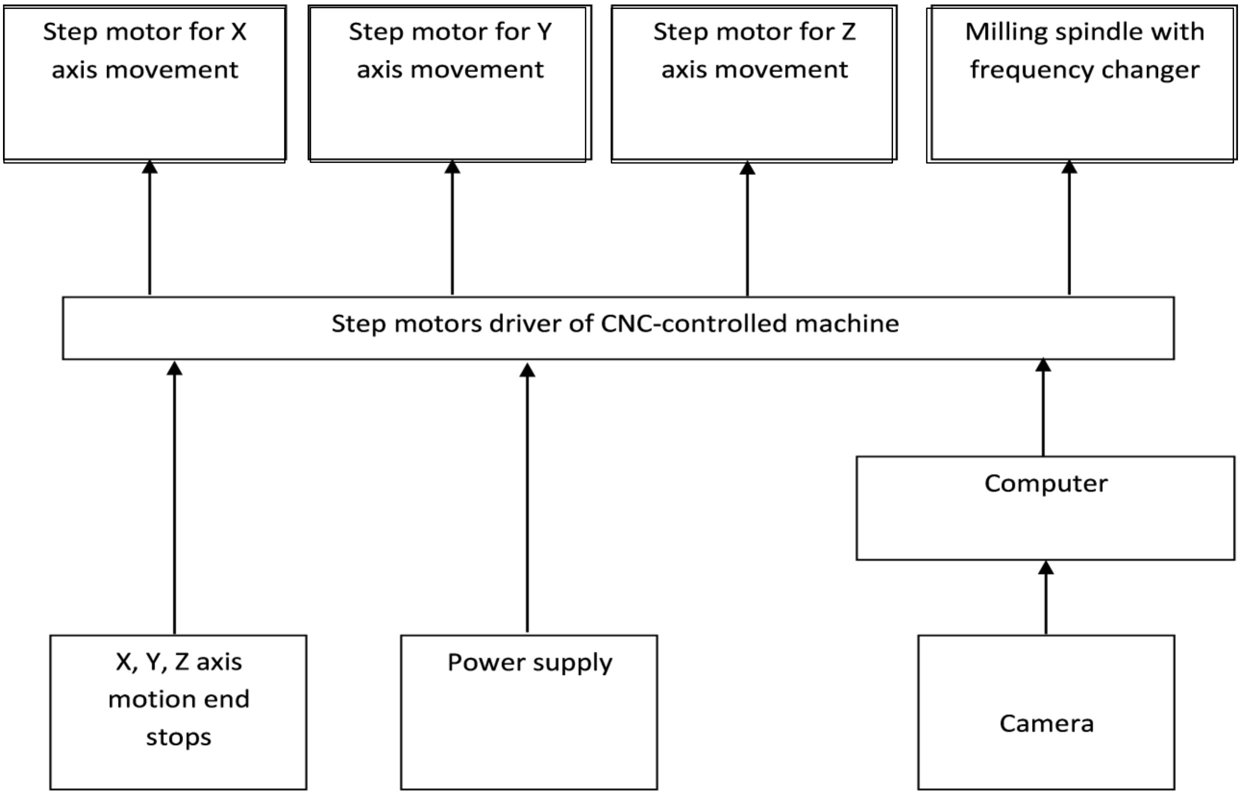

Figure 2 – Schematic structure of automated control system of tool feed in adaptive milling

Adaptive milling effectiveness was investigated in experiments on wood working with variable speed of cutter feed in Robot-controlled machine.

Three wood samples were used: pine, birch, oak with square work contour and variable speed of cutter feed see in (Figure 3).

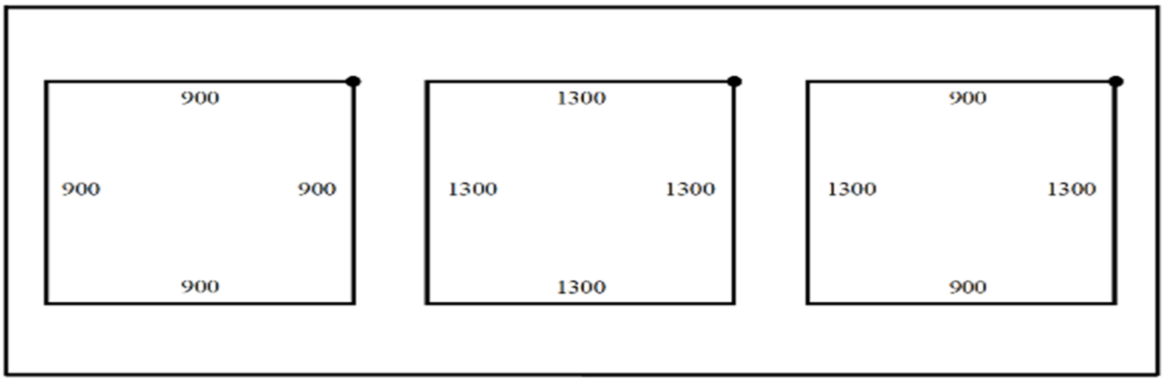

Figure 3 – Milling problem for Robot-controlled machine in experiment on variation of feed speed based on fiber orientation (feed speed is expressed in mm/min while milling pine wood surface)

Milling problem involved coned cutter pass of square contour see in (Figure 4–6). For each timber species milling along the square trajectory was performed in three different ways, based on optimal speeds for length-wise and cross-wise milling, defined during experiments. Square 1 was milled with constant minimal feed speed (in particular, for pine 900 mm/min, see in figure 3). Square 2 was milled with constant maximum (*out of optimum values*) feed speed (for pine 1300 mm/min.). Square 3 was milled with varying speed bearing in mind fiber directions (for pine two sections along fibers were milled at a speed of 900 mm/min, and two sections across fibers – at a speed of 1300 mm/min).

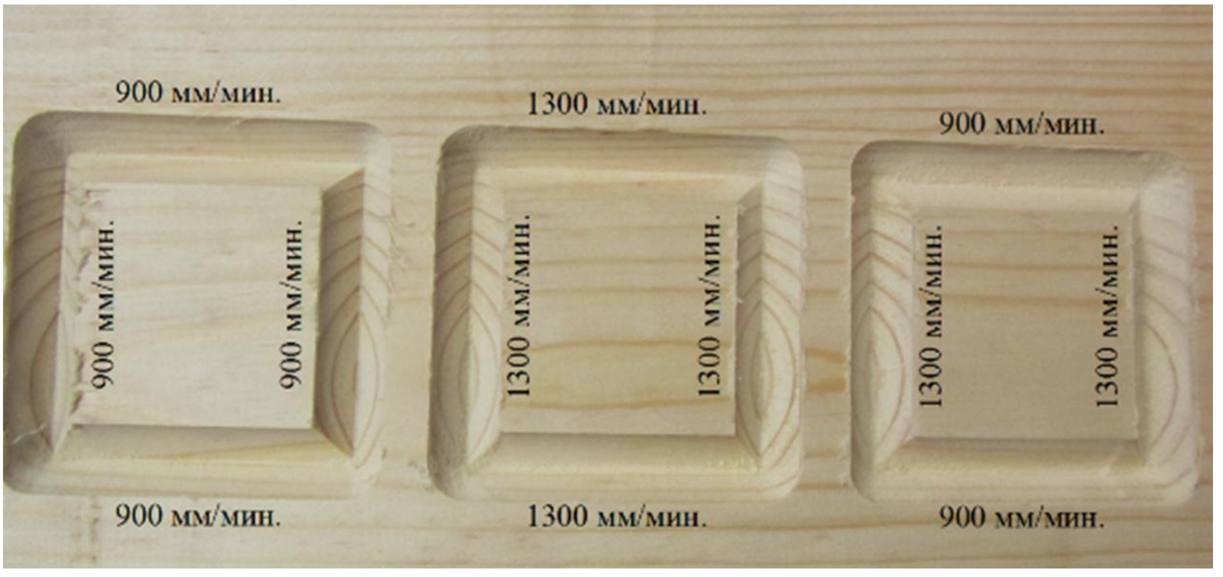

Figure 4 – Result of pine wood surface milling (supply rate along the milling contour sides is indicated in mm/min)

For birch and oak, milling of the first quadrate was also made with the minimum supply rate (1300 mm/min), milling of the second quadrate was made with the maximum supply rate (1700 mm/min), and milling of the third quadrate was made using two supply rate values (1300 and 1700 mm/min), depending on the wood species: for birch, the maximum supply rate of 1700 mm/min was used along the grain, for oak it was used against the grain. *Figures 4–6 show the results of milling of pine, birch and oak wood as per the described procedure.*

After milling, roughness of the twelve parts of the received grooves (the quadrate sides) was measured, and the average roughness value was calculated for each quadrate. Besides, total milling time of each quadrate was calculated based on the used supply rates. The measurement results are given in Table 1 where the cells with roughness and performance values that are the best for particular wood species are darkened.

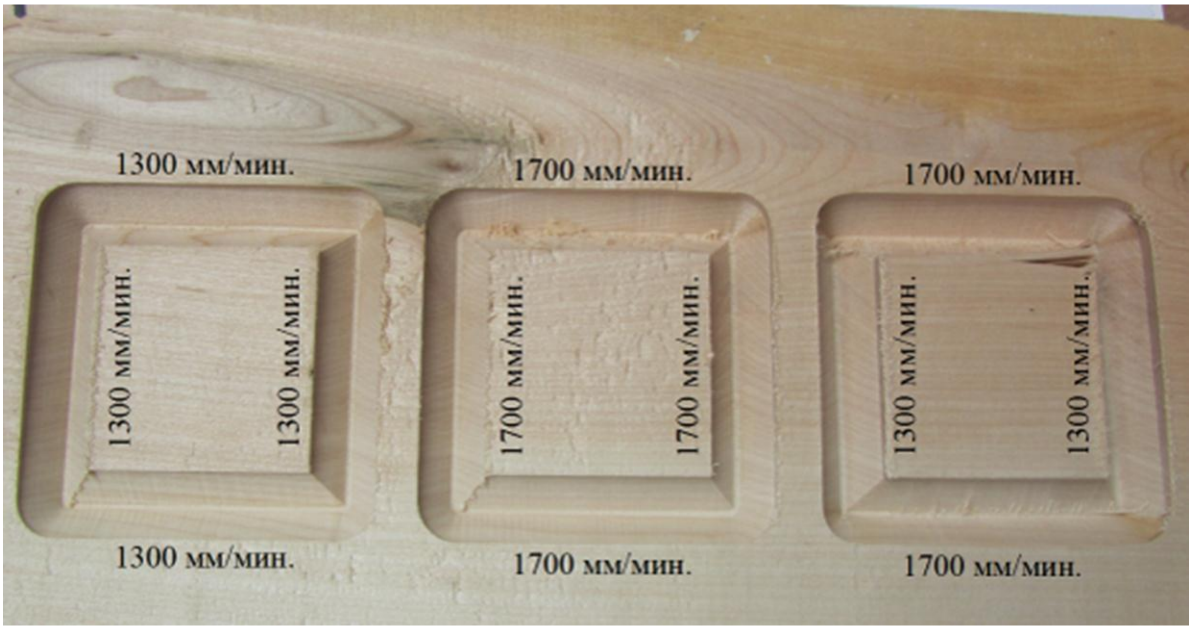

Figure 5 – Result of birch wood surface milling

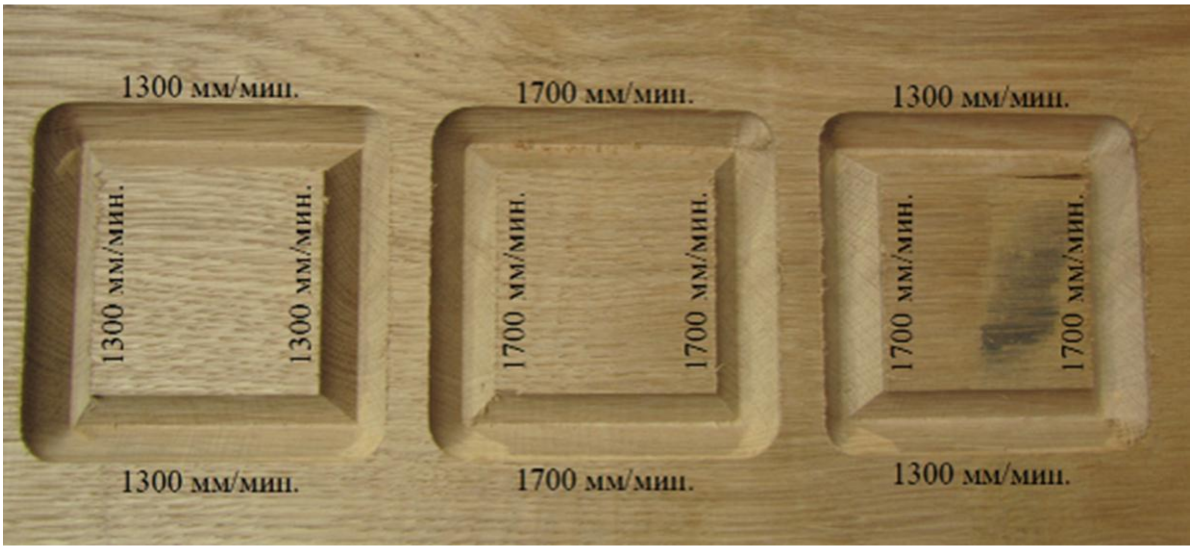

Figure 6 – Result of oak wood surface milling

Table 1 – Milling quality and efficiency with constant and variable rate of supply of the milling tool

Species	Quadrate	Average roughness, micron	Quadrate milling time, s
Pine	left (900 mm/min)	100	26,7
	middle (1300 mm/min)	115	18,4
	right (900 и 1300 mm/min)	85	22,6
Birch	left (1300 mm/min)	85	18,4
	middle (1700 mm/min)	95	14,1
	right (1300 and 1700 mm/min)	80	16,3
Oak	left (1300 mm/min)	115	18,4
	middle (1700 mm/min)	115	14,1
	right (1300 and 1700 mm/min)	110	16,3

Based on the review of the received results, the following conclusions can be made. If milling is made with lower supply rate (the left quadrate), high roughness values (due to the areas for which such supply rate is not optimal) and much time of the quadrate milling are received. If milling is made with higher supply rate (the middle quadrate), high roughness values (also due to the areas for which such supply rate is not optimal) and little time of the quadrate milling (that is positive) are received.

In regards to roughness reduction, the best alternative is the use of variable supply rate depending on the grain direction (the right quadrate). In this case, roughness is minimum, in particular, while milling pine, roughness is 18–35 % less than while milling with the constant supply rate. At this, time of milling of the quadrate is quite little in comparison with the cases with constant supply rate. Moreover, at the place of production,

lower supply rate is usually used, it is why acceleration of supply in separate areas results in significant increase in efficiency (in this case, by 23 % for pine).

For birch and oak, the use of variable supply rate also results in the reduction of roughness, in comparison with the constant supply rate, however, in a less degree (by 19 % and 5 % respectively).

It should be noted that at such milling with control of supply rate along the complex trajectory of supply of the milling tool, it would be possible to significantly reduce the percentage of shatters. This would allow to significantly reduce economic expenditures for additional operations concerning the surface recovery.

Conclusion:

Thus, in the process of adaptive milling of wood, taking into account direction of grain allows to reduce roughness, time of part processing and possibility of unacceptable surface damage.

Technology of adaptive milling can be used, in the first place, by small businesses, with small-batch and job-order production of furniture decorative fixtures. Under working conditions of these enterprises, there is no possibility to select mill conditions by experiment, as it is done at large furniture enterprises with large-batch orders.

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التحكم في تغذية أدوات العمل في ألة روبوت

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عقيل ثامر جواد

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المستخلص:

يتضمن نظام ضبط وحدة التحكم (PID) برنامج كمبيوتر مع برمجيات نظام تشغيل معينة ، تماما مثل أجهزة الإدخال / الإخراج وكابلات الموصل ، الاتصالات يمكن أن تؤدي إلى التسهيلات التي يتم تنفيذها عادة من قبل استخدام خادمOPC ، في حين التوافق هو الكائن الذي يشتمل في حلقة نظام التحكم. توافق النظام ومنسق ال (PID) هو التسجيل التام للحصول عليه من البراميتر بطريقة تكون قريبة من الاوبتيمل نتيجة لبديهية واجهة المستخدم، كومبيوترات عليه القدرة و خوارزميات تحديد نظام لا حدود لها.

كلمات مفتاحية: الة روبوت ، خادم OPC، وحدة التحكم (PID)

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An Adaptive Intrusion Detection System by using Decision Tree

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Abstract:

In recent decades, data security has turned into a new vision in data innovation as the quantity of PC protection breaks are presented to an explosion in the number of security incidents. An assortment of Sequence Discovery System have been utilized for shielding PCs and systems from pernicious system based or have based assaults by utilizing conventional measurable strategies to new information mining approaches in a years ago. In any case, the present monetarily accessible interruption identification frameworks are mark based that are not equipped for identifying obscure assaults. In this paper, we introduce another learning calculation for abnormality based system interruption identification framework utilizing choice tree calculation that recognizes assaults from ordinary practices and distinguishes diverse kinds of interruptions. Test comes about on the KDD-99 bench-mark organize interruption discovery data-set exhibit that the suggested algorithm resulted 98.5% Discovery rate in comparing with other executing techniques.

Key Words. Network Security, Sequence Discovery System, and decision tree.

1.Introduction. As propels in PC organize innovation grow for interchanges and business as of late, the rate of interruptions increment more than twofold consistently. Interruption discovery is the way toward distinguishing activities that endeavor to bargain the privacy, uprightness or accessibility of PCs or systems.

The utilization of information digging calculations for recognizing interruptions is currently considered to manufacture productive and versatile interruption identification frameworks (Sequence Discovery System) that identify unapproved exercises of a PC framework or system. Sequence Discovery System was first presented in [1], and later in 1986 an author [2] proposed a few models for Sequence Discovery System in light of insights, Markov chains, time-arrangement, and so on [2]. Irregularity based interruption identification utilizing information mining calculations, for example, Supervised Algorithms and Unsupervised Algorithms are fluffy rationale demonstrate, and hereditary calculation have been broadly utilized by analysts to enhance the execution of Sequence Discovery System [3]-[8]. Be that as it may, the present economically accessible Sequence Discovery Systems are mark based. Mark based Sequence Discovery System performs design coordinating strategies to coordinate an assault design comparing to known assault designs in the database and results “low False Positives (FP)”, yet it requires update the rules for the system to fit for recognizing obscure assaults. Then again, irregularity based Sequence Discovery System constructs models of ordinary conduct and naturally identifies atypical practices. Peculiarity identification procedures recognize new kinds of interruptions as deviations from ordinary utilization [9], yet the downside of these strategies is the rate of False Positives (FP). The utilization of information digging calculations for irregularity based Sequence Discovery System are to incorporate an insightful operator in the framework that can recognize the known and obscure assaults or interruptions.

Interruption recognition frameworks (Sequence Discovery System) assemble and break down data from an assortment of frameworks and system hotspots for indications of interruptions. Sequences Discovery System can be have based or organize based frameworks. Host-based Sequence Discovery System situated in servers to look at the inward interfaces and system based Sequence Discovery System screen the system traffics for recognizing interruptions . System based Sequence Discovery System perform bundle logging, continuous activity investigation of IP system, and tries to find if an interloper is endeavoring to break into the system. The real capacities performed by Sequence Discovery System are: (1) observing clients and frameworks movement, (2) evaluating framework design, (3) surveying the information records, (4) perceiving known assaults, (5) distinguishing unusual exercises, (6) overseeing review information, (7) featuring typical exercises, and (8) rectifying framework setup blunders. An assortment of Sequence Discovery System have been utilized for ensuring PCs and systems in a decades ago, yet at the same time there a few issues that ought to be consider in the present Sequence Discovery System like low location exactness, unequal identification rates for various sorts of assaults, and high False Positives. In this paper, another choice tree proposed for learning calculation to detect system assaults, which enhances the recognition rates and diminishes False Positives (FP) utilizing KDD-99 bench-mark arrange interruption location data-set in correlation with other existing techniques.

The remnants of the paper are sorted out as takes after. Segment 2 shows the different methodologies for irregularity based interruption discovery frameworks. Our proposed calculation for abnormality based system interruption recognition framework is presented in Section 3. In Section 4, the exploratory outcomes are communicated. At long last, our decisions and future works are specified in Section 5.

2. SEQUENCE DISCOVERY SYSTEM FOR ANOMALY DETECTION

In 1980, the idea of Sequence Discovery System started in [1]; a danger order demonstrate that builds up a security checking observation framework in light of distinguishing peculiarities in client conduct. In [1] model dangers are named outside infiltrations, inner entrances, and misfeasance. Outer infiltrations are interruptions in PC framework by outside interlopers, who don't have any approved access to the framework that they assault. Misfeasance is characterized as the abuse of approved access of both to the framework and to its information. In 1986, [2] specified a few models for business Sequence Discovery System improvement in view of insights, Markov chains, time-arrangement. In the mid 1980's, a research institute built up a Sequence Discovery Expert System that consistently observed client conduct and distinguished suspicious occasions [10].

Later SRI built up an enhanced adaptation of called the Next-Generation Sequence Discovery System [11], [12] that could work progressively for consistent observing of client action or could keep running in a cluster mode for occasional investigation of the review information, a review information is a record of exercises created by the working framework that are logged to a document in sequentially arranged request. The Sequence Discovery Expert System empower the framework to look at the present exercises of the client/framework/connect with the evaluated interruption discovery factors put away in the profile and afterward raise a caution if the present action is adequately a long way from the put away reviewed movement. In 1988, a measurable inconsistency based Sequence Discovery System was contributed by [13], which utilized both client and gathering based peculiarity recognition techniques. In this framework, a scope of qualities were viewed as typical for each characteristic and amid a session if a property fell outside the ordinary range then an alert raised. It was intended to recognize six kinds of interruptions: endeavored break-ins by unapproved clients, disguise assaults, infiltration of the security control framework, spillage, foreswearing of administration, and noxious utilize . (SPADE) [14] is a measurable oddity interruption recognition framework that is accessible as a module for SNORT that an open source organize interruption identification and counteractive action framework (NIDPS) created by Source fire [15], [16] .

An author proposed a similarity between the human safe framework and interruption identification that included breaking down a program's framework call successions to manufacture an ordinary profile [17], which broke down a few UNIX systems, based projects like send mail, ip, and so on. On the off chance that the arrangements veered off from the ordinary succession profile then it considered as an assault. The framework they created was just utilized disconnected utilizing beforehand gathered information and utilized a very basic table-query calculation to take in the profiles of projects. In 2000, [18] built up an irregularity based interruption discovery framework that utilized credulous Bayesian system to perform interruption recognizing on movement blasts. In 2003 [19] proposed a multisensory combination approach utilizing Bayesian classifier for characterization and concealment of false cautions that the yields of various Sequence Discovery System sensors were accumulated to create single alert . Around the same time, [20] proposed an oddity based interruption location conspire utilizing vital parts investigation (PCA), where PCA was connected to decrease the dimensionality of the review information and land at a classifier that is an element of the vital segments. In another paper , [21] proposed an irregularity based interruption identification utilizing concealed Markov models that registers the example probability of a watched arrangement utilizing the forward or in reverse calculation for recognizing odd conduct from ordinary practices. [22] Proposed characterization based peculiarity discovery utilizing inductive guidelines to portray arrangements happening in ordinary information. In 2000, [23] built up the Fuzzy Intrusion Recognition Engine (FIRE) utilizing fluffy rationale that procedure the system input information and produce fluffy sets for each watched highlight and afterward the fluffy sets are utilized to characterize fluffy guidelines to distinguish singular assaults . FIRE makes and applies fluffy standards to the review information to group it as ordinary or strange. In another paper, [24] displayed the odd system activity discovery with self-arranging maps utilizing DNS and HTTP administrations for organize based Sequence Discovery System that the neurons are prepared with ordinary system movement then constant system information is nourished to the prepared neurons, if the separation of the approaching system movement is more than a preset edge then it rises a caution. Another system have been proposed based irregularity recognition utilizing information mining methods created by Minnesota Sequence Discovery System in 2004 [25].

3. LEARNING ALGORITHM

A. The "Decision Tree (DT)" is intense and prominent information digging calculation for basic leadership and grouping issues. It has been utilizing as a part of numerous genuine applications like restorative analysis, radar flag arrangement, climate forecast, credit endorsement, and misrepresentation recognition and so on. DT can be developed from substantial volume of data-set with many characteristics, on the grounds that the tree measure is free of the data-set estimate.

A "Decision tree has three principle parts: hubs, leaves, and edges". Every hub is named with a characteristic by which the information is to be parceled. Every hub has various edges, which are marked by conceivable estimations of the characteristic. An edge interfaces either two hubs or a hub and a leaf .

B. Leaves are marked with a choice incentive for arrangement of the information. To settle on a choice utilizing a choice Tree, begin at the root hub and take after the tree down the branches until the point that a leaf hub speaking to the class is come to . Every Decision Tree speaks to an administerator set, which arranges information as per the properties of data-set. The DT building calculations may at first form the tree and after that prune it for more successful characterization. With pruning strategy, bits of the tree might be expelled or consolidated to diminish the general size of the tree. The time and space many-sided quality of developing a choice tree relies upon the extent of the informational index , the quantity of properties in the informational collection, and the state of the subsequent tree. Choice trees are utilized to arrange information with normal qualities. The ID3 calculation constructs choice tree utilizing data hypothesis, which pick part characteristics from an informational collection with the most astounding data pick up [26]. The entropy figuring is appeared in condition 1. Given probabilities p1, p2,..,pn for various classes in the informational index

"Entropy: $H(p1,p2,\ldots pn) = \sum_{i=1}^{\$}\left(pilog(1/pi)\right)$ " (1)

Given an index values, *D, H(D)* finds the measure of entropy in class based subsets of the informational collection. At the point when that subset is part into s new subsets *R = {D1, D2,… , Dn}* utilizing some characteristic, we can again take a gander at the entropy of those subsets. A subset of informational collection is totally requested and does not require any additionally split if all cases in it have a place with a similar class. The ID3 calculation ascertains the data pick up of a split by utilizing condition 2 and picks that split which gives most extreme data pick up.

"*Gain*(D,S)= H(D)- $\sum_{i=1}^{\$} p(Di)H(Di)$ " (2)

"Regression Trees" is a procedure of creating a twofold tree for basic leadership [28]. Truck handles missing information and contains a pruning system. The SPRINT (Scalable Parallelizable Induction of Decision Trees) calculation utilizes a contamination work called GINI list to locate the best split [29].

"*GINI* (D) = 1-∑ pj2 " (3)

Where, pj is the likelihood of class Cj in informational index D. The integrity of a split of D into subsets D1 and D2 is characterized by

"*GINISPLIT*(D) = n1/n(GENO(D1))+ n2/n(GENO(D2))" (4)

A. Improved Learning Algorithm

In a particular data-set, the first step the algorithm sets the weight's values for each example of data-set; W_i equal to 1/*n*, where *n* is the number of all examples in data-set .

After that the algorithm estimates the prior probability $P(C_j)$ for each class by summing the weights that how often each class occurs in the data-set. Also for each attribute, A_i, the number of occurrences of each attribute value A_{ij} can be counted by summing the weights to determine $P(A_{ij})$. In An algorithm C4.5 [27], which is an enhanced of ID3 version algorithm uses highest "*Gain Ratio"* in equation 3 for splitting issue that ensures a larger than average information that have been gained .

"*GainRatio(D,S)*= $\frac{Gain(D,S)}{\left(\frac{D1}{D}\ldots\frac{D\$}{D}\right)}$ "

(5)

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The C5.0 computation enhances the execution of building trees utilizing boosting, however is a way to deal with consolidating diverse classifiers. Be that as it may, boosting does not generally help when the preparation information contains a great deal of clamor. At the point when C5.0 plays out a grouping

another hand , the conditional probabilities $P(A_{ij} \mid C_j)$ are estimated for all values of attributes by summing the weights how often each attribute value occurs in the class C_j. Next, the algorithm uses these probabilities to update the weight's values for each example in the data-set. It's executed by multiplying the probabilities of the different features values from the examples. Suppose the example e_i has independent attribute values $\{A_{i1}, A_{i2},\ldots,A_{ip}\}$. We already know $P(A_{ik} \mid C_j)$, for each class C_j and attribute A_{ik}. We then estimate $P(e_i \mid C_j)$ by

$$\text{“}P(e_i \mid C_j) = P(C_j) \prod_{k=1\rightarrow p} P(A_{ij} \mid C_j)\text{”} \quad (6)$$

To update the weight, the estimation of likelihood for e_i in each class C_j. The probability that e_i is in a class is the product of the conditional probabilities. The posterior probability $P(C_j \mid e_i)$ is then found for each class for each attribute value .

The last step, the algorithm will calculates the information gain using updated weights and builds a tree for decision. The main procedure for the algorithm is described below :

Steps: Tree-Creation
Input:datat
a-set *D*
Output:
decision
tree *T*
Procedure:

1. Which are the weights in D, Wi=1/n, where n is the total number of the examples.
2. Figure out the prior probabilities P(Ci) for each class Ci
3. Compute the subjunctive probabilities P(Aij | Cj) for each feature values in D. “P(Aij | Cj) = P(Aij)/Ci
4. Figure out the posterior probabilities for each example in *D*. $\text{“}P(e_i \mid C_j) = P(C_j) \prod P(A_{ij} \mid C_j)\text{”}$
5. Update the weights by; “Wi= PML(Cj|ei)”
6. Discover the splitting attribute with the highest info. That gaining using the weights updated, W_i in *D*.T = construct the root node and label with splitting attribute.
7. For *T*, *D* = database created by applying splitting predicate to *D*, and continue steps 1 to 7 until each final subset belong to the same class or leaf node created.
8. After the decision tree construction is completed the algorithm terminates.

4. Experimental Results Analysis

A. Sequence Discovery Data-set

The KDD-99 data-set has been used as a part of the third International Knowledge Discovery and Tools for Data Mining Competition used for building a system interruption identifier, a prescient model fit for recognizing interruptions and typical system associations [30]. However, being impacted with numerous interruptions assaults and got much consideration in the examination group of versatile interruption discovery. The KDD-99 data-set challenge utilizes an adaptation of DARPA-98 data-set. In KDD-99 data-set, every case speaks to quality estimations of a class in the system information stream, and each class is named either ordinary or assault. The attack types in KDD-99 data-set sorted into five primary types as shown in Table 2.

1- Remote to User (R-2-L) is a strike that a remote customer acquires passageway of an area (customer/account) by targeting over a framework correspondence, which join send-letters. Client to Root (U-2-R) is an attack that an intruder begins with the passageway of a common customer record and a short time later transforms into a root-customer by abusing diverse gaps of the system. Most basic adventures of U-2-R assaults are customary cradle floods and stack module.

2- “Denial of Service (DoS)”: the source of the handling power or memory of a setback target unreasonably possessed or too full caused by DoS attacks, making it difficult to manage genuine requests. DoS attacks are requested in perspective of the organizations that an attacker renders unavailable to honest to goodness customers like Apache 2, arrive, mail bomb, back.

3- Probing: A gatecrasher with a guide of targeted and administrations that are accessible on a system can utilize the data to search for misuses. And is an assault that outputs a system to assemble data or find known system vulnerabilities. In KDD-99 data-set these four attack classes (DoS, U2R, R2L, and probe) are divided into 20 different attack classes that illustrated in Table 1.

TABLE 1
The Types of Attacks in KDD-99 Data-set

Four Attack Classes	20 Attacks Classes
“Denial of Service (DoS)”	“land, back, neptune, pod, smurt”
Remote to User	“imap, ftp_write, guess_passwd, , warezmaster multihop, spy, warezclient”
User to Root	“perl, loadmodule, buffer_overflow, rootkit”
“Probing”	“ipsweep, nmap, satan , portsweep“

There are 41 input qualities in KDD-99 data-set for each system association that have either discrete or nonstop esteems and partitioned into three gatherings . The primary gathering of characteristics is the fundamental highlights of system association, which incorporate the “span”, “model”, “benefit”, number of bytes from source IP addresses or from goal IP locations, and a few banners in Transfer Protocol. The second gathering of qualities in KDD-99 is made out of the substance highlights of system associations and the third gathering is made out of the factual highlights that are figured either by a period window or a window of certain sort of associations.

Table 2 demonstrates the quantity of cases of 10% preparing illustrations and 10% testing cases in KDD-99 data-set. There are some new assault cases in testing information, which is no present in the preparation information .

TABLE 2
Attack Types’ in KDD-99 Data

Attacks Type	Training Cases	Testing Cases
“legitimate”	97233	60423
“Denial of Service”	391445	237593
“Remote to User”	1326	8516
“User to Root”	62	80
“Probing”	4304	4276
Examples		310868

A. Analysis of Experiment

The evaluation of the performance for the enhanced algorithm for sequence discovery, 5-class have been executed to classy the anomalies using KDD-99 data-set. All the experiments were executed by using (Intel Core i7-2640M Processor 2.80 GHz processor with 8 GB of RAM). The results of the comparison of the enhanced algorithm along with ID3 and C4.5 as shown in Table 3 .

TABLE 3
41 Attributes for Comparison

Method	legitimate	Probe	DoS	U-2-R	R-2-L
New Algorithm (Data Rate %)	98.67	98.51	98.54	98.52	97.26
New Algorithm (False Positive %)	0.09	0.53	0.07	0.17	7.89
ID3 (Data Rate %)	97.67	96.45	97.51	43.23	92.67
ID3 (False Positive %)	0.11	0.66	0.05	0.18	10.12
C4.5 (Data Rate %)	98.45	97.90	97.56	49.23	94.70
C4.5 (False Positive %)	0.14	0.57	0.09	0.17	11.09

5. CONCLUSION

This paper presents another learning approach for peculiarity based framework intrusion revelation using decision tree, which modifies the weights of data-set in light of probabilities and split the data-set into sub-data-set until all the sub-data-set has a place with a comparable class. In this paper, the Sequence Discovery System using decision tree has been developed. The trial comes to fruition on KDD-99 benchmark data-set demonstrate that proposed estimation achieved high area rate on different sorts of framework strikes. The future research issues will be to test it extensively to get higher detection rate .

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نظام كشف التسلل التكيفي باستخدام شجرة القرارات

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المستخلص :
في العقود الأخيرة ، تحول أمن البيانات إلى رؤية جديدة في ابتكار البيانات حيث يتم تقديم كمية من فواصل حماية الكمبيوتر الشخصي إلى انفجار في عدد الحوادث الأمنية. تم استخدام مجموعة متنوعة من نظام اكتشاف التسلسل من أجل حماية أجهزة الكمبيوتر والأنظمة من الاعتداءات الخبيثة القائمة على النظام أو التي تعتمد على النظام من خلال استخدام الاستراتيجيات التقليدية القابلة للقياس إلى أساليب التعدين الجديدة للمعلومات في السنوات الماضية. وعلى أي حال ، فإن الأطر القائمة الحالية لتحديد الهوية التي يمكن الوصول إليها بصورة نقدية تستند إلى علامة غير مجهزة للتعرف على الاعتداءات الغامضة. في هذه الورقة ، نقدم حسابًا تعليميًا آخر لإطار تعريف عدم انتظام النظام القائم على الشذوذ باستخدام حساب شجرة الاختيار الذي يعترف بالاعتداءات من الممارسات العادية ويميز الأنواع المتنوعة من الانقطاعات. يظهر الاختبار على KDD-99 تنظيم قاعدة البيانات اكتشاف تنظيم مجموعة معارض أن الخوارزمية المقترحة أسفرت عن ٩٨.٥٪ معدل اكتشاف مقارنة مع تقنيات التنفيذ الأخرى.

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A model for human activities recognition using partially occluded images

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Abstract:

The demand for Human Activities Recognition (HAR) from still images and corresponding features categorization using appropriate classifier is never ending. HAR of partially occluded object using still image is highly challenging than extracting information from video due to the absence of any prior knowledge resembling frames stream .This new research domain dealing with the computer identification and subsequent classification of specific activities to develop understanding of human behaviors has diversified applied interests in surveillance cameras, security systems and automotive industry. We propose a new model for HAR with occluded missing part of human body in still image.(A template is built to complete the full appearance predictable by the system and their subsequent features classification is made.)The problems of HAR in still images are addressed using nonlinear Support Vector Machine classifier. The process of de-noising and chamfer matching are performed. This model is simulated with1200 still images of (64 x 128) pixels based on existing datasets such (INRIA and KTH). Using this model ,a recognition rate 86% is achieved for seven activities such as running, walking, jumping, clapping, jogging, boxing and waving .The excellent features of the results suggest that our method may constitute a basis for HAR with occlusion human body parts in still images promising for accurate features classification.

Keywords: Human activities recognition, still image, occlusion human body, features extraction.

1. Introduction:

Lately, HAR using still images via the inference of human pose became the key issue in precisely characterizing and classifying the behavioral pattern .The major difficulties in estimating human postures from still image using partially occluded human body in the scene provided further impetus in features categorization due to its relevance towards sundry applications [1]. One of the most popular approaches in this research domain is the pictorial structure framework [2,3]. Despite many efforts, optimal and efficient models for HAR are far from being developed.

Detail examinations of the activities from still images when the human body is partially occluded in the scene are our interests. However, training the activities recognition system with only available knowledge on the rigid parts of the image without any other prior information is indeed a challenging task. The main excitement lies in extracting the full feature from partially occluded object and then classifying it for the human body. Unlike other methods [4] that use the entire body and their full feature matching ,the present approach is completely new. Undoubtedly, there is a need to establish an optimal model for recognizing human activities with high level of accuracy which may be accomplished by a statistical measure based on the data likelihood. Recently, intensive experimentations are conducted on several datasets[5] to enable an understanding on how many activities recognition are supported.

In the last decade, human activities recognition of video images is widely studied. Several methods are proposed to improve the accuracy and robustness to cope with challenging video .Many algorithms are developed to achieve accurate results with high recognition rate as much as 100% for single activity .The most important factor is the unavailability of prior knowledge where the generated current information can be compared with the previous one as a frame in video sequences .Although, the human activities recognition using still image being poor in terms of prior knowledge even then good and powerful features are extracted from the human body [6].For effective and influential features the object should be integral part of the image .In this viewpoint we propose a method to complete the occluded object in still image then extract features and classify the activities to get them recognized.

The recognition of human interaction with other objects in realistic video demonstrated a correlation between the entities and behavior of the human parts [7].Based on this interaction pattern recognition, a method is developed to keep track of the hand part with corresponding distance of the head and this kind of feature is found to be suitable when some activities including drinking, smoking, eating, and so on are needed .This paper aimed to represent the object trajectory by drawing the path for each object to compare with the position of human part. This approach can efficiently reveal a relation between object and human parts. The occlusion of some parts of the human body does not cause any problem provided the activity must be related and compatible with those particular objects such as smoking and drinking.

Bo and Ram [8] introduced an approach to automatically detect and track multiple with possible partially occluded human in two activities of walking and standing pose given by one camera .Essentially, a human body is treated as an assembly of body parts. This method learnt by boosting a number of weak classifiers which are based on edge-let features. The occluded parts of the body are solved by joints likelihood model including an analysis of possible occlusions. Sung J. and Mark S. proposed another method [9] comprised of detecting the heel strike using gait trajectory model containing many striking features such as robustness to occasion, camera view and low resolution. This head movement based method is conspicuous and sinusoidal in which the region of interest is extracted as feature from silhouette image. The number of times the heel strike is regarded to find the trajectory model which is effective when used for HAR in video stream.

The detection of human behavior remains a challenge in computer vision and HAR especially due to highly articulated body postures, changes in viewpoints, varying illumination condition and clustered background. To avoid these difficulties most of the previous studies on HAR often considered on low-articulated postures. Trung and Phong [10] proposed a new method to detect human region from still images using raw edges. The object fixed in the still images is mostly obtained from pedestrians with varieties of occurrences inside the image .This approach produced better results than sliding window-style method for detecting human activities .The method relies on characteristics of boundaries and interest points that combine different image processing techniques such as image mean algorithm and probability for choosing a human region .INRIA challenged dataset is used because it possesses multi-activities with several view-points in addition to the occlusion clusters under varying length condition indoor and outdoor scenes.

Precisely, each HAR system must pass through several stages processing such as removal of noise, segmentation, feature extraction and classification. Mehmet and Alper established a technique composed of a fuzzy inference system and an edge detection and dilation unit for removing the noise from image to address the speckle noisy image [11]. An efficient and accurate feature selection and classification method is introduced by Habil [12] to achieve a superior classifier .This approach is capable of learning the system by running it simultaneously in an online manner to estimate the parameters within a nearest neighbor with Gaussian mixture model. Halime [13] developed a Cellular Neural Network model to calculate the adaptive iterative value via wavelet transform and spatial frequency for segmenting the mutable region inside an image .An integrated automatic image capture strategy for automatically analyzing large numbers of image via evaluation software is used .A learning based framework for action representation and recognition on the description of an action by time series of optical flow motion features is reported by Michalis and Vasileios [14]. The Gaussian mixture modeling is used to cluster the activities .Excellent experimental results are achieved by using Weizmann, KTH, UCF sports and UCF you tube action datasets.

In this paper, we address the problems of HAR in still image having partial occlusion using one of the most popular classifier called Support Vector Machine(SVM).High accuracy is achieved for the features classification when applied to the images having a missing part of the human body or occlusion .The model is simulated on INRIA and KTH datasets with several still images using Matlab programming. This work renders three major contributions .Firstly, the completion of the occluded object (a part of human body) that occurs out of the scenes when the object is hidden .The features are then extracted from this occluded part to suggest the remaining parts of the body. Secondly, the proposed new features are highly compatible with the nature of activities preferred for classification .Lastly ,a high recognition rate is achieved using the non-linear SVM classifier for seven activities such as walking, running, jogging, waving, jumping, clapping, and jumping.

2. Methodology

To achieve Human Activities Recognition system we take in our consideration five main stages first start with preprocessing stage that consist of remove the noise from given image and segment it to get the object (human body) extracted from background image, then feature extraction to extract the important features needed in the final stage with classification to classify the activities using non-linear SVM classifier.

2.1 Noise Removalor De-noising

The presence of noise in image is one of the prime problems in computer vision .Noise appears when the images are taken using fixed camera, two sources imaging device and surveillance camera .Removal of noise from image is prerequisite for processing .Several methods are suggested to handle this difficulty .Among all these methods, the most simplistic and powerful is the standard total variation de-noising introduced by Rudin-Osher Fatemi called ROF model [15].Following the classical form of a regularization approach, we minimize the objective function consisting of data-fidelity combined with regularization term and describe it in a discrete framework. Denoting N as set of nodes of the pixel grid of size h and the image coordinates by x which is a function of (x,y), the following optimization problem need to be solved to obtain smoothed data$\upsilon$from noisy input f,

$$\min_{\upsilon}\left[\left(\sum_{x\in N}\frac{1}{2}\omega(x)(\upsilon(x)-f(x))^2\right)+\lambda R(\upsilon)\right] \quad (1)$$

Where the $\lambda R(\upsilon)$ is the regularization and $\omega(x)$ is the weight term used to account for the locally varying noise variance. Rescaling of the regularization parameter λ yields $\omega(x)$=1. The regularization parameter $(\lambda > 0)$ controls the amount of smoothing. The regularization term for ROF takes the form,

$$\lambda R(\upsilon) = \sum_{(x,y)}\|\nabla_{\upsilon}(x,y)\| \quad (2)$$

$$= \sum\_((x,y))▒√(〖(\upsilon(x+h,y)-\upsilon(x,y))〗^2 + (\upsilon(x,y+h)-\upsilon(x,y))^2 \quad (3)$$

According to this formula, 3 x 3 window is used to cover all given image except corner area that can Stretching and shrinking based on (h) factor to find the averaging values.

2.2 Segmentation

The most significant part in HAR is the segmentation without which features cannot be extracted from an object (part of human body). Conventionally, the segmentation is based on the gradient of the image acting as the stopping term and cause unsatisfactory performance in noisy images[16]. A new technique is proposed that does not consider the edge information but utilizes the difference between the regions inside and outside of the curve .This is one of the most robust and widely used techniques for image segmentation of human body. In the preprocessing stage an object inside the scene is segmented using an energy function defined by,

$$\mathbf{F(c)} = \int_{\mathbf{in(c)}}|\mathbf{I(x)} - \mathbf{C_{in}}|^2\,\mathbf{dx} + \int_{\mathbf{out(c)}}|\mathbf{I(x)} - \mathbf{C_{out}}|^2\,\mathbf{dx} \quad \mathbf{(4)}$$

Wherex $\in \Omega$ (the image plane)$\subset R^2$, $I:\Omega \longrightarrow Z$ is a certain image feature with intensity of color, or texture. Here ,C_{in} and C_{out} are the mean values of image features inside[in(c)] and outside[out(c)] the curve C. By considering image segmentation as a clustering problem the two different segments (clusters) are minimized. This procedure is not efficient for segmented object within inhomogeneous global region but powerful for local minimization such as human body being usually homogenous.

The parameters used inside (local C_{in}) and outside(global C_{out}) of human body and I make this method acceptable in term of still image.

2.3 Chamfer Matching(complete missing part)

Chamfer matching is a popular technique for finding the set alignment between two edge maps. Let U={u_i} be the best of template and V={v_j} be the set of query image edge maps. Then, the chamfer distance between U and V is defined as the average of distance between each points u_i ∈U and its nearest edge in V and is expressed as,

$$d(U,V) = \frac{1}{n}\sum_{u_i\in V}\min_{v_j\in V}|u_i - v_j| \quad (5)$$

Assuming W is a warping function defined on the plain of image and s is a parameter with 2D Euclidean transformation, such thats $\in SE(2)$,$s = (\theta, t_x t_y)$. Here θ is the in-plane rotation angle ,t_x and t_y are the translation along x axis y axis, respectively. The action on image points is follows the transformation,

$$W(x,s) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} x + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \quad (6)$$

This action can check the entire edge of the shape for one template at different angles and for the best alignment parameters $\hat{s} \in SE(2)$ between two edge maps onegets,

$$\hat{\mathbf{s}} = \mathbf{arg\,min}_{s \in SE(2)}\, \mathbf{dcm}(\mathbf{W}(\mathbf{v},\mathbf{s}),\mathbf{V})$$
(7)

where W(U,s)={W(u_i,s)}.

The matching cost within chamfer matching can be computed efficiently by distance transform image $DT(x) = \min_{v_j} \in V\,|x - v_j|$by specifying the distance from each pixel to the nearest edge pixel of V. The evaluation of linear time distance transform of O(n) is obtained via $dcm(U,V) = \frac{1}{n}\sum_{u_i \in U} DT(u_i)$.

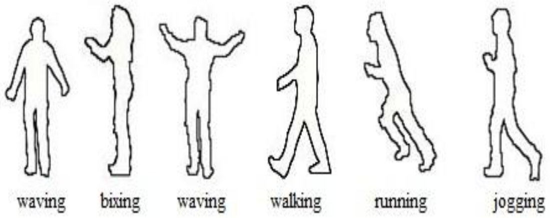

Figure 1 : Full body template for six activities used by our method.

Let M={P_1,P_2,…,P_N} is a template model in which each $P_{i,j}$∈ {1,…,N} is a member of the template (or contour) representing the shape of the i-th part of the object. To obtain the template model M, a set of K templates (vector) λ={T_1,T_2,…,T_k} considering all possible shapes of the human body are collected as shown in Figure 1. Each template $T_j \in T_{,j}$∈ {1,…,k} is centered in a fixed size window and then divided into N parts to create N part templates T_{j1}, T_{j2}, …,T_{jN} as illustrated in Figure 2. We assume that the number of parts (N) and their location l_i=(x_i,y_i), I∈ {1,…,N} can be determined in advance. For each type of parts (i-th part), the corresponding template set {T_{ji}}, j∈ {1,…,K} are aligned based on their location li and clustered by using a K-means algorithm. The set pi consists of mean templates signifying the centerscf{T_{ij}}.

Figure 1 depicts the template model M representing shape of the human body. The full human body structure is decomposed into four different parts (N=4) including head-torso or upper, bottom (legs), left and right. The layout of these parts is displayed in Figure 2. The K-means algorithm [20] used 5,8,6 and 6 parts for defining the three main layout left-right, bottom and top which represent the number of clusters.

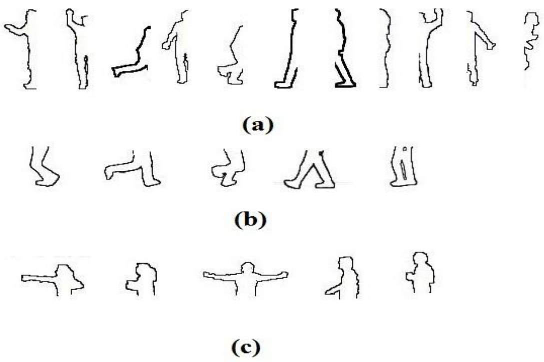

Figure 2 : Parts of template (a) Left-Right part, (b) Bottom part and (c) Upper part.

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The total number of 5+8+6+6=25 templates (each template is different from another according to its side for example right or left side got six shapes while down side got shapes) is used to cover 5*8*6*6=1440 postures which represent the total number of features in this model. Local appearance features are extracted at the key points (joints). The key points satisfy all the human body and overall activities for the human .Therefore ,the selection of joints as key points makes it reasonable. Each template must have at least four key points to reconstruct the human body.

After defining the template model M, the sets of key points are detected to determine the shapes of the object. A candidate region called detection window at location L=(x,y) is a collection of sets of key points $G(I, M)$ is given by the expression,

$$G(I, M) = \cup_{i=1}^{N} S^{li}(I, P_i) \quad (8)$$

where ∪ represents the redundant union operator on subset in which the elements of individual sets can be duplicated in the union set, $S^{li}(I, P_i)$. such P is the set of key points representing the shape of the i-th part located at L_i which is obtained from,

$$S^{l}(I, \mathcal{T}) = \{e(t) \in E(I) | t \in T^{l}\} \quad (9)$$

where e(t) is the closest point of t in entire point, E(I). Note that, every template T∈Pi is already been sampled at the same number of locations.

This method is better than others in terms of high accuracy when detecting objects included in the still image .However, the calculation is rather time consuming because of the mathematical rigor ,especially if more activities considered online .The approach is considered to be effective for the estimation of the missing parts of the human body widely used in many security and vision applications.

2.4 Features Extraction

In acquiring features we use sampled patch P which is additionally augmented with vector V_p that consisting of associated joints J_K. The functional dependence based on squared distances yields,

$$\mathcal{H}(P) = \frac{1}{|P|} \sum_{p \in P} \| V_{p,k} - M_k \|^2 \quad (10)$$

where M_k denotes the mean, and V_P consider a vector of joints in one block then can get vector of features $\mathcal{H}(P)$.

The probability of the class, $P(c/L_T)$ and probability of vectors ,$P(V/L_T)$ are stored. The unary potential at location x for the joint K is given by,

$$\Phi_K(J_K(X)) = \sum_{Y \in \Omega} \frac{1}{|T|} \sum_{T \in \mathcal{T}} p(c = k | L_T(p(Y)).\, p\left(X - Y | L_T(p(Y))\right) \quad (11)$$

where J∈ {0,1}.

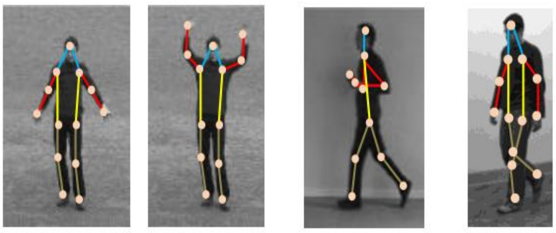

Figure 3 : Simple images from KTM dataset showing stick structure.

Figure 3 depicts the detail mechanism of extracting features from detected object inside the image. Six parts of the body including Torso, Upper leg, Lower leg, Upper arm, Fore-arm and Head make easier to construct the human body for extracting the features according to the structure of pose tree. The tree starts with root in the head joint and continue to the terminal leaf in feet and wrist. The distance among joints in torso and shoulder is calculated to complete entire body features.

2.5 classification

In order to identify the activities ,SVM classifier were used to classify each activity according to given vectors of feature extracted from an object, the non-linear classifier clustering the features of such activities in to the groups and these groups tends to be small and far of each other be suitable margin to recognize the activity of specific features. These features determine the activity belong to which class which are already arrange accordingly as shown in Figure (4)

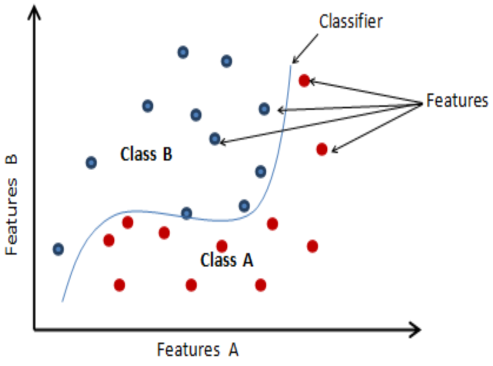

Figure 4: classifier architecture

For more description non-linear formulation of SVM was used to classify *w* features to put it within class 0 or 1 of *n* template to all shapes as illustrated bellow:

$$\left[\frac{1}{n}\sum_{i=1}^{n} max\left(0,1-y_i(w.x_i-b)\right)\right]+\gamma\|w.\|^2$$

Such kernel used with non-linear SVM is y where represented as $y \in \{2^{-2}2^{-1}\dots 2^{1}2^{2}\}$ as class A or B for each pair activitie.

3. Results and Discussion

Classification of human activities depends on number and types of features extracted from the object .In the proposed method we extract the full features from an image derived from the entire body by detecting the occluded or missing parts. Figure 5 displays the achieved recognition rates of the occluded human body parts.

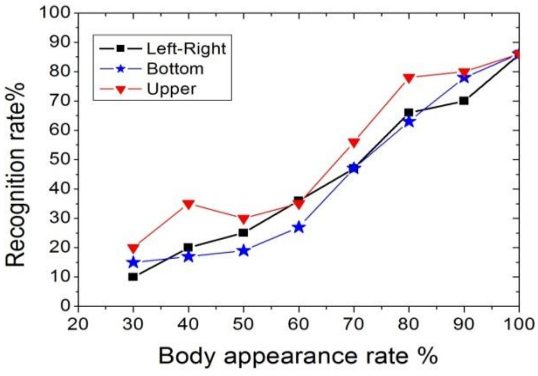

Figure 5: Recognition rates of three body parts with the appearance of each part.

Recognition rate can only be observed beyond 30% because the system cannot recognize less than a certain threshold of extracted features from the part of the body and need to satisfy a minimum number .For instance, head has no features to extract compare to other parts of human body. Meanwhile, the feature types used in this research do not conform to the head of the body which is only 12% of the human body.

Superior recognition rate depends on the positive detection rate and the choice of corresponding features in advanced. In case, the missing part from the body is too much then the positive detection is low and there by harmonizing with correct estimated template become difficult. Figure 5 shows positive recognition with corresponding samples of body part .Here, features are extracted from each part by making a relation between them and the amount of occluded parts .Less recognition rate occurs when the appearance is low enough making it difficult to extract good features needed by the classifier.

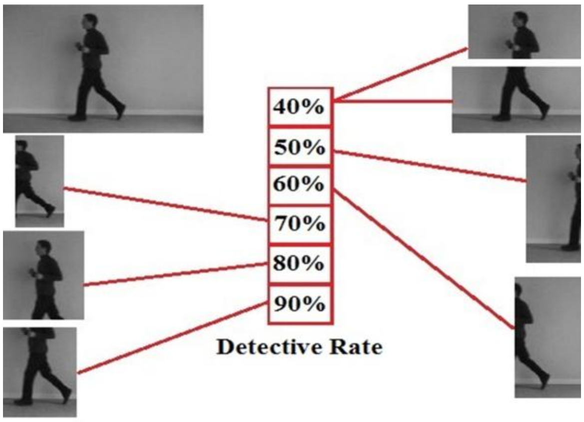

Figure 6 : Positive detection rates with corresponding body parts in training system.

These detective rate are computed experimentally when training the system then become fixed for testing algorithm .There are several images in KTH dataset where the detection of some body parts is very difficult. For example, with arms due to wearing of different clothes hiding the arms and some human position when tilted make them impossible to detect .This imparts additional difficulty in the complete detection of entire body parts using the proposed model. In KTH dataset there are different activities such as walking, jogging, running, waving, boxing and clamping. All these activities are performed via 25 different characters for both sexes with different scenarios including outdoors, indoors, outdoors with scales and outdoors with different clothes. These features are inputted in our proposed non-linear SVM classifier to achieve high-quality and accurate recognition rate.

The recognition rate with proposed method (86 %) is found to be highest with the full extraction of powerful features and the corresponding detection of the human activities .The results obtained by us using the same two datasets as reported earlier display highest recognition rate compare to that of previous researchers. The full color high resolution images from the dataset of INRIA and gray-look low resolution from KTH data set used in this work are shown in Figure 7.

Figure 7 : Two different datasets used in our system.

To characterize the activities as much as possible it is important to detect each part of the human body, especially when using some occluded or missing body part we are trying to estimate the whole body according to a given appearance .Detection and estimation of such appearance is significant to built and suggest a suitable template in advance. The features extracted from available incomplete parts of human body renders low recognition rate which need to be enhanced to achieve highest possible recognition rate. The relation between positive detection of objects and features extracted from human body is illustrated in Figure 8. The most important features of human body appearance are observed to be extracted from 55 to 82%.

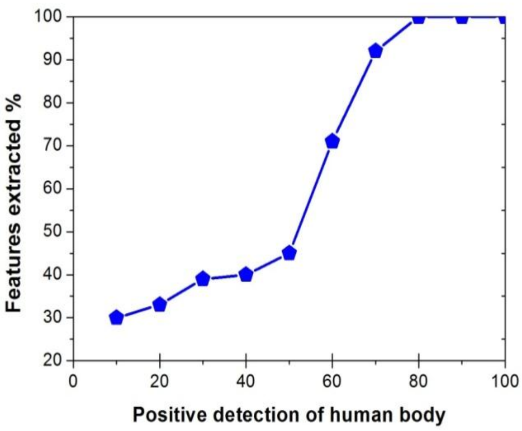

Figure 8 : Relation between feature extraction and human body appearance.

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The results for the confusion matrix with supervised classification are furnished in Figure 9. Interestingly, for activities such as walking, running and jogging the system is capable of recognizing the features extracted from two parts of the body (arm and leg) and a high recognition rate is accomplished. In contrast, the activities such as boxing and waving achieves relatively lower recognition rate because the features are extracted from one part of the body (arm).

We evaluate the accuracy of our method with seven activities using two different datasets

	Running	Walking	Boxing	Jogging	Waving	Clapping	Jumping
Running	0.84	0.01	0.04	0.03	0.0	0.0	0.0
Walking	0.12	0.90	0.02	0.04	0.02	0.3	0.0
Boxing	0.0	0.02	0.86	0.0	0.13	0.1	0.2
Jogging	0.04	0.14	0.01	0.86	0.0	0.0	0.0
Waving	0.0	0.03	0.0	0.02	0.81	0.13	0.0
Clapping	0.0	0.0	0.0	0.1	0.14	0.79	0.1
Jumping	0.0	0.2	0.0	0.0	0.0	0.1	0.87

Figure 9: Confusion matrix showing recognition rate for seven activities.

because some activities are available only in a particular dataset and some belong to another. Training set with INRIA dataset is found to be much more computationally time consuming than with KTH because each image is changed to gray scale followed by noise reduction, segmentation and so on. Meanwhile, training with INRIA dataset consisting of 780 images of (64 × 128) pixels format for each activity takes longer time to learn the machine.

4. Conclusion

A new method is proposed for recognizing activities in still images for occluded human body or missing part out of the scenes .We design the templates for some activities with different sides to complete and estimate occluded body parts for extracting powerful features from stick structure which are divided into six parts according to the pose tree of joints of human body. The classifier is used in three main stages including preprocessing (foreground extraction), segmentation (background subtraction) to extract useful features from object and sort out these features by the classifier (classification). The process of de-noising and chamfer matching allows us to obtain high-quality and accurate features. Our method achieves the highest recognition rate as much as 86% for seven activities including running, walking, boxing, jogging, waving, capping and jumping. This method despite time expensive is capable of extracting features from partially appearance human body parts and fully estimated body to classify the activities using non-linear SVM classifier using two public datasets KTH and INRIA .The model is simulated using MATLAB programming for several images in high pixels format. We assert that the appropriate implementation using suitable human computer interfaces critically depend on such image analyses useful for widespread applications from surveillance to security systems to automation.

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نموذج للتعرف على الانشطة البشرية باستخدام صور مغلقة جزئيا

فضيلة صبري ابو الماش
دائرة البعثات والعلاقات الثقافية

المستخلص :

ان الحاجة الى انظمة التعرف على الانشطة البشرية (HAR) يتم من خلال الصور الثابتة التي ازدادت في الآونة الاخيرة وما يقابلها من تصنيفات للمواصفات واستخلاصها وتصنيفها بالمصنفات القياسية المتوفرة. انظمة التعرف على الانشطة البشرية التي تهتم بالجزء المفقود من الصورة يحوي على تحدي كبير وذلك لقلة التعامل مع الانظمة التي تعتمد على الصور الثابتة اما الصور المتحركة والفيديو فهي شائعة الاستعمال. لذلك التحدي في الصور الثابتة كون المعلومات المسبقة للصورة لا تتوفر ويلزم التنبؤ بها.

اهمية البحث تأتي من المصالح الرئيسية التي تعتمد على كامرات المراقبة في الشركات الامنية والمصانع. ففي هذا البحث نقترح نموذج جديدا لتمييز الانشطة البشرية للجزء المفقود من الجسم في صورة ثابتة بعد التنبؤ بالجزء المفقود ثم استخراج المواصفات للشكل وبعد تصنيفه نحصل على دقة مناسبة. اشكال معينة بنيت لإتمام الاجسام في الصورة الذي يلزم لاستخراج المواصفات الكلية للجسم بسبب انه لايمكن للنظام التنبؤ الا في حالة اكمال الجسم. مشكلة التصنيف تم معالجتها عن طريق مصنف قياسي معروف (SVM). بعض الاجراءات اتبعت للصور قبل المعالجة وذلك عن طريق ازالة الضوضاء من الصورة وكذلك في نظامنا تم استعمال ١٢٠٠ صورة ثابتة بدقة (٦٤*١٢٨) بكسل من قاعدة بيانات قياسية (INRIA, KTH) . معدل الدقة المنجز هنا تقريبا ٨٦% مع سبعة فعاليات بشرية هي: الركض، الجري، المشي، القفز، النشر، الملاكمة، حني اليد. المواصفات التي تم الحصول عليها تعتبر واعدة بالنسبة الى الانظمة ذات الصور الثابتة.

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A Surveyof human face detection methods

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Abstract

The aim of this research is to produce a critical survey on detecting and extracting human face with the features over the past 10-15 years. Face Detection is one of the most common techniques in various future visual applications, such as teleconferencing, facial recognition systems, biometrics and human computer interface , not only because of the difficult nature of the face as an object, but also because of the myriad applications that require face detection application as a first step .

Finally, we offer a facial detection technology based on skin color segmentation as well as the facial features inside it. The determination of the human face as elliptical area was achieved. Meanwhile, several techniques were used such as enhancement, thresholding, edge detections and binarization techniques to achieve the aim of the suggested method. The facial features are detected and extracted inside the elliptical area; these features can be categorized in three parts: Nose, Mouth and lips localization. The features are detected and extracted in the human face based on image processing techniques.

Keywords: Face Detection, Facial Features, Image Based Methods, Feature Based Method, Image Processing

1. Introduction

One of the most concerted biometric techniques is a face detection, which can be used in many applications such as Identification of persons, access to government institutions, measurement of breathing and surveillance [1]. Human facial extraction techniques are the most important and decisive step within the image processing, artificial intelligent and pattern recognition. The concepts of these techniques are looking for significant information in the human image [2].

This paper provides a review of the human face detection methods as well as the facial features. The area of the face and features detection become a common in the fields of the research due to their importance and multiple applications. Facial detection and extraction techniques suffer from many limitations that may be have a clear impact on these techniques, such as skin color, facial hair , wearing glasses and facial expressions. Additionally light conditions and the head movements in an image are the most common and difficult problems [3]. The human face is changeable and cannot be easily detected or recognized, this problem complicates computer work with the help of the recognition or detection system [4]. So many researchers have been looking into this area to find advanced ways to develop this field [5]. The methods proposed for human face detection can be categorized into two fields: **Feature-based methods** and **Image -based methods**. Where **Feature-based methods** are classified in three groups*:* **Active shape model, low level analysis and feature analysis methods** while **image based methods** are classified in three groups: **Neural network**, **Linear Sub Space** and **Statistical Approach methods [6].**

Table 1: Illustrated Various Face Detection methods

1-Feature Base Methods	Active shape Model	Snakes
		Templates
		PDM
	Low level analysis	Skin color
		Motion
		Edge
2-Image Based Methods	Neural Network	
	Linear subspace	Eigen Face
	Statistical Approach	PCA
		SVM

2. FEATURE BASE METHODS

Recognize the objects based on their features. Detect and recognize the human face from the other things based on the human features such as eyes, mouth and nose, and then used this features to detect human face. The statistical classifiers are helps to distinguish between the face and non-face. Although there are many method has been suggested to extract the facial features which illustrated in the literature, most of them faced the limitations such as the lights , low resolution occlusion and noise.

Most research and studies have shown that skin color is an important element of facial detection among other objects although different people have different color skin and it is more cleared when people race is also a measure of evaluation [7].

2.1 Active Shape Model

2.1.1 Snakes

Snakes or Active contours are usually implemented to locate the boundary of the head, and then the features related to the located boundary can be established by these contours. This can be accomplished by placing the starting point of the snake at the proximity around the head's borders, after the snake stands on the nearby edges and then assumes the shape of the face. F snake is defined as:

$$E_{snake} = E_{internal} + E_{external} \quad (1)$$

The F internal is the internal energy that presents the part that depends on the intrinsic properties of the snake and defines its natural evolution, while the external energy F external responds the internal energy and allows the contours to deviate from the natural evolution and eventually assume the shape of nearby features. Although this method is perfect in detecting human face, it may be faced two limitations such as: a snake doesn't work very well in recognize non convex features because their tendency to link minimum curvature. Also, sometimes part of contour gives incorrect features for some images [8],[4]

2.1.2 Deformable template-based approach

One of the major reasons that makes computer aided detection or recognition system complicated, is the difficulty in recognizing the variable structures in human face [4]. One simple solution to this problem would be based on matching the test image with a template. This approach was modified by [9], in this approach, the authors used the template-based approach to locate the human face based on selecting four features (whole face, nose, mouth and eyes) as template per person.

The recognition technique was achieved by comparing the test image with the database images. Correlation was computed for each feature template; a vector of matching scores for each feature was calculated. The test subject was then classified as the subject with the highest score. This approach introduced a significant method to store the face area by discrimination ability (eyes, nose and mouth in order). This method succeeded in recognizing a number of people, it had some limitations because it deals only with the static and frontal images, so it was unable to deal with head movements. Additionally the illumination must be controlled; the same powerful light was used for the test and data base images.

2.1.3 PDM (Point Distributed Models)

The flexible model (PDMs) was created by [10]. It is quite different from the other models in the application process. The PDMs are compact parameterized descriptions of a shape based upon statistics. The bounding shape of PDM is categorized into a group of labeled points, and then the differences of these points can be described over a training set that contains objects of various sizes and poses.

One of the proposed methods was suggested by

Wiscott et al. [45], they based on a well-known feature as a method of matching the diagram to a flexible package proposed. The difference in lighting and contrast played important role in success the "Gabor filters" to remove the noise and the changeability in the image. A graph is generated for each face separately as follows: on the face, a group of fiducially points were chooses. Each fiducially point is a node of a completely connected diagram, called with "Gabor filters" then apply a window around the fiducially points.

This technology has really enhanced facial recognition performance under variations of pose, angle, and expression.

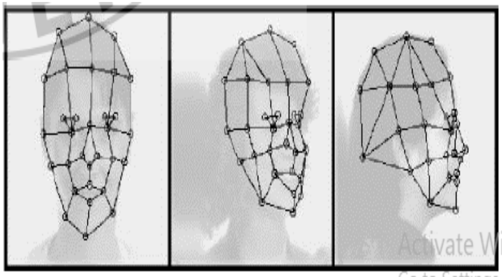

Figure1: Grids for face recognition[46]

The face PDM was applied to present the global appearance of the face with the facial features such as (nose, eyes, mouth and eyebrows). Moreover it can be used to detect multiple faces and has several implementations such as coding and facial expression interpretation [4],[11]. On other hand the detection of face features automatically is considered as one of the difficulties in this approach.

2.2 Low level Analysis

2.2.1 Skin –color method

Skin color segmentation can be considered as an important feature to detect human face as well as feature extraction. For this purpose, It can be considered as a rapid technique than others. The color is continual orientation under specific lighting conditions. This effects makes motion valuation much easier, the reason was only the translation model is needed to estimate the motion [12]

Skin color can be considered as a significant feature that can be used to detect human face. It has proved effective because it gives a fast detection and provides robust results enough to perform real-time tracking [13]. This process depends on the color of the human face which can be tracked and distinguished from the color of other objects in the scene and the color of the background.

[14] proposed a new approach to detect human face as well as facial feature, the two major steps that has been proposed to detect the human face in the suggested images are : Initially, the skin region was segmented from the image using RGB, YCbCr, CEILAB (L*a*b) and HSV color models. The skin region was tested using color models and the knowledge of geometrical properties of human face while the non-skin region was removed using color models with thresholds. The test results showed the distinctive effect of this algorithm in detecting human face.

The skin color segmentation was used from many researchers to detect and recognize the human face from the color images such as [15]-[16]-[17]-[18]-[19]

However significant limitations of this method appeared when there is a wide variety of skin colors, light conditions and there are problems with grey level images. Sometimes it can be used as a first step in face detection, but is not suitable for high level feature extraction and analysis.

2.2.2 Motion base

When using the video sequences, the moving object can be detected based on the motion information [20]. The simply thresholding accumulated frame can be used to extract the Animated silhouettes such as body parts and faces. The facial features can be located and extracted by frame differences besides face regions [21].

2.2.3 Edge

A Laplacian edge detection operator was performed to detect the facial features (nose, eyes, mouth and chin contour) from gray level images [22]. In [23] introduced an algorithm to detect further head outline in addition to the locations of facial features that includes eyes, eyebrows and mouth from a grey-scale image where the edge detectors were used to obtain the face curve. [24] proposed a novel method to detect human face in thermal image as well as the tip of the nose.

The aim of this project was extracted human face from the background and then determined the area under the tip of the nose after detect the nose region. This area was used to measure the respiration rate because it is more affected than other regions by breathing process

Initially, the human face was enhanced by using average-low-passfilter to reduce the noise from the image. The prewitt operator was used to segment the face boundaries from the background. The prewitt operator was used to extract the boundaries in horizontal and vertical directions of the human face. After extract the boundaries of the human face, the nose region was extracted based on the eye regions as well as the temperature of this area. Then the area (region of interest ROI) under the tip of the nose was detected and extracted as circle regions. Then the respiration signal was extracted by averaging the pixel values within the ROI for each image and the resulting information was plotted against time.

In general the facial features such as eyebrows and lips seem darker than surrounding regions in the face, then this techniques can be implemented for local minima, meanwhile the local maxima has been implemented to marker the bright facial spots like nose tips then the gray-scale thresholding can be performed for final detection [23]-[25]- [26].

3. IMAGE- BASE METHODS

3.1 Neural network

[27] proposed another suggestion method for detecting human face using a neural network.

This method was suggested to deal with the light problem in the face detection process. The image was segmented into several regions. The neural network was also used to classify the regions into two groups, either face or no face. Pre-processing steps were applied for each region by using a light correction algorithm and histogram equalization techniques. The light correction needed a statistical analysis for the background color to be approximately estimated across the image. While the histogram equalization techniques were used to adjust the histogram of the image in a variety of ways either by averaging the histogram for an image or enhancing the contrast of image. The light condition and skin color problems remained problems with the face detection technique.

The development of this method was suggested by [28]. The facial expressions was detected and located in the human face. All the extracted features from the human face were inputted to Neural-Network to detect the decision of being Neutral face or non- Neutral.

Facial expression analysis automatically combines a number of tasks which includes face detection, facial features extraction and facial features represented. Based on this analysis the face was classified as Neutral face or Non-Neutral. After the face region was extracted by using skin-color technique. Detection and extraction were focused on the six facial point features from the image which were less affected by noise than other parts of the images. The features includes: -two pupil centers, two eyebrow inner end-points and the two corners of the mouth. The distance between these points was calculated. Further, the features of the eyes and the mouth were extracted by an ellipse fitting algorithm and the histogram technique respectively. All the features were inputted to Neural-Network to detect the decision of being Neutral or non- Neutral. This approach was an important technique to reduce the error rates for face recognition, because universal expressions are one of the important reasons for inaccuracy in the recognition process. This approach could detect the face and extract the facial expression and recognition.

3.2 Eigen faces method as a Linear Sub Space

[29] was present an early example of employing eigen-vectors in face detection and face recognition. The recognition based on a simple neural network for face images.The basis of the eigenfaces method is the Principal Component Analysis (PCA). Eigen-faces and PCA have been used to represent the face images efficiently [30].

[31] suggested using eigenface technique to detect and recognize the human face from the color image. This technique was based on calculation of the Euclidian distances between the new eigenface and the previous one. The smallest Euclidean distance is person resembles the most. Although the results showed that sometimes failure occurs, but it have success rate about of 94.7% when applied it to the large database.

Two approaches were suggested from [3] to detect the position of the human face area using the genetic algorithm and detecting the facial features using the eigenface technique. The authors tried to solve several problems such as the orientation of the face, skin color and the lighting effect. The eye regions play an important role on detecting the facial region and facial features based on the characteristics of eye regions

Genetic algorithm and the eigenface techniques were also applied. The possible face regions were determined by using the genetic algorithm, whilst the fitness of these regions was determined using eigenface. . To reduce the searching space, the human eye regions were selected by testing all the valley regions in an image using a genetic algorithm. The pair of possible eye candidates was used to segment the face regions.

The size of human face (Z_{face}) is proportional to the distance between two eyes (E_{eye})as in Equation (2)

$$Z_{face} = 1.8 \quad E_{eye} \qquad (2)$$

The relationship between the facial features (the eyebrows, eyes, nose, and mouth) play significant role in determined the possible face region.. The facial features are then extracted from the detected face regions. The eigenfaces was used to calculate the fitness value for each candidate face. A suitable candidate region was chosen after a number of repetitions, and then features were extracted from this region. The selected face was then further verified by measuring the symmetries and determining the existence of the different facial features.

The significant progress in the performance of this method was proposed by [32]. He suggested the algorithm to detect human face by calculate the eigenvalues and eigenvectors of the face images. This calculation depended on locating the human face as well as the eyes in the image uses Principal Component Analysis. This method was succeeded in detecting human faces, and feature extractions in simple and complicated background.

Most researchers have used eigenface and eigenvector to locate the human face and facial features from the color images such as [34]-[35]-[36]-[37].

3.3 Statistical Approach

3.3.1 Principle Component Analysis (PCA)

PCA technique was introduced firstly by [38] which based on the Eigen- faces. [38]. [36] suggested using PCA technique to detect and recognize human face. In this approach the eigen-face was extracted for each image in the training set by represented each image in the training set as a linear combination of weighted eigenvectors.

These eigenvectors are obtained from covariance matrix of a training image set. Weights are found after selecting the most important set of faces. The classification is done by measuring minimum Euclidean distance that was achieved by projected each test image onto the subspace spanned by the eigen-faces to perform the recognition and detection process [39]. This technique has the advantage to keep the important information of image and rejects the redundant information therefore it considers as an important technique in extraction the facial features from the images. The PCA was used to reduce the large dimensionality of observed variable to the smaller fundamental dimensionality of independent variable without losing much information. This technique was widely used from many researchers such as [40],[41].

3.3.2 Support Vector Machine (SVM)

This technique is considered as a linear classifier, in the last 50 years the SVMs become a widely applied classification technique. It played an important role in object detection, and recognition [15]. This technique was work by firstly scanned the images to detect the faces, then scaled the faces image into the image`s patches. The filtered image (output image) is used as input to the recognition classifier which the code expression into different dimension. The SVM trained after selected the facial features [42]. [1] proposed a new technique to detect and recognize human face by using Support vector machines (SVM). They tried to solve the problem of face detection and face recognition by using SVM with binary tree. They tried to compare SVM with other techniques; the experimental results showed that the SVMs are a better learning algorithm than the nearest center Classification approach (NCC) criterion for face recognition.

A modified method to detect the human face in visible domain using the skin colour was proposed by [42]. They also used a skin colour to detect the human face region. They suggested another technique to reduce the effect of significant limitations of the skin colour by using a high-pass filter. This filter of the wavelet transforms was used to highlight the edges of the selected areas.

Vertical, Horizontal, and filter-like projections were represented as feature vectors .The element vectors were then categorized as face or non-face using either vector support techniques or dynamic programming. Dynamic programming is a classification technique that is used to calculate the best possible path matching between the original feature vector and template feature vector.

The Euclidean distance was used to measure the similarity between these two vectors. The decision is made as face or no-face depends on thresholding the resulting distance.

In this approach the authors attempted to demonstrate that the use of vector machine technique is more appropriate than dynamic programming in face detection process as it is more accurate, faster and cheaper.

[43] tried to achieve sufficient speed and accuracy in SVM techniques; they tried to build a component-based face detector in online face recognition system using SVM classifiers.

Finally, a method was suggested to detect human face as well as feature extraction, human face was detected based on skin color segmentation and then extracted as ellipse region. The nose region was located and extracted as rectangular region in the elliptical facial area. This technique was achieved by threshold and binarization the extracted area and then looking for the longest vertical line.

As the Mouth region represented the bottom part of the face therefore, it was detected based on thresholding the elliptical area, then the mouth area was located by searching the area under the tip of the nose. The longest horizontal line represents the center line of the lips. This area was extracted as rectangular area. The lips were detected in the mouth region based on the threshold techniques as well as the points of the lips corners [44]. Fig.2 shows the extraction of the human face as an elliptical region, the nose as rectangular region while the mouth as an elliptical region with lips.

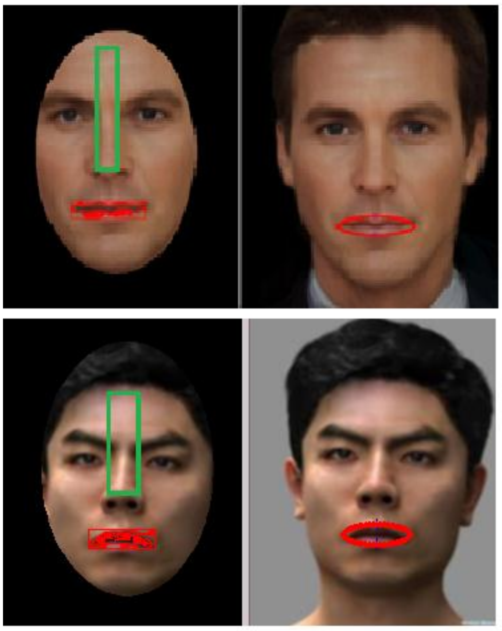

Figure2: Detecting The Human Face As Elliptical Area As Well As The Mouth And The Nose Regions In The Human Face

4. Conclusions

Detecting facial features within human face is an important task when dealing with image processing techniques. Facial detection methods can be sorted through a number of methods depending on how they are used and operated.

In this paper, they are classified into feature-based methods and image-based methods. The paper has introduced the advances has made to improve the performance of face detection with facial features.

Finally, we suggested a method to detect and extract human face as elliptical area from the background. The facial features within the face area were located and extracted sequentially. Image processing such as (enhancement, binarazation, thresholding segmentation and edge detection) played an important role to achieve the aims of our proposal.

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مسح لطرق تحديد وجه الإنسان

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المستخلص :

الهدف من هذا البحث هو إجراء مسح نقدي لكشف واستخراج الوجه البشري مع الميزات على مدى السنوات العشر إلى الخمس عشرة الماضية. يعد اكتشاف الوجه أحد أكثر التقنيات شيوعًا في التطبيقات البصرية المستقبلية المختلفة ، مثل المؤتمرات عن بعد ، وأنظمة التعرف على الوجه ، والقياسات الحيوية ، وواجهة الكمبيوتر البشرية ، ليس فقط بسبب الطبيعة الصعبة للوجه ككائن ، ولكن أيضًا بسبب عدد لا يحصى من التطبيقات التي تتطلب تطبيق اكتشاف الوجه كخطوة أولى.

وأخيرًا ، نقدم تقنية الكشف عن الوجه استنادًا إلى تجزئة لون البشرة بالإضافة إلى ميزات الوجه الموجودة داخلها. تم تحديد تصميم الوجه الإنساني كمنطقة إهليلجية. وفي الوقت نفسه ، تم استخدام العديد من التقنيات مثل التحسين والعتبة وحواجز الكشف وتقنيات الثنائيات لتحقيق الهدف من الطريقة المقترحة. يتم الكشف عن ملامح الوجه واستخراجها داخل منطقة بيضاوي الشكل. يمكن تصنيف هذه الميزات في ثلاثة أجزاء: توطين الأنف والفم والشفاه. يتم الكشف عن الميزات واستخراجها في الوجه البشري على أساس تقنيات معالجة الصور.

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Use of logistic regression to study the most important factors affecting the incidence of tuberculosis

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Abstract :
What distinguishes this research Is to focus on the concept of logistic the regression and its characteristics and how to build the model analytical descriptive approach was used to describe the logistic regression How to estimate Its Features And its use as an important tool in studying the influencing factors in the injury of tuberculosis the sample included 299 patients.

The first topic: methodology of research

1.1 Introduction: -

The models of logistic regression have been noticed in many years. They are widely used in life experiences, which are among the main concerns of the countries of the world for their relation to human life and development through finding the best way to provide the best services in the fields of pharmaceuticals, vaccines, vitamins, pesticides, hormones and others. The method of analyzing the path of efficient statistical methods in the analysis of data may enable the researcher to identify and clarify the potential negative relationships of a group of factors and to determine the direct and indirect impact of the phenomenon of the total study and thus helps to derive logical explanations of the phenomenon and more efficient in the analysis of data. The importance of logistic regression analysis is highlighted by the ability to study the effect of several factors on a particular phenomenon in a direct or indirect manner and we use logistic regression in the response variables.

2.1 Research Objective

There are two main objectives of research : The first objective is represented by the concept of logistic regression and the methods of estimating the parameters of this model.
The second objective is to apply the logistic regression in the health aspect of the tuberculosis phenomenon to identify the most important causes of this disease

3 .1 Research sample

This study was applied to data collected from Al-Hussein Hospital through tests of chest diseases for each patient. A group of doctors specializing in chest diseases was used to classify the most important factors affecting the disease. A sample of 299 patients was taken from the center of the chest 2017/2/3 and until 2017/5/1

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4.1 Research Hypotheses

There are several hypotheses for the logistic regression model, including:

1-Variable period (0.1) variables can be continuous or discreat or binary versions or multiple.

2 - There is a relationship between the dependent variable and explanatory variables take the following pictures:

$$\text{pr}\left(Y_k = {}^{1}/_{x}\right) = \frac{\exp(B\acute{X})}{1+\exp(B\acute{X})} \quad . \ . \ . \ (1)$$

3-There is no correlation between random errors (independence of errors)

4 - There is no correlation between random error and explanatory variables

5 - There is no correlation between the explanatory variables with each other in a complete way if the variables that have a relationship between them should be correlation completely.

6- The random variable (y_i) is assumed to be distributed by Bernoulli y ~B(θ) in the mean E (y_i)= θ and variance V (y_i)= $\theta\,(1-\theta)$.

7-The expected value of the random error is zero, since Pr (x) [1-pr (x)] and the variance of the random error U_n expands based on the bar

The second topic: Theoretical side.

1-2 Logistic Regression[2][3] [5]

It is common in human, social and economic studies that the dependent variable is a separate variable to take the fixed value or more. This is a significant criterion for the researchers of their attempts to employ linear regression analysis (simple or multiple), which is somewhat restricted by requiring that The dependent variable is a quantitative variable that is connected rather than descriptive and separate.

Quantitative models are important models in natural, engineering and social studies. The choice of an appropriate model depends on the nature of the data, especially when the variable of response is binary, ie, the occurrence of the phenomena, such as death or life. This response is affected by the existence of a set of independent variables that affect the response variable. The nonlinear models are among the best models representing such phenomena. The best models are the probability model and the logistic model, which is the focus of our research. Logistic model of commonly used nonlinear models.

(Lea 1997), which is seen in such cases, although there are many statistical methods developed to analyze data with descriptive variables (qualitative)
(Function Analysis Disctianant) such as analysis of the functions of excellence

However, logistic regression has many features that make it suitable for use in such situations. Logistic regression is a useful way to illustrate the relationship between independent variables (age, sex, etc.) and the variable of the answer, the probability, which takes two different values.

An example of a cancer diagnosis is that the two values for the response variable are either infected or not.

The importance of logistic regression when compared with other statistical methods (linear regression and differential analysis) is that logistic regression is a more powerful tool to provide a test of the significance of transactions. It also gives the researcher an idea of the effect of the independent variable in the binary dependent variable.

In addition, the logistic regression calculates the effect of independent variables, allowing the researcher to conclude that a variable is stronger than the other variable in understanding the emergence of the desired result, and that regression analysis can include independent qualitative variables as well as the interaction of independent variables in the dependent variable The advantage of using logistic regression is that it is less sensitive to deviations from the normal distribution of the study variables, compared to other statistical methods such as differential analysis and linear regression, and the logistic regression Biddable exceed many restrictive assumptions to use the method of least squares (OLS) in the linear regression which makes the ultimate logistic regression analysis the best method in the case of the binary variable value. (Binary Logistic Regression), which we use in our multinomial logistic research used in the case of the multivalent nominal variable (more than two values).

There is also a third type of logistic regression called ordinal logistic regression, which is used In cases where the dependent variable is a class variable, we use the two-valued dependent variable in values (0,1) without any other coding.

There are also several definitions of the regression model:

The logistic regression model can be defined as a model used to predict the probability of an event by fitting the data on the logistic curve. Logistic regression uses several expected variables that can be numerical or factional. For example, logistic regression in marketing is used to calculate the consumer's tendency to buy a product or refrain from purchasing. Logistic regression is used broadly or broadly in medicine and social sciences.

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It is also possible to define the statistical method used to examine the relationship between the dependent variable and one or more independent variables, that is, the binary-value variable and one or more independent variables of any kind called Binary Logistic Regression.

It is also known as the type of regression used to predict the value of dependent binary or class dependent variables depending on the set of independent variables mixed, such as continuous variables or measurements, and the other section in the form of intermittent qualitative or class variables.

2-2 Characteristics of the logistic regression model[1][5]

1.This model does not put any preconditions on the explanatory variables

2.The model does not specify which vectors belong to the new observations, but also determines the probability of this affiliation. It can also be used to analyze the binary and multivariate descriptive variable.

3.The maximum possible method (ML) is used to estimate its parameters and therefore the quality conditions are met in these variables

4.ease of calculations used in the form Model.

With these characteristics, logistic regression becomes one of the most appropriate models for the analysis of binary and multivariate descriptive variable .

3-2 Method of building the regression model[1][3]

1.Achieving the relationship between the binary nominal variable and the nominal independent variable by means of a single analysis using the Chi-square and the correlation test

2.To achieve the correlation between the variable logarithm of the binary nominal and the independent quantitative constant variable by the scheme of dispersion between the two variables where the relationship must be positive

3.Analysis of the linear relationship between independent variables

2-4 Analysis of simple and multiple logistic regression[2][4]

The analysis of logistic regression is used in epidemiological and medical studies, in which the quantitative and qualitative independent variables that affect the probability of the resulting variable are determined when the logistic regression is applied.

2-4-1 Required for simple logistic regression[1][3]

Quantitative or nominal independent variable such as weight, height, marital status and gender. It is a binary variable, such as a disease (yes, no), gender (male, female) and others.

2-4-2 Required in multiple logistic regression[2][6]

Two or more independent quantitative or nominal variables such as weight, height, marital status and gender. One variable my name is a dual-type follower such as an illness (yes, no) or gender (male, female) and others.

2-5 Logistic regression model[2][4][6]

The logistic regression model is defined as one of the regression models where the relationship between the y variable and the explanatory variables (x_1, x_2, ..., x_k) is nonlinear where the y variable of the binary response takes the values 0,1 and the success is the probability of __i Or failure Failure to respond to the probability of 1-__i

so the dependent variable y follows the Bernoulli distribution and the probability density function will be as follow:

$$P(Y = Y_i) = \theta_i^{y_i} (1 - \theta_i)^{1-y_i} \quad . . . \quad (2)$$

$y_{i=0,1}$ Since

Y_i : binary variable dependent response

θ_i : The probability of a response when $y_{i=}1$

Therefore, signing the dependent variable represents the probability of a response

$$E(y_i) = p(y = 1) = \theta_i \quad . . . \quad (3)$$

The variance of the dependent variable by Bernoulli distribution is

$$V(y_i) = \theta_i (1 - \theta_i) \quad . . . \quad (4)$$

Let $X_\circ, X_1, X_2, \dots\dots X_p$ be a set of explanatory variables and let n have many observations of these variables that are matrix X

$$X = (X_{ij})nxp \quad . . . \quad (5)$$

(n) i = 1,2 .... n represents the sample size . (P) J = 0.1,2 .... P represents the number of parameters

If $y_{i=} [y_1, y_2, \dots . y_n]$ is a random sample of the binary response variable and $y_i \in \{0,1\}$ This leads to a regression model given as follows

$$y_i = \theta_i + \varepsilon_i \quad . . . \quad (6)$$

represents the regression function (probability of response)

$$\theta_i = p(y = 1) = \frac{e^{X_i \underline{\beta}}}{1 + e^{X_i \underline{\beta}}} \quad . . . \quad (7)$$

β: vector of information dimensions (p * 1)

Xi = {X_{i0}, X_{i1}, ⋯ X_{ip}} A class vector of explanatory variables

(1 * p):

ε_i: The error limit that will have an average of zero as in the following formula:

$\varepsilon_i = y_{i-} \theta_i$

$$E(\varepsilon_i) = E(y_i) - E(\theta_i) = \theta_i - \theta_i = 0 \quad . . . (8)$$

Either the error limit variance is equal to the variance of the adopted binary response variable.

$$V(\varepsilon_i) = V(y_i) = \theta_i (1 - \theta_i) \quad . . . (9)$$

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The error threshold follows the Bernoulli distribution with an average of 0 and the variance (θ_i-1) θ_i

It is noted that the variance of the error limit depends on the values of the response probability θ_i i.e. on vector values X_i and there are Variance of error non homogenoce .

2-6 Methods for estimating the parameters of the logistic regression model

To estimate the parameters of the logistic regression model, it was based on the weighted low squares method (WLS), which is used in regression analysis to address some analysis problems. The maximum possible method (MLE) is also used, and the maximum possible method requires repetitive methods for its calculation. Therefore, initial values of β_1 and β_0 are required.

Before analyzing these methods, it is important to know that binary data such as success and failure appear in most areas of study. The analysis of log regression is most often used to examine the relationship between intermittent responses and total explanatory variables. There are those who discussed logistic regression (Agresti 1990, 1989) Cox & Snell) Since in binary data the distribution of random error (U_i) is sporadic, it is an abnormal distribution, but a binomial distribution is distributed. If the error is distributed naturally, the variance is not equal in all aggregates. In the case of discrete response models, the model is as follows:

$$Y_i = \beta_\circ + \beta_1 X_{i1} + \dots + \beta_k X_{ik} + U_i \quad . . . \quad (10)$$

X_i : independent Variable .

$\beta_\circ, \beta_1, \beta_k$ Regression equation parameters. U_i: is a random error

Y_i : dependent variable

Since the variable (Y_i) distribution of any Bernoulli there are only two values, so the random distribution of error (U_i) distribution is sporadic and does not continue any

$Y_i = 1$; $Y_i = 0$; $E(Y_i) = P_i$; $Q_i = 1 - P_i$

While

P_i: Response Ratio When $Y_i = 1$; Q_i Response Ratio When $Y_i = 0$

$$\mathrm{Var}(U_i) = E(U_i)^2 - [E(U_i)]^2 \quad . . . \quad (11)$$

The compensation shall be $\mathrm{Var}(U_i) = P_i Q_i \quad . . . \quad (12)$

Therefore, the Variance error in this case is nonhomogeneous and the ordeal least square method cannot be used to estimate the parameters of the linear model in the case of dichotomous binary data distributions.

2-6-1 Weighted least squares method (WLS)[1][3][6]

In WLS, the relationship between the dependent variable (Z) and the independent variables is as follows:

$Z = X\beta + U . . . (13)$ Where as

U: A vertical direction Rank (r * 1) represents random errors

X: matrix of independent variables of order [r * (k + 1)]

β: Parameters of vector [(k + 1) * 1] ; Z: vector axis of rank (r * 1)

Since the expectation and variance of variable Z is

$$E(Z_i) = X\beta \quad . . . \quad (14)$$

$$\mathrm{var}(Z_i) = \frac{1}{n_i p_i q_i} \quad . . . \quad (15)$$

$$Z \sim N\left(X\beta, \frac{1}{n_i p_i q_i}\right)$$

The estimated formula of the model parameters can be derived by multiplying the two ends of the model by the inverse of the symmetrical country matrix (p), which is at the Ranke of (r * r). We obtain the following model

$$P^{-1}Z = P^{-1}X\beta + P^{-1}U \quad . . . \quad (16)$$

$$P = \begin{bmatrix} \frac{1}{\sqrt{n_1 p_1 q_1}} & . & . & 0 \\ . & . & 0 & . \\ . & 0 & . & . \\ 0 & . & . & \frac{1}{\sqrt{n_r p_r q_r}} \end{bmatrix}$$

We use the OLS method to find β estimates as it comes

$$U = Z - X\beta$$

$$U'U = (Z - X\beta)'(Z - X\beta)$$

And compensation

$$(P^{-1}U)'(P^{-1}U) = (P^{-1}Z - P^{-1}X\beta)'(P^{-1}Z - P^{-1}X\beta)$$

$$(P^{-1}U)'(P^{-1}U) = \left(Z'p'^{-1} - \beta'X'P'^{-1}\right)(P^{-1}Z - P'X\beta)$$

And unzip the brackets we get

$$(U'P'^{-1}P^{-1}U) = (Z'P'^{-1}P^{-1}Z - Z'P'^{-1}X\beta - \beta'X'P'^{-1}Z + \beta'X'P'^{-1}X\beta)$$

$$(U'W^{-1}U) = Z'W^{-1}Z'W^{-1}X\beta - \beta'X'W^{-1}X\beta$$

$$(U'W^{-1}U) = Z'W^{-1}Z - 2\beta'X'^{W^{-1}}Z + \beta'X'W^{-1}X\beta$$

Taking the first derivative for β we get:

$$\frac{\partial U'W^{-1}U}{\partial \beta} = -2X'W^{-1}Z + 2X'W^{-1}X\beta$$

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On the basis of the first derivative of zero, we obtain:

$$X'W^{-1}Z = X'W^{-1}X\widehat{\beta}$$

By multiplying the equation by $(X'W^{-1}X)^{-1}$ we obtain an estimate of the values of $\widehat{\beta}'S$)

$$\widehat{\beta} = (X'W^{-1}X)^{-1}X'W^{-1}Z \quad . . . \quad (17)$$

W=P'P

$$W^{-1} = \begin{bmatrix} n_1p_1q_1 & . & . & 0 \\ . & . & 0 & . \\ . & 0 & . & . \\ 0 & . & . & n_rp_rq_r \end{bmatrix}$$

) r*1 (Where : Z is a vertical direction bar

$$Z = \begin{bmatrix} \ln \frac{p_1}{1-p_1} \\ . \\ . \\ . \\ \ln \frac{p_r}{1-p_r} \end{bmatrix}$$

The weighted least squares method achieves estimates when the Pi (probability of response) value is zero or one, so the method is processed so that the value of the Zi variable is as follows:

$$Z_i^{\ *} = \ln \left[\left(P_i + \frac{1}{2n_i}\right) \Big/ \left(q_i + \frac{1}{2n_i}\right) \right] \quad . . . \quad (18)$$

$$\text{Var}\ (Z_i^{\ *}) = \left[(n_i + 1)(n_i + 2) \Big/ n_i^{\ 3} \ \left(P_i + \frac{1}{n_i}\right)\left(q_i + \frac{1}{n_i}\right) \right]$$

$$\text{Var}\ (Z_i^{\ *}) = \frac{1}{W_i^{\ *}}$$

$W_i^{\ *}$: New weighted weight

The estimates can be calculated as follows:

$$\widehat{B} = \left(X'^{W^{*-1}}X\right)X'W^{*-1}Z^{*} \quad . . . \quad (19)$$

2-6-2 Maximum likelihood method MLM[2][5] [6]

This method is based on the finding of β values, which are estimates of vector β, which makes the function at its extreme end, and assuming that we have r of independent random variables $(Y_1, Y_2, \dots, Y_r)$; (n_i, p_i) and (Y_i)that represents the sum of successes in each end of the function at (n_i) the end of and that (K) of the independent variables in each set of totals, the probability density function of (Y_i) :

$$P_i\ (Y_i = y_i) = C_{y_i}^{n_i} P_i^{y_i} (1 - P_i)^{(n_i - y_i)} \quad . . . \ (20)$$

i=1,2,….,r

y_i=0,1,2,….,n_i

$$E(y_i) = n_ip_i \quad . . . \quad (21)$$

$$\text{var}(y_i) = n_ip_i(1 - p_i) \quad . . . \quad (22)$$

The p_i response rate is estimated as follows:

$$p_i = {y_i}/{n_i} \ ;$$

$$Q_i = 1 - p_i = 1 - y_i/n_i = (n_i - y_i)/n_i$$

The Maximum Likelihood function for the co-distribution of data (Yi) is by formula

$$L(P) = \prod_{i=1} C_{y_i}^{n_i}\ P_i^{\ y_i}(1 - P_i\ )^{n_i - y_i} \quad . . . \ (23)$$

$$L(P) = \prod_{i=1} C_{y_i}^{n_i} \left[\frac{p_i}{1-p_i}\right]^{y_i} (1 - p_i)^{n_i} \quad . . . \quad (24)$$

$$\ln L(P) = \sum_{i-1}^{r} \left[\ln C_{y_i}^{n_i} + y_i \ln \left[\frac{p_i}{1-p_i}\right] + n_i \ln(1 - p_i) \right] \quad . . . (25)$$

Since (pi) the probability of success is affected by the independent explanatory variables according to the logistic model (in the form of matrices) :

$$P_i = \frac{\exp(X'_i\ \beta)}{1 + \exp(X'_i\ \beta)} \quad . . . \quad (26)$$

$$1 - P_i = \frac{1}{1 + \exp(x'_i\ \beta)} \quad . . . \quad (27)$$

When compensating the formulas (26) and (27) in (25) we get the following formula:

$$L(P) = \sum_{i-1}^{r} \left\{ \ln C_{y_i}^{n_i} + y_i x'_i\beta + n_i \ln\left[\frac{1}{1 + \exp(x'_i\beta)}\right] \right\} \quad . . . \quad (28)$$

We use the Newton Raphson method to find the *Maximum Likelihood* estimates according to the formula

$$t_{s+1} = t_s - G^{-1}g_{(s)} \quad . . . \quad (29)$$

t_{s+1} :represents the vector of β parameters to be estimated

t_s:The vector represents the initial values of the parameters

$g_{(s)}$:The first derivative vector of the logarithm represents the function Likelihood

$$g_{(s)} = \frac{\partial \ln L}{\partial \hat{\beta}} = \begin{bmatrix} \frac{\partial \ln L}{\partial \beta_\circ} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial \ln L}{\partial \beta_k} \end{bmatrix}$$

G: Matrix The expected value of the negative value of the second derivative of the logarithm is the Maximum Likelihood function

$$G=\left[\frac{\partial^2 \ln l}{\partial \widehat{\beta_\iota}\, \partial \widehat{\beta_\iota}}\right] =$$

$$\begin{bmatrix} -E\frac{\partial^2 \ln l}{\partial \beta_\circ^2} & -E\frac{\partial^2 \ln l}{\partial \beta_\circ \partial \beta_1} & -E\frac{\partial^2 \ln l}{\partial \beta_\circ \partial \beta_k} \\ . & -E\frac{\partial^2 \ln l}{\partial \beta_1^2} & -E\frac{\partial^2 \ln l}{\partial \beta_1 \partial \beta_k} \\ . & . & -E\frac{\partial^2 \ln l}{\partial \beta_k^2} \end{bmatrix}$$

Thus, the formula for finding estimates is as follows:

$$\hat{\beta}_{s+1} = \hat{\beta}_s + \left[\frac{\partial^2 \ln l}{\partial \hat{\beta}_j\, \partial \hat{\beta}_j}\right]^{-1} \frac{\partial \ln L}{\partial \hat{\beta}_s} \quad . \; . \; . \quad (30)$$

The original Newton-Raphson formula in the case of the logistic model to find the Maximum Likelihood estimates is

$$\hat{\beta}_{s+1} = \hat{\beta}_s + (x'vx)^{-1} x'^{(y-\hat{y}_s)} \quad . \; . \; . \quad (31)$$

$\hat{\beta}_{s+1}$ *Vertical direction of the estimate values in the cycle (s+1) at grade ((k+1)*1)*

$\hat{\beta}_{s+1}$ Vertical direction of the estimate values in the cycle (s) at grade ((k+1)*1)

X: matrix of independent variables in the cycle (r((k + 1)*

*V: a diagonal matrix of the rank (r*r)*

$$V=\begin{bmatrix} n_1\hat{p}_1\hat{q}_1 & 0 & 0 & 0 & 0 \\ 0 & n_2\hat{p}_2\hat{q}_2 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & n_r\hat{p}_r\hat{q}_r \end{bmatrix}$$

Estimation of the vector of parameters (β) is discontinued when the difference from the previous cycle and the subsequent cycle is very small and approaches zero.

The third topic: The applied side

1-3 Description of data Description of data

The data of the research were collected from Al-Hussein Hospital in Dui Qasr Governorate through tests of chest diseases for each patient. A group of specialized doctors were used to classify the most important factors affecting the disease. A sample of 299 patients was taken from Al- 2017/2/3 and until 2017/5/1 are:

y: - The condition of the patient where he takes two values (infected 1) and (not infected 0) which represents the dependent variable.

X_1 : Age of patient

X_2 : patient sex (male = 1) and (female = 2)

X_3: Classification of the environment of the casualty (rural=1) and (city= 2)

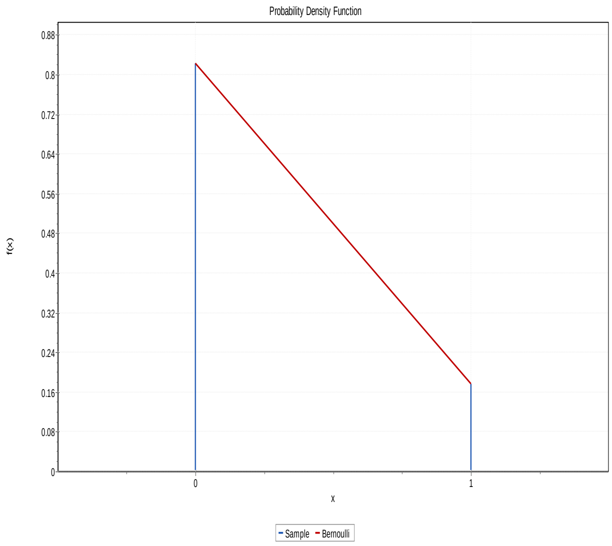
: X_4 : Smoking (Smoker 3) and (Non-Smoker 4)

To follow the test data Of dependent variable .

To determine the distribution of variable y variable data (the binary response variable) we will adopt the ready-made program
(EASY FIT) test (Anderson-Darling), which shows that these data are distributed according to Bernoulli's distribution with their estimation parameter (p = 0.17726) as shown in Table (1).

#	Distribution	Parameters
1	Bernoulli	p=0.17726
2	Binomial	n=1 p=0.17726
3	D. Uniform	a=0 b=1
4	Geometric	p=0.84943
5	Poisson	□=0.17726
6	Hypergeometric	No fit
7	Logarithmic	No fit (data min < 1)
8	Neg. Binomial	No fit

Table (1-4) Results of the test of good ness of fit to the variable data adopted in the application



When the dependent variable is plotted, it is determined by the Bernoulli distribution function as follows

2-3. Statistical analysis

These data are analyzed through the statistical program SPSS to deter mine the importance of variables and their impact on chest diseases through the following tables:

Unweighted Cases[a]		N	Percent
Selected Cases	Included in Analysis	299	100.0
	Missing Cases	0	.0
	Total	299	100.0
Unselected Cases		0	.0
Total		299	100.0

Table 2 shows sample size

From the above table, we note that the number of persons (sample items) who took the information for the sample of the research 299 and the missing data (observation) was equal to 0.

Observed			Predicted		
			The patient's condition		Percentage Correct
			Uninfected	Injured	
Step 0	The patient's condition	Uninfected	246	0	100.0
		Injured	53	0	.0
	Overall Percentage				82.3

Table (3) shows the dependent variable

The above table shows that the number of people who did not have chest diseases from the study sample 246 and people with chest diseases was 53

.

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 0	Constant	1.453	.105	231.569	1	.0781	.224

Table (4) shows the child's test

By comparing the value of a child test with the value of the kay square test at a significant level of 0.001 and the k-1 = 3 freedom level of 187.326, we find that the value of the child generated is greater than the square value of the square test. Therefore, Dependent on the dependent variable.

Iteration		-2 Log likelihood	Coefficients				
			Constant	x1	x2	x3	x4
Step 1	1	279.546	.512	.0014	.231	.123	.116
	2	276.342	.531	.0011	.713	.265	.164
	3	276.089	.891	.0013	.521	.275	.218
	4	276.127	.871	.0017	.338	.267	.281

Table (5) between the number of iterations maximum repeatability capabilities

From the table above, we note that the Maximum Likelihood are stabilized at the fourth frequency, which is used to interpret the results statistically

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	276.34	.67	.18

Table (6) Cox test

The table shows the Cox test, which is a corresponding test for the Kay box test, so that the independent variables explain 69% of the logistic regression model and the remaining 31% are included within the error limit, ie, there are other variables with high impact that are not included in the logistic regression model.

		B	S.E.	Wald	Df	Sig.	Exp(B)
Step 1[a]	x1	.0120	.120	.217	1	.083	1.834
	x2	.327	.231	1.745	1	.095	.993
	x3	.126	.628	.884	1	.074	1.003
	x4	.542	.231	.777	1	.089	.781
	Constant	.628	1.008	.329	1	.054	.451

Table (7) shows the parameters of Maximum Likelihood potential

From the above table, we note that the second variable, which represents the sex of the patient, has a greater effect on the injury process through the logistic regression model. Then the third variable, which represents the environment classification of the injured (rural = 1) and (city = 2) The first age is the patient who represents the least average error

The fourth subject : CONCLUSIONS & RECOMMENDATIONS

The main conclusions and recommendations reached will be discussed in this section

4-1.Conclusions:

1.Data on the associated variant of chest disease and data distribution was followed by Bernoulli distribution.
2.by the value of a father test that was greater than the value of the scale of the Kay Square test this indicates the existence of the impact of each of the four variables and the age of the patient and the sex of the patient and the classification of the patient's environment and smoking on chest diseases.
3.through the Cox test that the four variables have interpreted 69% of the impact of chest disease.
4.Classification of the patient's environment and gender of the patient had a greater impact than the rest of the variables on the incidence of chest diseases.

4-2.Recommendations:

In Based on the conclusions reached by the researcher in this research, he recommends the following:
1.In the applied side can be used other variables not mentioned in our applied study had a significant role in the process of chest disease.
2.To develop the statistical data collection base in the Ministry of Health to obtain real, realistic and accurate data so that the results are good and satisfactory, which helps us to develop this field to reach the desired goal.
3.Other models such as the Cox model, the model of the analysis can be used, and the comparison with the logistic model.
4.The use of other methods for estimating the logistic regression function such as the bizi methods in statistical analysis.
5. Using nonlinear regression models of various types to analyze health and biological phenomena

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استخدام الانحدار اللوجستي لدراسة الاصابة بمرض التدرن

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المستخلص :

ان ما يميز هذا البحث هو التركيز على مفهوم الانحدار اللوجستي وخصائصه وكيفية بناء النموذج له اذ تم استخدام المنهج الوصفي التحليلي في توصيف الانحدار اللوجستي وكيفية تقدير معالمه واستعماله كـــــأداة مهمة في دراسة العوامل المؤثرة في إصابة بمرض التدرن حيث تضمنت عينة البحث ٢٩٩ مـــريــض.

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Evaluating the Quality of Health services provided to the population of The Arab Republic of Egypt

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Abstract

Egypt is the most populous country in the Middle East and it was ranked 111 among the ranks of 188 countries in the human world report of 2016 so it was necessary to adopt effective programs for human development especially that direct to health, because health has become a standard of well-being in developed countries. Among the population targets in Egypt is raising the health properties of citizens and provide a good health program able to protect them from the risk of diseases. One of these efforts is the establishment of the general Authority for social health insurance as a health experiments. Its health services cover more than 59% of the total population in 2017 which means that it included more than half of the population. It becomes the most important health establishment in the country which protects people from the risk of diseases. One of the advantages of this experiments, it's not completely free and it's not completely high cost.

Key words: Health Insurance - Health Care - World Health Indicators - Health Programs - Disease.

Preface:

Human health is a grant from God Almighty, which must be preserved and protected from disease and its causes, so that the individual remains strong and able to perform his religious and mundane duties, which Allah created for him.

To maintain health one must have a clear background on the health culture, so that it does not flounder in dealing with its health, body and diseases, which does not resort to traditional recipes, and not based on scientific and medical, because they are counterproductive, Comprehensive health coverage directly affects the health of the individual, as health services enable him to increase his productivity and
contribute actively to the well-being of his family and society. These services also ensure that children can go to school and learn. Comprehensive health coverage is a critical component of sustainable development, poverty reduction and an essential component of efforts to reduce inequities in society.

The research is divided into eight sections as follows:

1- Problem of study

2 - Questions of the study

3 - Study hypotheses

4- Previous studies.

5. Contextual framework.

6. Objectives of the study.

7. Data and methodology.

8. Conclusion and recommendations.

1- Problem of study:

One of the three components of the population problem in Egypt is the low population characteristics. At the top of these low characteristics is the low level of health of citizens, which has abnormal repercussions in the spread of diseases such as (C virus and kidney failure and .... the main locomotive to raise the level of health in Egypt is the General Authority for Health Insurance(HIO) because its medical services cover more than half of the
population 59% in 2017 If these services upgraded to the required level, it will lead to a positive and significant change in the health level of citizens in Egypt and therefore It was necessary to **evaluate the performance of the medical activities provided by the General Authority for Health Insurance(HIO)** In order to determine how to improve their performance in the case of negative indicators or to maintain their role as in the case of proving the quality of health care provided to the beneficiaries and to examine the shortcomings of the health services provided and how to address them and determine the geographical locations that have not been served healthily Enough.

2- Study Questions:

1. How effective are the health services provided to beneficiaries in the public health of the State?
2. Are the physical, human and therapeutic resources available to the HIO sufficient to provide medical services to the beneficiaries?
3. Are the funds allocated to the health care provided by HIO to the beneficiaries sufficient to provide the service in a suitable manner?

4- Is the mortality rate in health insurance hospitals low?

5-How much will beneficiaries be in 2025?

3 - Study hypotheses:

1- Medical services provided to all geographical locations in the Arab Republic of Egypt.

2 - There is a lack of performance of medical service providers.

3. There is a difficulty in reaching specific medical standards according to the World Health Organization.

4 - There is a negative impact of some medical activities provided to each other.

4. Previous studies:

Several studies dealt with the extent of the health insurance umbrella. These studies concluded that social health insurance should be included for all social groups. Other studies also examined the obstacles and problems of social health insurance. These studies concluded that health insurance still faces technical, administrative and financial problems. The importance of performance improvement in hospitals and solving the problems facing it and the development of a plan and time period for this development. There are other studies concerned with comparing the systems applied in social health insurance to some countries. It concluded that to develop the health sector in the country, the health sector should be organized, the role of the service providers should be reviewed and the comprehensive coverage and distribution justice should be implemented. Improve quality and suitability with demand by respecting health controls and standards.

There are studies to assess the current situation of social health insurance and concluded that the health insurance status is still continuing as it is in the forefront and bears the burden of the high cost of incurable diseases.

5. Contextual framework:

The theoretical framework is important because it explains a large number of variables affecting the quality and efficiency of the medical service provided to beneficiaries. These variables affect and are directly and indirectly affected by each other.

1- Medical service outlets.

2. Population groups receiving medical service.

3- Type of medical service provided.

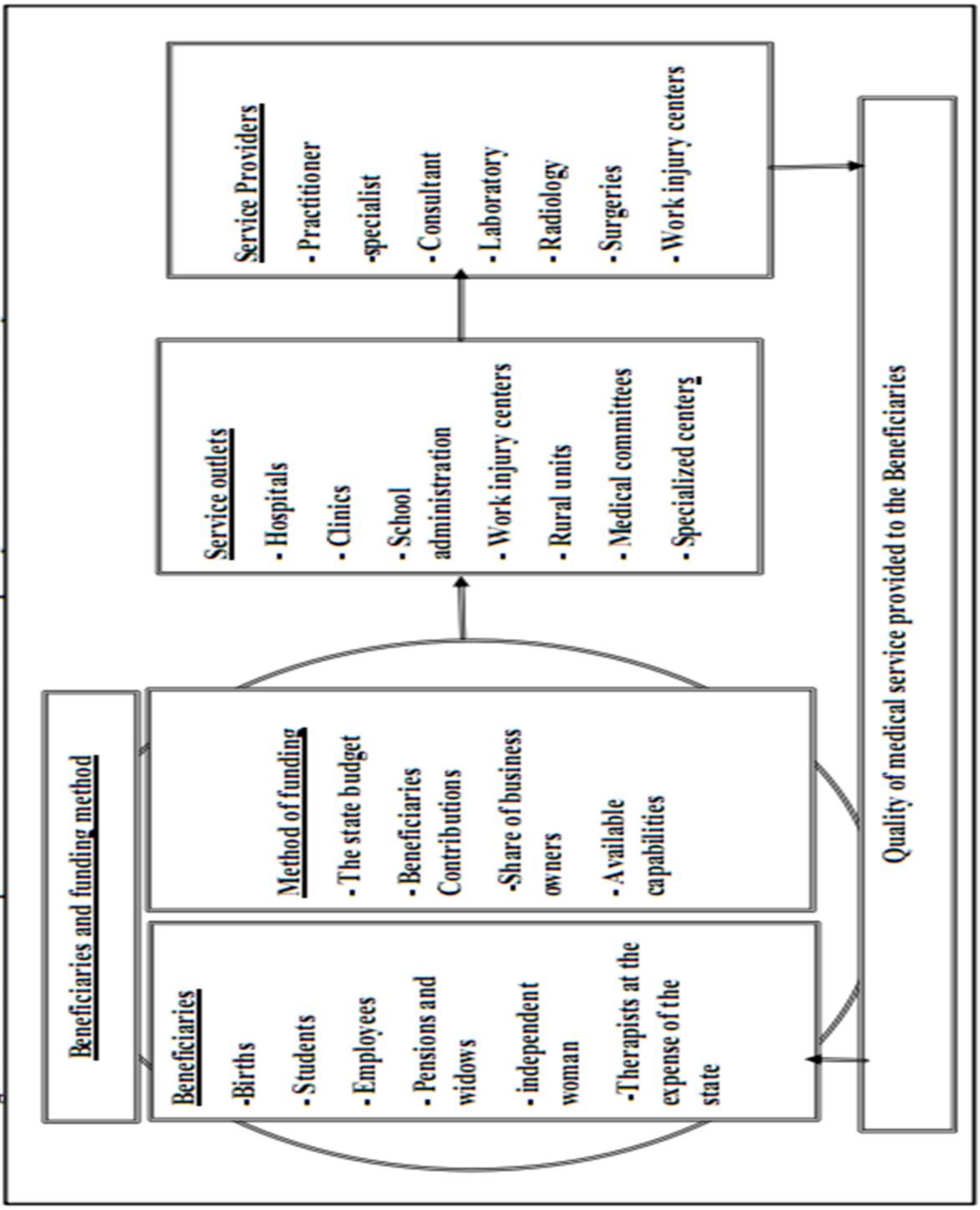
4 - Method of funding the medical service provided.

5. Cost of medical services.

6. Time required for medical service.

7. The material and human resources necessary to provide the service.

Figure 1: The relationship between the determinants and quality of health service delivery.



Source: A proposal prepared by the researcher.

6- Objectives of the study

A. General Objective:

Conducting an evaluation study of the health care activities provided by HIO for the inhabitants of the Arab Republic of Egypt according to international and local indicators to ensure the provision of a distinguished health service to all health insurance associates in the republic without distinction.

B. Sub-objectives:

1- Research the shortcomings of the health services provided.

2. Identify geographical locations that have not been sufficiently served.

7- Data and methodology

Performance evaluation criteria were used for the medical activities provided in accordance with Egyptian law and compared with the WHO standards, including the following:

- Performance evaluation criteria for medical activities provided.

- **Prediction of time series with ARIMA model.**

First Evaluation of medical activities provided to the population in the Arab Republic of Egypt:

Here, the population will be divided into a number of segments: students / births / workers in the public, private sectors and government / Independent women / work injuries.

The medical services should be evaluated for each of the previous segments as follows: Service of the practitioner / Specialist / beds service in hospitals.

❖ School students:

1- Percentage of vaccinations given to school students and kindergartens ranges between 41% and 100% for all students in all governorates of the Republic according to the data of the report of HIO, 2012.

2 - Criteria to evaluate the quality of the performance of medical service provided to school students according to human and physical potential:

*** - Practitioner service:** Cairo governorate is one of the most urban governorates that need to increase the number of practicing doctors by 307 practicing doctors because the lack of the number of practicing doctors will lead to a terrible accumulation of clinics in the governorate, which needs to display 15 cases on the doctor within one minute which Indicates the poor quality of medical service provided to students in that governorate, which is considered the capital of Egypt. In order to achieve the ideal standard that Egypt seeks for its students and to improve the quality of the medical service provided to them, the number of practicing physicians should increase to 848 doctors to participate in the number of students who attend it during the year 2500 students, 8 students per day and one student every hour. In the order of North West Delta (Alexandria and Matrouh) where the quality of medical service is not good because it is impossible to present 9 cases to the practitioner within one minute. Requiring an increase in the number of practicing physicians to 500.

❖ Births:

1 - availability of a reception department working 24 hours found no problem in the availability of reception departments in all governorates except Ismailia, South Upper Egypt and Luxor, due to the lack of the establishment of hospitals in those governorates, but there are only clinics and medical complexes provide treatment services for newborns and in the case of One of the newborns is exposed to any emergency condition. The medical service is provided by either the third party contracting with the Commission or through the transfer to the nearest hospital in the neighboring governorate and closest to their place of residence.

❖ Workers in the private, public and government sectors:

1 - Evaluation of medical services provided to employees in the private, public and government sector According to the available physical resources of the numbers of comprehensive clinics there is a convergence between the standard of the Ministry of Health and what is on the ground as the case in the North West Delta, Port Said and Minya There is a large disparity To be taken into clinics are provided to serve the beneficiaries in the following governorates of Gharbya , Giza and Cairo finally there are some governorates where there is no problem in serving beneficiaries of the workers in accordance with Law 32 and Law 79, where there is an abundance of clinics The governorates of Dakahlia, Beni Suef, Kafr El-Sheikh, Beheira, Damietta, Suez, Fayoum, and Sohag .

❖ Pensions and widows:

*** -** Practitioner service: There is an abundance of practitioners in all governorates and some rural and upper governorates. Therefore, we found that the most governorates that need to increase the number of practitioners are Qaliubiya, Gharbia, Dakahlia, Beheira, Menia, Menoufia, Fayoum, Ismailia and Beni Suef. Respectively, in order to provide the service in full and in a short time without long waiting in the restroom until the presentation to the practicing doctor.

* Specialist service: There is an abundance of numbers of specialists in the following specialties (internal / chest / heart / general surgery / nose and ear / brain and neurological / emphysema / skin and genital) and there is no problem in the numbers on these disciplines.

❖ independent women:

* - Practitioner service: we found that there is no problem in the availability of doctors practicing in all the urban governorates, and rural governorates where the service of the best practice only in three branches are respectively as follows: Minya / ELbheira / Dakahlia.

❖ Beneficiaries of work injuries:

* Practitioner service: There is a great shortage in the number of general practitioner compared to the number of participants with work injuries and the most governorates that need to increase the number of doctors is Cairo, followed by North Upper Egypt and Northwest Delta. Doctors to the ideal standard of 2000 cases per year, bringing the total number of practicing doctors to 7599 doctors distributed in all governorates.

* Specialist service: There is an abundance of specialists and there is no shortage, where there is one or two cases in the year for the specialist, which indicates that there is no problem in the time required to provide service for that category.

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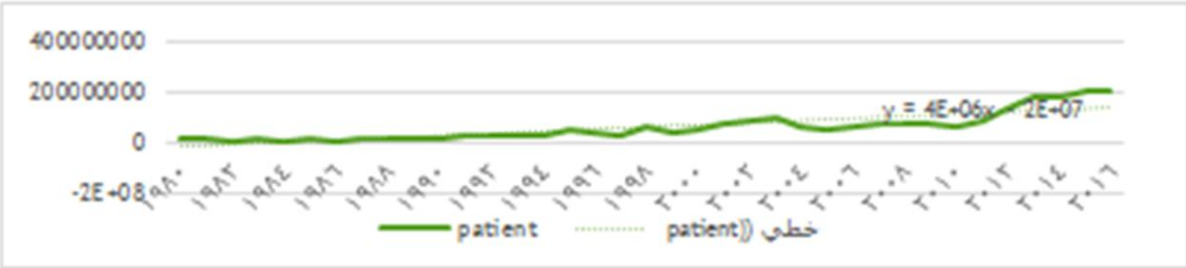
Second Time series prediction with ARIMA model:

Figure 2 Number of patients:



The previous figure shows that the time series has a general direction of the increase and suffers from the problem of unit root, therefore, it is difficult to rely on the previous series to obtain accurate measurements. Unit root was tested in the expanded Dicky- Fuller method.

Since the unit root test is not significant at the 10% level of -1.611, we reject the alternative hypothesis and accept the null hypothesis that the series is not static at 10%, and it is possible to obtain a stable series by the first difference.

$$Y_t=\log (y_{t-1}) +ar (1)+ ma(1) + e_t$$

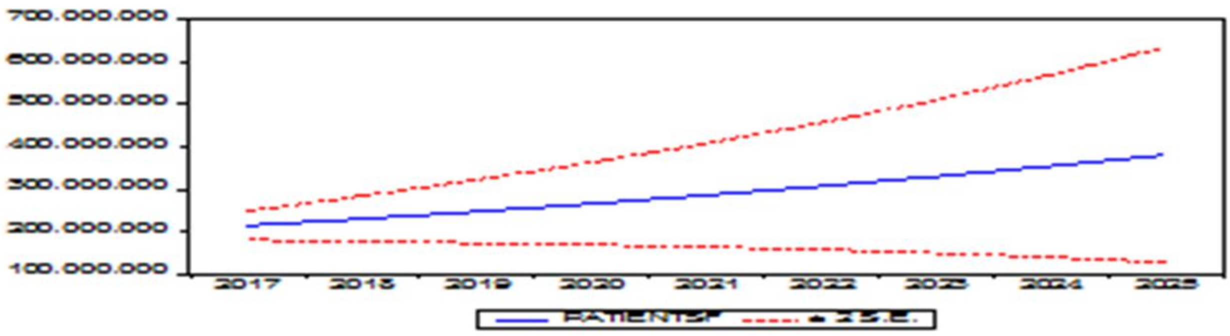
The value of R_Square of 0.92 and the value of Durbin Watson = 2.06 indicates the quality of the prediction model and the predictability value of the policy stability until 2025 is as follows:

Table 2: Number of expected patients during the period from 2017 to 2025.

years	Max	expected	Min
2017	2.70	٢.٤٢	١.٥٧
2018	3.07	٢.٦٧	١.٥١
2019	3.48	٢.٩٥	١.٤٤
2020	3.92	٣.٢٦	١.٣٦
2021	4.41	٣.٦١	١.٢٧
2022	4.94	٣.٩٩	١.١٥
2023	5.52	٤.٤٢	١.٠٢
2024	6.17	٤.٩٠	٠.٨٥
2025	6.88	٥.٤٣	٠.٦٦

Source: According to researcher.

Figure 3: The minimum, highest, and expected number of patients.



A - Examination of the coefficients of the function of autocorrelation and partial correlation

The coefficients of the autocorrelation and the partial correlations were observed within 95% confidence intervals. This indicates the suitability of the model to analyze the series data.

B) The L Jung-Box Q test

We found that the test was 0.36, indicating the randomness of the residual.

- As for the analysis of the residual, we found that the quality criteria for the model and according to EVIEWS program as follows:

***- SERIAL CORRELATION:**

We accepted the null hypothesis and the absence of auto-correlation of the first degree in the residual, where the probability is 0.20

***- Heteroskedasticity:**

Here we will use the Arch effect method. We accept the null hypothesis because the probability is 0.72.variance is constant and is achieved when the probability is greater than 0.05.

***- Normality Test:**

We found that the residuals follow the normal distribution of the probability of 0.1342.

***- Checking model's residuals:**

By examining the auto-correlative function of the residuals, it was found that it was interrupted after the first time unit and that the partial auto-correlation function decreased gradually. This results in the errors being followed by random changes. The first difference of the stacks obtained from the model follows the process of moving averages with parameter Θ =١ Of the first level

p= -0.5 Thus, the following tests are performed:

Test 1 (Θ= 1 VS $\Theta \neq 1$)

Therefore, we accept the null hypothesis that the value of the parameter Θ is not significantly different from the one, the errors in the model follow purely random changes.

Test 2 (p= -0.5 VS $p \neq -0.5$)

For the second test of the auto-correlation coefficient, we accept the null hypothesis that p is significantly different from -0.5 and this confirms the suitability of the model.

***- Random forms**

In order to enhance trust, the residuals were drawn against time, and from this we found the dispersion of the planks around the semi-stable medium. As we observe when drawing the errors against the estimated value, there is no correlation between the residuals and the estimated values.

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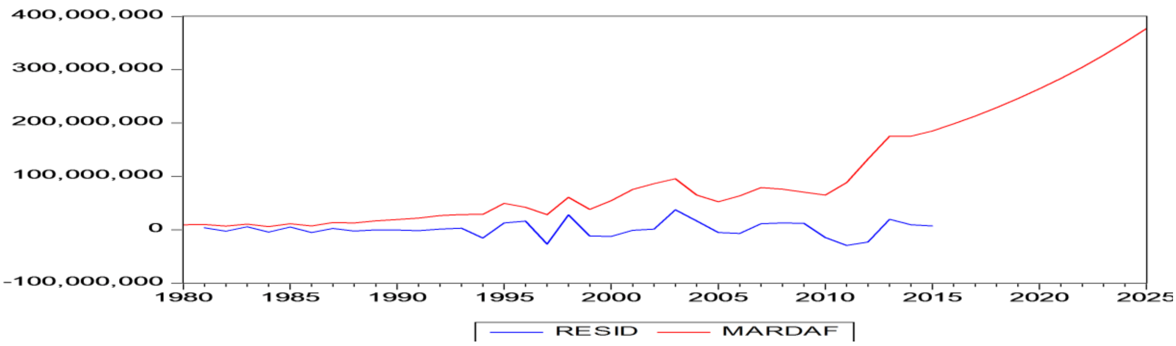

Figure 4: The correlation between the residuals and the estimated values

Model quality standards:

It is clear from the previous model that its quality standards are as follows:

Table 3: Model quality standards

Measure	Value
Theil inequality coefficient	٠.١١٥٠٥٩
Mean Abs .Percent Error	٢٧.٠٧٨٤١
Mean Absolute Error	١٣٤٠٩٥٨٧
Root Mean squared Error	١٩٢٩٣٨٧٧

Source: According to researcher.

*- predictive power of the model:

There is no doubt that the prediction is an important and prominent role in the decision-making process because by predicting the numbers of health service providers predict their needs of possibilities Such as the number of doctors, nurses, pharmacists, service workers, technicians and other health professionals

Table 4: Forecasting the physical and human potential of 2025 according to the standards of Egyptian law.

Max	Expected	Min	Egypt standards	Statement
٢٧٥٢٠٠٠	٢١٧٢٠٠٠	٢٦٤٠٠٠	١٠٠٠/٤	beneficiary /Number of beds
١٧٢٠٠	١٣٥٧٥	١٦٥٠	٤٠٠٠٠/١	beneficiaries /Number of clinics
٦٨٨٠٠	٥٤٣٠٠	٦٦٠٠	١٠٠٠٠/١	Beneficiary /Dental Specialist
٢٧٥٢٠٠٠	٣٦٢٠٠	٢٦٤٠٠٠	١٠٠٠/٤	Beneficiary /Nurse
٤٥٨٦٧	٣٦٢٠٠	٤٤٠٠	١٥٠٠٠/١	Beneficiary/Internal Specialist
٣٤٤٠٠	٢٧١٥٠	٣٣٠٠	٢٠٠٠٠/١	Beneficiary/Specialist in General Surgery
٢٢٩٣٣	١٨١٠٠	٢٢٠٠	٣٠٠٠٠/١	beneficiary /Specialist of nose and ear
١٧٢٠٠	١٣٥٧٥	١٦٥٠	٤٠٠٠٠/١	Beneficiary /Specialist Chest
٢٧٥٢٠٠٠	٢١٧٢٠٠٠	٢٦٤٠٠٠	١٠٠٠/٤	Beneficiary /Service Agent

Source: According to researcher.

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Table 5: Predicting the physical and human potential of 2025 according to WHO standards:

Max	Expected	Min	WHO standards	Statement
١٠٣٢٠٠٠٠	٨١٤٥٠٠٠	990000	١٠٠٠/١٥	beneficiary /Number of beds
٦٨٨٠٠	٥٤٣٠٠	6600	١٠٠٠٠/١	beneficiaries /Number of clinics
١٣٧٦٠٠٠	١٠٨٦٠٠٠	١٣٢٠٠٠	١٠٠٠/٢	Beneficiary /Nurse
٣٤٤٠٠٠	٢٧١٥٠٠	٣٣٠٠٠	٢٠٠٠/١	Beneficiary /Pharmacist
١٣٧٦٠٠٠	١٠٨٦٠٠٠	١٣٢٠٠٠	٥٠٠/١	Beneficiary /Service Agent

Source: According to researcher.

8- Conclusion and Recommendations

This section presents the main findings of the study:

1- There are governorates that have not been served as well as Luxor governorate.

2- not to include health insurance for all segments of society, and this leads to the exposure of this category of disadvantaged health insurance to the risk of destitution and disease because of the costs of treatment and increases the risk of development of the disease because of its inability to pay the cost of treatment, which may deprive treatment and may lead to Worsening disease or death.

3- There is a significant decrease in the physical, human and therapeutic potential available in HIO in the Arab Republic of Egypt, which affects the medical service provided to the beneficiaries.

4- The proportion of medical examinations and vaccinations given to school students and kindergartens exceeds 90% in most governorates.

8- Recommendations

Based on results, the study recommends the following:

1 - Establishment of a hospital in Luxor to accommodate the sick cases in that governorate and to reduce the cases transferred to other governorate to receive the required health service.

2 – We must raise the level of medical service provided to citizens in order to benefit the desired development in the State.

3. Re-distribution of health services according to population needs.

4. Social health insurance must play an active role in raising the level of job satisfaction and social security among individuals.

5 - Inclusion of health insurance for all segments of society, including housewives and irregular labor, because of their inability to afford the financial burdens related to the costs of treatment.

6 - We need to follow up the technical performance of medical activities in hospitals through specific quality programs in advance, which contributes to solving the problems that occur on the surface and the development of a time plan for this program.

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تقييم جودة الخدمات الصحية المقدمة للسكان بجمهورية مصر العربية

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جامعة القاهرة
معهد الدراسات و البحوث الإحصائية
قسم الإحصاء الحيوي و السكاني

المستخلص :

فى هذا البحث تم دراسة مدى جودة الخدمات الصحية المقدمة للسكان بجمهورية مصر العربية عام ٢٠١٧ من قبل الهيئة العامة للتامين الصحى الإجتماعى بالإضافة الى استخدام نموذج ARIMA فى التنبؤ بإحتياجات السكان من الامكانيات البشرية و المادية اللازمة لتقديم خدمة صحية افضل خلال عام ٢٠٢٥.

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Comparison of four non-Bayesian methods to estimate the scale parameter for Modified Weibull distribution by using the Simulation

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Abstract.,
In this paper, four methods were obtained to estimate the scale parameter of Modified Weibull distribution using complete data, which are the Modified Moments Estimator (MME), the Maximum Likelihood Estimator (MLE), White Estimator (WE) and Least Squares Estimator (LSE), Monte Carlo simulation is used to compare these four estimators with respect to the Mean Square Error criteria (MSE), and the results on simulated samples of the comparison showed that for all the varying sample size in this study, and in all cases for the four methods The MLE method is best followed by the OLSE method then the WE method and the MME method .

Key Words. Modified Weibull distribution, Modified Moments Estimator, Least Squares Estimator, White Estimator.

Introduction

The probability density function of any random variable t having a modified Weibull distribution (MWD) with scale parameter $\alpha > 0$ and both shape parameters $\beta \geq 0$ and $\lambda > 0$ is given by

$$f(t;\alpha,\beta,\lambda)=\begin{cases}\alpha(\beta+\lambda t)t^{\beta-1}\exp(\lambda t-\alpha t^{\beta}e^{\lambda t}) & , t\geq 0\\ 0 & , o.w\end{cases} \quad (1)$$

If $\beta = 0$, the resulting distribution is called the type 1 extreme-value which is also known as a log-gamma distribution when $\lambda = 0$ then MWD reduces to the two-parameter Weibull distribution. Also when both $\beta = 2$ and $\lambda = 0$ then MWD reduces to one-parameter Rayleigh distribution. The modified Weibull model was developed by Xie et al.(2003) [1] this lifetime distribution is an important feature for reliability analysis.

Vasile et al. [2] studied the method of Bayes to estimate the parameters of the MWD and Upadhyaya and Gupta [3] using Markov chain Monte Carlo simulation to study the Bayes analysis of the MWD. Ateya [4] study the estimation problem of the censored sample of order statistics generalized from MWD.

The cumulative distribution function and reliability function respectively are

$$F(t;\alpha,\beta,\lambda)=1-\exp(-\alpha t^{\beta}e^{\lambda t}) \quad (2)$$

$$R(t)=\exp(-\alpha t^{\beta}e^{\lambda t}) \quad (3)$$

We review four methods which are the Modified Moments Estimator (MME), the Maximum Likelihood Estimator (MLE), White Estimator (WE) and Least Squares Estimator (LSE), These methods are compared in Section 6, using the mean square error (MSE) criteria, all these four methods which are using to estimate the scale parameter for Modified Weibull are non-Bayesian methods .

2. Modified Moments Estimator (MME)

(Whitten and Cohen), in (1982), proposed a new modification on moment method [5], using the following equation

$$E\left(\hat{F}(t_{(i)})\right)=F(t_{(i)}) \qquad i=1,2,...,n \quad (4)$$

$$t_{(1)} \le t_{(2)} \le . \quad . \quad . \le t_{(n)}$$

Where $t_{(i)}$ is the i's order random variable, $\hat{F}(t_{(i)})$ is estimated unbiased for function distribution $F(t_{(i)})$ and by replacement $F(t_{(i)})$ by the plotting position formula

$$P_i=\frac{i}{n+1}, i=1,2,...,n \qquad (5)$$

$$F(t_{(i)})=\frac{i}{n+1}, i=1,2,...,n \qquad (6)$$

Then

$$F(t_{(1)})=\frac{1}{n+1} \qquad (7)$$

From equations (2) and (7), we get

$$\exp(-\alpha t_{(1)}{}^{\beta} e^{\lambda t_{(1)}})=1-\frac{1}{n+1} \qquad (8)$$

By taking the natural logarithm of equation (8), we get

$$\hat{\alpha}_{MME}=\frac{-Ln\left(1-\frac{1}{n+1}\right)}{t_{(1)}{}^{\beta} e^{\lambda t_{(1)}}} \qquad (9)$$

Where the symbol $\hat{\alpha}_{MME}$ indicates the estimate of the scale parameter α by using MM method.

3. Maximum Likelihood Estimator (MLE)

The likelihood function for three-parameter Modified Weibull distribution (1) is [1].

$$L(\alpha,\beta,\lambda;t_1,t_2,...,t_n)=\alpha^n \prod_{i=1}^{n}(\beta+\lambda t_i)\prod_{i=1}^{n} t_i^{\beta-1} \exp\sum_{i=1}^{n}(\lambda t_i-\alpha t_i{}^{\beta} e^{\lambda t_i}) \qquad (10)$$

Taking the logarithm of the likelihood function, so we get the function

$$\ln L=n\ln\alpha+\sum_{i=1}^{n}\ln(\beta+\lambda t_i)+(\beta-1)\sum_{i=1}^{n}\ln t_i+\sum_{i=1}^{n}(\lambda t_i-\alpha t_i{}^{\beta} e^{\lambda t_i})) \qquad (11)$$

The partial derivative of the log-likelihood function with respect to unknown parameters α is

$$\frac{\partial \ln L}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n} t_i^{\beta} e^{\lambda t_i} \qquad (12)$$

We place the partial derivative for log-likelihood with respect to α to zero as follows

$$\frac{n}{\hat{\alpha}}-\sum_{i=1}^{n} t_i^{\beta} e^{\lambda t_i}=0 \qquad (13)$$

$$\hat{\alpha}_{MLE}=\frac{n}{\sum_{i=1}^{n} t_i^{\beta} e^{\lambda t_i}} \qquad (14)$$

Where the symbol $\hat{\alpha}_{MLE}$ indicates the estimate of the scale parameter α by using MLE method.

4. Least Squares Estimator (LSE)

The idea of this method is to minimize the sum of squared differences between observed sample values and the estimate expected values by linear approximation [6].

$$\varepsilon_i=y_i-\beta_0-\beta_1 x_1-\beta_2 x_2 \qquad (15)$$

$$\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}[y_i-\hat{y}_i]^2 \qquad (16)$$

$$\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}[y_i-\hat{\beta}_0-\hat{\beta}_1 x_1-\hat{\beta}_2 x_2]^2 \qquad (17)$$

By using the CDF of modified Weibull distribution (2) which are as follows

$$1-[F(t_i)]=\exp(-\alpha t_i^{\beta} e^{\lambda t_i}) \qquad (18)$$

By taking the double logarithm of above equation getting

$$\ln(-\ln[1-\{F(t_i)\}])=\ln\alpha+\beta\ln t_i+\lambda t_i \qquad (19)$$

Comparing the above equation with the simple linear model

$$Y=\beta_0+\beta_1 x_1+\beta_2 x_2+\varepsilon \qquad (20)$$

We get

$$Y=\ln(-\ln[1-\{F(t_i)\}]), x_1=\ln t_i, x_2=t_i,$$

$$\beta_0=\ln\alpha, \beta_1=\beta, \beta_2=\lambda \quad .$$

$$\varepsilon=\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i \qquad (21)$$

By taking the sum square of above equation for the two sides to reach

$$\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}[\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i]^2 \qquad (22)$$

$$\frac{\partial(\sum_{i=1}^{n}\varepsilon_i^2)}{\partial\alpha}=-\frac{2}{\alpha}\sum_{i=1}^{n}[\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i] \qquad (23)$$

We place the partial derivative $\sum_{i=1}^{n}\varepsilon_i^2$ to zero as follows

$$-\frac{2}{\alpha}\sum_{i=1}^{n}[\ln(-\ln[1-\{F(t_i)\}])-\ln\alpha-\beta\ln t_i-\lambda t_i]=0$$

$$\hat{\alpha}_{LSE}=\exp\left(\frac{\sum_{i=1}^{n}\ln(-\ln[1-\{F(t_i)\}])-\beta\sum_{i=1}^{n}\ln t_i-\lambda\sum_{i=1}^{n}t_i}{n}\right) \quad (24)$$

Where the symbol $\hat{\alpha}_{LSE}$ indicates the estimate of the scale parameter α by using LSE Method and $F(t_i)$ is Empirical Distribution Functions which is [7].

$$F(t_i)=\frac{i-0.5}{n} \quad (25)$$

5. White Estimator (WE)

This method is mainly based on the reliability function of the distribution whose parameters are to be estimated and the formula of the function converted to a formula similar to the Linear Regression Equation, and its characteristics are to use its estimators as primary estimators for other estimation methods.

This method is applied to the modified Weibull distribution by taking the natural logarithm of both sides of the formula (3) we get the following formula

$$-\ln R(x)=\alpha t^{\beta}e^{\lambda t} \quad (26)$$

Comparing formula (26) with the following linear regression formula [8].

$$z=\phi y+\varphi \quad (27)$$

$$z=-\ln R(t),\ \phi=\alpha,\ y=(t^{\beta}e^{\lambda t}),\ \varphi=0 \quad (28)$$

$$\hat{\phi}=\frac{\sum_{i=1}^{n}(y_i-\bar{y})(z_i-\bar{z})}{\sum_{i=1}^{n}(y_i-\bar{y})^2} \quad (29)$$

Where

$$\bar{z}=\frac{\sum_{i=1}^{n}z_i}{n}=\frac{\sum_{i=1}^{n}-\ln R(t_i)}{n} \quad (30)$$

And

$$R(t_i)=1-F(t_i)=1-\frac{i-0.5}{n} \quad (31)$$

Set $(\hat{\phi})$ in form (28) we obtain

$$\hat{\alpha}_{WE}=\hat{\phi} \quad (32)$$

Where the symbol $\hat{\alpha}_{WE}$ indicates the estimate of the scale parameter α by using WE Method.

6. Monte Carlo Simulation

The Monte Carlo simulation is using to compare the MSE for the scale parameter modified Weibull distributions with respect to MME, MLE, WE and OLSE methods, in this study we used the three models. The first model is α=0.1,β=0.2,λ=2,the second model is α=0.05, β=0.2,λ=0.1 and the third model is α=0.3, β=2,λ=0.2.the probability density functions with this models are illustrated in Fig. 1

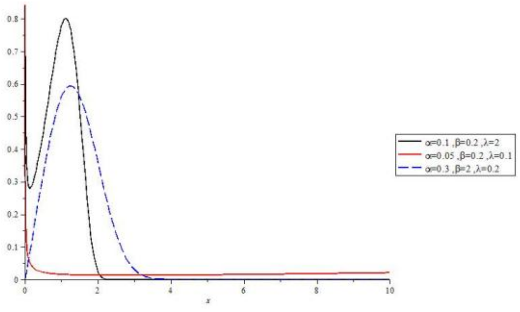

Fig.1. Different models shapes of pdf versus time

The MSE for any model in this Simulation is calculated using 24,800 simulated samples. All computations in this simulation are performed using MATLAB R2014a. We consider sample sizes n=10, 25, 100 and 200. We can generate random numbers from the modified Weibull distribution by using the inversion of the cumulative distribution function. And we replicate the data of experiment N times (N=200,500,1000,2000,2500) with sample size n, the results of simulation presented in the following Tables.

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Cr. / Meth. / n	MSE				N
	MME	MLE	WE	LSE	
10	2.383729236800643	0.001438063444104	0.002239672831175	0.002669346419856	200
	33.648011790890713	0.001765433113177	0.003885925331741	0.003269032572172	500
	21.252011566744255	0.001655171755509	0.004702689841331	0.002667558871810	1000
	31.479130706661955	0.001602443785124	0.004870285091825	0.002507155344652	2000
	23.075840594685047	0.001561226500369	0.006141199825043	0.002628589177462	2500
25	6.033424239420221	0.000616354782930	0.000841312007086	0.001453861618977	200
	4.507147322274402	0.000545467405829	0.001085909024811	0.000897824828307	500
	19.206956103971507	0.000475412286613	0.001391672857426	0.000806102267597	1000
	6.086150606366833	0.000473905174187	0.001444151975628	0.000862067567216	2000
	19.350973297827583	0.000490254983197	0.001973022131112	0.000777758467862	2500
100	3.210931476253547	0.000071962327029	0.000227684493831	0.000130169924409	200
	1.454521076404451	0.000102951778319	0.000275130880454	0.000179175351622	500
	10.616429106555705	0.000093776771464	0.000316419393898	0.000162374177441	1000
	24.339393251617938	0.000098610585834	0.000342047813986	0.000159220442238	2000
	99.336813474673278	0.000104208303217	0.000468051412648	0.000170623720589	2500
200	2.214592525395552	0.000054626588130	0.000096813656970	0.000087189024627	200
	5.975713459530112	0.000046840887403	0.000126351090577	0.000081744652007	500
	20.413847579365985	0.000051966435075	0.000162230952909	0.000087498830131	1000
	12.057602293630540	0.000050726119845	0.000175632920196	0.000084772316627	2000
	9.972721383806443	0.000050804962080	0.000239815413910	0.000083709803411	2500

Table-1. MSE of scale parameter α for the first model

Cr. / Meth. / n	MSE				N
	MME	MLE	WE	LSE	
10	0.600528542955062	0.000362103793363	0.000682870997850	0.000657123460946	200
	8.648731876098426	0.000574184732385	0.000912874105925	0.001075636703398	500
	30.235692929472133	0.000729490743855	0.001093849682803	0.001255720751272	1000
	23.147495988702524	0.000739364551927	0.001179560068047	0.001235487059464	2000
	97.329825822911943	0.001036060095879	0.001675616166989	0.001677181513686	2500
25	0.560890317361752.	٠.٠٠٠١٢٧٧١٦٥٣٧٦٦٠	0.000952598840050	0.000240620231339	200
	1.296706222958869	0.000173725581724	0.000247168907412	0.000296538842959	500
	5.190199546287034	0.000202547441944	0.000318077071852	0.000342345089545	1000
	24.519165732042783	0.000223687261805	0.000352924856757	0.000384973562539	2000
	16.470487995151860	0.000121449654185	0.000487054189287	0.000208214775373	2500
100	14.133282096560173	0.000025527706921	0.000042877140125	0.000039033745534	200
	6.999907208554048	0.000035479752785	0.000065512298778	0.000057606322895	500
	3.966748072195498	0.000045260998636	0.000079070454860	0.000073347671188	1000
	438.7659200985143	0.00004785263240	0.000088558994225	0.000077234908500	2000
	359.1210829682811	0.00006449266160	0.000118367166696	0.000103925437500	2500
200	24.097332481215879	0.000013911561315	0.000024075051749	0.000023895975042	200
	26.759433362973123	0.000018244664363	0.000036646460792	0.000031587280336	500
	17.992704050140315	0.000020490848670	0.000039537033232	0.000034800882159	1000
	531.3921693138716	0.00002270771970	0.000044770081252	0.000038863877600	2000
	430.5269491736749	0.00003088326490	0.000060449409516	0.000051601324300	2500

Table-2. MSE of scale parameter α for the second model

Cr. / Meth. / n	MSE				N
	MME	MLE	WE	LSE	
10	92.783635257522718	0.021375659212246	0.024583355922621	0.033511280002231	200
	900.8764969978296	0.026903098460800	0.032863467813329	0.041021218490000	500
	4032.351352442018	0.026240629047000	0.039378588580908	0.042246888000000	1000
	1249.939960709109	0.026092726897000	0.042464162449683	0.047452379245000	2000
	9905.778740941878	0.034568533836000	0.060322182011600	0.061281843026000	2500
25	56.574535102180100	0.003309256212745	0.007029355824195	0.006213497462386	200
	469.6859933465812	0.004983251232800	0.008898080666838	0.089833525741000	500
	291.0073221553217	0.007531189153100	0.011450774586706	0.013178561010400	1000
	898.4793785900660	0.007865722885000	0.012705294843266	0.013474168028600	2000
	1279.291462242854	0.010626346911000	0.017533950814360	0.018584726738000	2500
100	15.632592952906153	0.000977621525935	0.001686915251769	0.001557690728678	200
	27.297885386289721	0.001348569043347	0.002495441162018	0.002265631504168	500
	40.967280881829538	0.001650572281862	0.003081280689277	0.002644477362376	1000
	15945.93289070901	0.001736131620000	0.003215367422850	0.002805816320000	2000
	13172.30320540781	0.002322145640000	0.004331281405101	0.003760887680000	2500
200	1500.818106616001	0.000511532437000	0.000851435380450	0.000873104024000	200
	6312.182944410619	0.000602154237100	0.001080440933797	0.001045570960600	500
	527.8395062493505	0.000724162434800	0.001370912560041	0.001256029543400	1000
	19061.14861102400	0.000813865600000	0.001592103002813	0.001388968840000	2000
	15444.74049969535	0.001104891550000	٠.002153481173084	0.001837551020000	2500

Table-3. MSE of scale parameter α for the third model

7. The conclusion

Note that we can make the following comments for the results in the above tables :

(1)The results of simulation presented in Table 1 is following conclusions may be summarized: the MLE is the best method and comes after LSE except when n=10,25 with N=200 according to the MSE criterion.

(2)The results of simulation presented in Table 2 is following conclusions may be summarized: the MLE is the best method and comes after LSE except when n=10 with N=500,1000,2000,2500 and when n=25 with N=500,1000,2000 according to the MSE criterion.

(3)The results of simulation presented in Table 3 is following conclusions may be summarized: the MLE is the best method and comes after LSE except when n=10 and when n=25 with N=500,1000,2000,2500 and when n=200 with N=200, according to the MSE criterion.

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مقارنة بين أربع طرق لابيزية لتقدير معلمة القياس لتوزيع ويبل المعدل باستخدام المحاكاة

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المستخلص

في هذه البحث ، تم الحصول على أربع طرق لتقدير معلمة المقياس لتوزيع ويبل المعدل باستخدام البيانات الكاملة، وهي مقدر العزوم المعدلة (MME) ومقدر الامكان الاعظم (MLE) و مقدر وايت (WE) ومقدر المربعات الصغر (LSE).وقد استخدمت محاكاة مونت كارلو للمقارنة بين هذه المقدرات الأربعة وفقا لمعيار متوسط مربعات الخطأ (MSE)، وقد أظهرت النتائج على عينات المحاكاة للمقارنة أن لجميع حجوم العينات المتفاوتة في هذه الدراسة ، وفي جميع الحالات عن الطرق الأربعة بان طريقة MLE هي الأفضل يتبعها طريقة OLSE ثم طريقة WE وطريقة MME .

قواعــد النشــر

- تعنى مجلة القادسية لعلوم الحاسوب والرياضيات بنشر البحوث العلمية الرصينة ذات العلاقة بعلوم الحاسبات ،الرياضيات ،الإحصاء والمعلوماتية والفيزياء الحاسوبية والتي لم تنشر أو تقدم للنشر سابقا .
- تخضع البحوث المقدمة للتقييم العلمي من لدن اختصاصين من داخل القطر وخارجه .
- يقدم البحث مطبوعا بنظام العمودين على ورق ابيض جيد قياس (A4) وبمسافة مضاعفة وبنظام الـ word حصرا وان يكون نظام office 2010 وان يكون حجم الخط المستخدم في طباعة البحث (١٠) ونوعه Time New Roman ماعدا العنوان واسم الباحث يكون حجم الخط (١٢)bold ونوعه Time New Roman ، أما الجداول والإشكال فيكون الخط bold ونوعه Time New Roman وعند وجود المعادلات في البحث يجب إضافتها باستخدام محرر المعادلات .
- على الباحث (أو الباحثيين) تقديم ملخص لبحثه باللغتين العربية والانكليزية يتضمن عنوان البحث واسم الباحث أو الباحثيين وعناوينهم بحدود (١٥٠-٢٠٠) كلمة .
- على الباحث (أو الباحثيين) ادراج البريد الالكتروني ويفضل ان يكون بريد رسمي .
- استخدام الباحث (أو الباحثيين) ذات البيانات الخاصة به (اسم الباحث ، المرتبة العلمية ، جهة الانتساب ، البريد الالكتروني الرسمي) والمستخدمة في بحوثه السابقة .
- يرتب البحث كما يأتي الخلاصة ، المقدمة ، المواد وطرائق العمل ، النتائج والمناقشة ، الخلاصة باللغة الثانية تتضمن عنوان البحث، اسم الباحث ومكان عمله .
- يتم ذكر المصادر في البحث بإتباع أسلوب الترقيم حسب أسبقية ذكر المصدر وتذكر المصادر في النهاية على الوجه الأتي :
  اسم الباحث (أو الباحثيين) عنوان البحث اسم المجلة ، المجلد ، العدد ، رقم صفحتي بدء وانتهاء البحث ، سنة النشر بين قوسين .
- تنشر البحوث باللغة الانكليزية فقط وان يقدم الباحث أربع نسخ من البحث (ورقية + اقراص CD) .
- بعد الانتهاء من عملية التقييم والتصويبات وعند القبول النهائي يقدم البحث على قرص CD (office 2010 + pdf) مع نسخة ورقية نهائية .
- أن لا تزيد صفحات البحث المقدم للنشر عن عشر صفحات وبنظام العمودين وفي حالة تجاوز عدد صفحات البحث اكثر من ذلك يتم دفع خمسة الاف دينار عراقي لكل صفحة زيادة وان لايتجاوز العدد الاجمالي للبحث ٢٠ صفحة .
- تعتمد المجلة تصنيف (Mathematics Subject Classificatio) في نشرها للبحوث العلمية .
- يقدم الباحث التصنيف المعتمد في المجلة لموضوع البحث .
- اجور التقييم والنشر للمجلة كالاتي :
  اولا :- اجور التقييم (٣٠٠٠٠) الف دينار عراقي .
  ثانيا :- اجور النشر حسب اللقب العلمي للباحث وكالاتي :
  ١- المدرس المساعد والمدرس (٥٠٠٠٠) الف دينار عراقي .
  ٢- الاستاذ المساعد (٧٥٠٠٠) الف دينار عراقي .
  ٣- الاستاذ (١٠٠٠٠٠) الف دينار عراقي .

ملاحظة : عند تقديم البحث يدفع الباحث مبلغ (٣٥٠٠٠) الف دينار عراقي غير قابل للرد وفي حالة قبول نشر بحثه في المجلة يدفع بقية الاجور حسب لقبه العلمي .
كما ويدفع مبلغ (١٠٠٠٠) الاف دينار عراقي غير قابل للرد اجور استلال ، وفي حالة اعادة فحص الاستلال للبحث مرة اخرى يعاد دفع المبلغ (١٠٠٠٠) كأجور اعادة استلال .

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الرقم المعياري الدولي 0204 - 2074

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رقم الايداع في دار الكتب والوثائق في بغداد (١٢٠٦) لسنة(٢٠٠٩)

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